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Colonel
A. P. STOLBOSHINSKY.

COURSE ON ARTILLERY.

Volume 8

LAW OF PROBABILITIES.

DISPERSION IN FIRING.

Chief editor,
major general of engineering/artillery service.
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Colonel A. P. Stolboshinsky. "The Course on Artillery", volume 8, Law of probabilities. Dispersion in firing.

In this volume are explained questions dealing with the law of probabilities, theory of errors and dispersion in direct and range firing.

The volume can be recommended as a text book for students in artillery schools. In addition it may be of assistance to officers in the Soviet Army in their individual work.

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Part I.

PRINCIPLES OF THE LAW OF PROBABILITIES.

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Chapter 1.

Probability of an event.

1. Subject of the law of probabilities.

Every occurrence, taking place in nature or in society, inevitably ends in some conclusion. Depending on the character of the action of the causes, producing this or other occurrence, the ultimate conclusion (result) of the occurrence may be accidental or non-accidental.

Let us show by example, in what conditions does the ultimate conclusion of an occurrence, bear on accidental character. Let us take a playing die (cube) marked one to six and throw it up above a table. The die will inevitably fall onto the table. The ultimate result of the fall - one or another score number is obtained - the result is accidental. It is not possible to determine beforehand what score will appear as a result of the throw of the die. When the throw is repeated, another score may be obtained, although the same die is thrown over the same table by the same person. This is explained by the fact that the outcome of this or other score is governed by a large number of causes: the position of the die in the hand of the thrower, the height of the throw, the position of the die at the instant of contact with the surface of the table, the kind of such a surface etc. Any of these causes can be subdivided into a series of smaller causes, each one of which can have, although very minute but nonetheless some influence on the result of the throwing of the die cube. Thus, for example, the position of the die in the hand of the thrower is determined by: with what surfaces and how fully does the die make contact with the palm, what parts of the palm and in what way they touch the die, how tightly is the hand closed at the same time, and so on. It is quite obvious that all these causes can not be accounted for, and with every individual throw their combined action, certainly, bears an accidental character. Therefore when the throws are repeated, different scores are obtained as a rule.

4.

This example demonstrates sufficiently the nature of accidental results of an occurrence. As can be seen, an occurrence, in which are inherent accidental results, can possess several such results; which one of these would take place in a test, is not possible to determine, as any one of these ultimate results may or may not take place.

This is the first distinctive feature of accidental results of an occurrence, as a non-accidental result can be one only and it is possible to pre-determine what kind of a result it will be. It is therefore for this reason that the non-accidental result of an occurrence is thus termed.

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4. occurrence consists in that when this occurrence is repeated, in other words, when the trial is repeated under conditions as similar as possible to each other, a completely definite rule can be established, to which these accidental results conform; the more complete and clearer the rule, means that the greater the number of such repetitions have been made.

In artillery firing practice it is constantly necessary to deal with accidental results of occurrences of such a nature.

Taking a more typical example - dispersion of shells. The occurrence of the flight of the shell inevitably terminates in the fall of the shell onto the ground. The result of the fall of the shell - the position of the point at which the fall has taken place - is an accidental result. A repetition of the shot under the same conditions (gun, shell, charge, mounting etc) will give, with very rare exception, another point of impact which does not coincide with the first. This is explained by the fact that the direction and the distance of flight of the shell, and hence also the position of its point of impact, are determined by a number of causes: the shape and weight of the shell, the weight and ballistic properties of the gunpowder charge, position of the barrel at the instant of firing, the speed and direction of the wind, the air temperature etc. For individual shots the shells will differ slightly from each other in weight and shape, the charges will differ in weight and properties of the gunpowder, the angles of projection and sight setting, will be different, the speed and direction of the wind will vary etc. It is not possible to take into consideration all these changes at every individual firing, and therefore the deviations of the shells due to dispersion are accidental in character. If a sufficiently large number of shots be made under identical conditions, it is possible to reveal a sufficiently definite rule for
5. the disposition of individual points of impact with respect to the mean (centre of dispersion) - a well known law of dispersion of shells: non-uniformity, symmetry and limitation of such a dispersion. Such a rule can not be established from the results of a small number of shots, which latter can even lead to incorrect deductions.

When we say, that any one of the several accidental results of some occurrence may exist or may not exist, this does not mean that the degree of possibility of obtaining (appearance) of such results is the same in every case. On the contrary, as a rule, this degree of possibility or, as we will term, <u>probability</u> of obtaining various ultimate results of occurrences, with which we have normally to deal, is not the same. In fact, we know, for example, that the dispersion of shells is not uniform - towards the centre of the ellipse of the dispersion the shells fall more frequently (more densely), whilst towards the outside they are less frequent. It is evident that the probability (the degree of possibility) of obtaining the point of impact of a shell in the vicinity of the centre of dispersion is greater than the probability of obtaining the point of impact removed from that centre.

In order to make it possible for comparisons of probability of individual accidental results of a given occurrence to be made, and hence to be able to anticipate them, it is necessary to know the numerical (quantitative) expression for the law which governs these results.

<u>The subject which treats quantitatively of the law governing accidental results of occurrences, is known as the theory of probability.</u>

Fig.1.
(1) - Mean trajectory.

In the example of the above dispersion, such a numerical (quantitative) rule is expressed by a well known dispersion scale (fig.1) which gives in percentages the numerical values of probabilities of

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obtaining, with one shot, the points of impact of a shell within definite limits, expressed by the values of mean deviations (Bδ). This scale, in particular, confirms that the probabilities of obtaining different accidental results of the same occurrence, are not identical. It (this scale) enables the calculations necessary for the artillaryman to be made, such as: probability of under-shooting, over-shooting, hitting, missing etc.

The theory of probability has a wide application in the most diverse branches of knowledge: science of social economy, astronomy, physics, meteorology etc. and, in particular in artillery fire. In all cases, where it is applied, the theory studies quantitatively the law governing accidental results of the occurrence in question.

It must be pointed out, that the theory of probability does not investigate the qualitative side of these laws.

The application of the theory of probability to artillery fire allows to anticipate scientifically the results of firing; to establish rules for firing; which, when systematically applied, ensure that the optimum results are obtained; allows methods of firing to be found, which produce hits on the target with the minimum expenditure of shells and in the shortest time. In other words it allows in one way or another to influence the results of firing.

Hence also the following formulation is true: the theory of probability is a branch of mathematics whichs develops numerical rules, from the systematic application of which, optimum results may be reasonably expected. The obtaining of the optimum results only with systematic application of the rules, based on the theory of probability, are conditional on that the quantitative laws of accidental results of occurrences become apparant only when a large number of trials is made.

The theory of probability, as any mathematical science, makes use of a number of axioms, assumptions and theorems, the establishment, conclusion and analysis of which, as well as the possibility of their practical application (mainly to artillery fire) form the context of the given Course.

2. An event.

In the theory of probability, an event is the term given to an accidental ultimate result of an occurrence, independently from the degree of significance of such a result.* The under-shooting, over-shooting, hitting and missing in gunfire, one or other score when throwing a die, a win or a lose in a lottery, - all these are events. Events are generally denoted by capital letters in Latin: A, B, C, D etc.

All mathematical training originates from some basic conception: arithmetic - from the conception of numerals; geometry - from the conception of a point, a line, a plane etc. As the theory of probability treats of the law governing accidental ultimate (finite) results of occurrences, the basic conception of the theory of probability is the event.

Events are termed singly-possible, if under given conditions one of them inevitably takes place. For example, in one rifle shot, a hit and a miss are events, singly-possible. In two shots from a rifle, the singly-possible events will be: two hits, a hit with the first shot and a miss with the second shot, a miss with the first shot and a hit with second shot, and finally, two misses.

If there are a series of singly-possible events of an occurrence, then an individual trial leads to but one of these events, and the appearance of one such event excludes the appearance of all the remaining events of the same series.

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* It must be pointed out, that this term is accepted only in the theory of probability, as generally, an event can not be set apart from the occurrence.

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7. Events, the appearance of one of which excludes the appearance of all the remaining events, in the series of singly-possible events, are termed mutually-exclusive (incompatible) events. Thus, in the case of two shots from a rifle, the four events enumerated above are mutually exclusive, as in obtaining, for example, two hits it is not possible with the same shots to obtain two misses or one hit and one miss: the two hits have excluded in this trial, the obtaining of any of the three remaining events.

If there are only two mutually exclusive events, they are termed dependent (opposite or complimentary) events. Obtaining of an under-shoot and an over-shoot, a hit and a miss in gun fire, a win and a lose in a lottery, are dependent events.

Apart from the mutually exclusive events, there are independent events; this takes place when the occurrence of one of them does not preclude the possibility of appearance of the remainder. For example, a shell may hit a target and not explode; the hitting and the non-explosion of the shell are independent events.

In firing on ricochet, it is possible to obtain a ricochet in front of the target; in this case, the ricochet and the under-shoot are independent events. In the same way, a ricochet and an over-shoot (non-ricochet and under-shoot, non-ricochet and over-shoot) are also independent events.

3. Probability of an event.

In section 1 of this chapter it has been mentioned, that the probability of an event, as the degree of possibility of its appearance, can be expressed numerically. A natural question arises, what kind of numeral is it and how to determine it.

Before we pass on to the numerical expression for the probability of an event, it is necessary first to establish which events should be considered as equally possible.

Events are considered equally possible if there is no basis to suppose that in given circumstances there exist some reasons, as a result of which the appearance of one of the events is made more possible than that of another.

EXAMPLE. In a closed box there are 10 completely identical balls which are marked in sequence: No.1, No.2, No.3 etc. up to No.10.
8. If these balls are thoroughly mixed and then, without looking, we withdraw one of the balls from the box, any one of the ten balls may be the one we withdraw. This is so, because in such a trial there is no basis to suppose that the appearance of any one ball with a particular number, is more likely than the appearance of another.

A more descriptive representation of singly-possible, mutually exclusive and equally possible events can be obtained with examples using playing dice, balls etc.. Hence the conclusions and illustrations of a great many of the basic assumptions of the theory of probability are made with such examples.

Let us make use of the example with the balls, in order to establish a rule (a general formula) for the mathematical expression of the probability of an event.

Let us assume, that there are 4 balls in each of three closed boxes, and in addition that in the box No.1 there are 3 white balls and 1 black ball, in box No.2 - 2 white and 2 black balls, and finally, in box No.3 - 1 white and 3 black balls (fig.2).

Fig.2.

The balls differ only in colour and are thoroughly mixed in their boxes. The trial consists in that one ball is to be taken out of the box, blindfold.

Let us see, how the probability of the white ball being selected out of each box individually, can be numerically expressed.

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8. Let us commence with box No.1. There are 4 balls in this box. According to the conditions of our test, any one of the 4 balls may be selected. Thus, the number of chances for the selection of a ball from the box No.1, is altogether 4. These chances (selections) are singly-possible (there being no other balls in the box), independent (one ball only is taken out) and equally possible (the balls differ only in colour and are picked up blindfold).

Our interest is the selection of the white ball. There are 3 such balls in the box. Therefore, out of 4 equally possible chances of selecting a ball (independently of its colour), there will be three chances which will favour (as is customarily referred to in the theory of probability) the selection of the white ball.

This can be explained in another way. Let us suppose, that 4 persons have each, simultaneously placed a hand into this box, and each person, without looking has picked up a ball. Let us assume, that the person taking out the white ball, wins, and the one taking out the black ball, loses. It is evident, that when the hands are withdrawn from the box simultaneously, three of them will hold a white ball, namely 3 persons will have a win. It is also evident,

9. that in a repetition of such a procedure (game), out of 4, 3 persons will always win, since, so long as the relationship of the white balls to the black in the box No.1, does not change, the selection of the white ball by one of the participants of this trial will always be favoured by 3 chances out of the 4 possible ones.

Let us now turn to the box No.2. There are also 4 balls in this box, but 2 of them are white. Following a similar reasoning, we conclude, that out of 4 possible chances of selection of the ball, 2 chances will favour the selection of the white ball.

And, finally, in the box No.3 there is only 1 white ball out of 4. Consequently, out of 4 possible chances of selection of the ball, 1 chance will favour the selection of the white ball.

Thus, the selection of the white ball:

- from box No.1 has 3 chances out of 4 in favour;
- from box No.2 has 2 chances out of 4 in favour;
- from box No.3 has 1 chance out of 4 in favour,

or applying it to the example of a game, if the balls are picked up:

- from box No.1, there will always be 3 wins out of 4;
- from box No.2, there will always be 2 wins out of 4;
- from box No.3, there will always be 1 win out of 4.

Consequently, prior to the playing of the game, each player will have:

- from box No.1 - 3/4 wins;
- from box No.2 - 2/4 wins;
- from box No.3 - 1/4 wins.

Therefore, the probability (the degree of possibility) for the selection of the white ball, or the probability of a win, from the box No.1 can be expressed by the fraction 3/4; that from box No.2 - by the fraction 2/4; that from box No.3 - by the fraction 1/4, i.e. by the relationship of the number of chances favouring the selection of the white ball from a given box, to the number of all the equally possible chances for the selection of a ball independent of its colour.

We will show, that expressing the probability of selection of the white ball by such a relationship is true also when we are dealing with a different quantity of white and black balls in a box.

Let us suppose, that there is a box No.4 in which are contained 3 white and 3 black balls, and let us compare the probability of selection of the white ball from this box, with the probability of selection of the white ball from box No.2 (fig.3).

Fig.3.

In both these boxes, the white balls comprise a half of the

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total number of balls, i.e. the relationship between the number of chances favouring the selection of the white ball, and the number of all the possible chances for the selection of a ball is here the same. And if this is so, then the probability of selection of the white ball from box No.4 is equal to the probability of selection of the white ball from box No.2. In actual fact:

$$\frac{3}{6}\frac{1}{2} \quad \text{and} \quad \frac{2}{4}\frac{1}{2}$$

Comparing the probabilities of selection of the white ball from boxes No.1, No.2 and No.3, we see, that these probabilities viz., 3/4, 2/4 and 1/4 are <u>directly proportional</u> to the total number of chances favouring the selection of the white ball:

$$\frac{3}{4}:\frac{2}{4}:\frac{1}{4} = 3:2:1$$

in other words, depending on by how much the number of chances, favouring the selection of the white ball from one box, is <u>greater</u> (<u>less</u>) than the number of chances, favouring the selection of the white ball from another box, so by the same measure, will, the numerical symbol for the probability of selection of the white ball from one box, be <u>greater</u> (<u>less</u>) than the numerical symbol for the probability of selection of the white ball from another box. Hence, in the mathematical expression for the probability of selection of the white ball from a given box, the number of chances, favouring the selection of such a ball, is therefore contained <u>in the numerator.</u>

Comparing the probability of selection of the white ball from the boxes No.1 and No.4, where the number of favourable chances is the same (3 and 3), but the number of all the possible chances is different (4 and 6), we see, that the probabilities of selection of the white ball from these boxes are <u>inversely proportional</u> to the numbers for all the possible chances:

$$\frac{3}{4}:\frac{3}{6} = 6:4,$$

in other words, depending on by how much the number of all the possible chances of selection of the ball, independently of its colour, from one box, is <u>greater (less)</u> than the number of all the possible chances of selection of the ball from another box, so by the same measure, will, the numerical symbol for the probability of selection of the white ball from one box, be <u>less</u> (<u>greater</u>) than the numerical symbol for the probability of selection of the white ball from another box. Hence, in the mathematical expression for the probability of selection of the white ball, the number of all the possible chances have been placed by us <u>in the denominator.</u>

On the basis of the outlined proportionality it can be asserted, that the relationship of the number of chances, favouring the selection of the white ball, to the number of all the possible chances actually does mathematically express the probability of the selection of the white ball whatever the proportion of the white balls to the black.

Expanding this deduction to any accidental event, we accept, that the <u>mathematical probability of an event is equal to the relationship of the number of chances, favouring the selection of this event, to the number of all possible chances;</u> at the same time the chances are assumed to be singly-possible, mutualy exclusive and equally possible, i.e.,

11.

$$P = \frac{m}{n},$$

where P - (the initial letter of the Latin word <u>probabilitas</u>, which means probability) = probability of the event;

m - number of cases, favouring the occurrence of the event;

n - number of all possible cases.

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11. Generally, the word "mathematical" is omitted and the term used is, simply: "probability of the event".

EXAMPLE. What is the probability for the occurrence of the score 6 with a single throw of the die?

Solution. The number of instances, favouring the occurrence of the score 6, is one, as only one side of the die bears such a score. The total number of possible cases is six, as any of the six sides may come uppermost. Hence the probability for the occurrence of this score 6, is

$$P = \frac{m}{n} = \frac{1}{6}.$$

The number of cases favouring the occurrence of an event, can not be greater than the number of all the possible cases, and therefore the probability of an event can not be greater than unity. The probability, equal to unity, corresponds to a definite event, i.e. an event which in given circumstances can not fail to occur. Thus, for example, the selection of the white ball from the box, which contains only 4 white balls, is a definite event. The probability of selection of a white ball from such a box, is

$$P = \frac{4}{4} = 1.$$

The number of cases, favouring the occurrence of an event can not be less than zero, and hence the probability of an event can not likewise be less than zero. The probability, which is equal to zero, corresponds to an impossible event, i.e. such an event, which in given circumstances can not take place. For example, the selection of a white ball from a box, which contains only 4 black balls, is an impossible event. The probability of selection of a white ball from such a box, is

$$P = \frac{0}{4} = 0.$$

This denotes, that the probability of an event is always expressed by a real fraction, as its magnitude is contained within the limits of unity and zero:

$$0 \leqslant P \leqslant 1.$$

12. Probability of an event may also be expressed as a percentage. An event which is impossible - its probability is 0%, an event which is definite - its probability is 100%.

When employing the above established formula for the expression of the probability of an event, the rough errors will not be excluded if no thorough account is taken of cases which are mutually exclusive, and equally possible.

EXAMPLE 1. There are 5 completely identical balls in a box: 2 red and 3 white; the balls are numbered as in fig.4. A man withdraws blindfold one ball and may in doing so win or lose. He wins if he withdraws a ball marked No.1 or a red ball. What is the probability of a win?

Fig.4.

Solution. The total number of possible instances for the withdrawal of a ball is 5. The number of instances favouring a win appears at first sight to be 4: two balls marked No.1 and two red balls. From this it appears to follow that the probability of a win is

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12. But such a solution is incorrect, as two instances, favouring a win, namely the withdrawal of a ball marked No.1 and the withdrawal of a red ball, are independent instances. It is possible to withdraw a red ball and at the same time a ball marked No.1.

For a correct solution it is necessary to determine the number of mutually exclusive instances, favouring a win. There are 3 such instances: the withdrawal of a white ball marked No.1 and the withdrawal of 2 red balls.

The required probability is therefore

$$P = \frac{3}{5}.$$

4. Frequency of an event.

In order to show the rule to which final, random results of an occurrence conform, it is necessary to deal with results of a sufficiently large number of trials, i.e. to determine, how frequently this or that event has been happening, out of the total of the possible events of the given occurrence.

Dividing the number of occurrences of the event, which we are considering, by the number of all the trials (experiments) made, we obtain the answer to the above question. For example, if in 20 shots there are 4 hits, then, dividing 4 by 20, we establish, how frequently hits were obtained during such firing.

The relationship of the number of occurrences of the event to the number of all the trials is termed the frequency of an event.

In the example given above, the frequency of hits is equal to

$$\frac{4}{20} = \frac{1}{5};$$

this frequency shows, that on the average 1 hit was obtained for every 5 shots.

13. In a general form, the frequency of an event is expressed by the formula

$$r = \frac{M}{N},$$

where r - frequency of an event;
M - number of its occurrences;
N - number of trials made.

EXAMPLE. In 12 shots there were 3 undershoots. What is the frequency of the undershoot?

Solution. Number of occurrences of an event M = 3, number of trials made N = 12, hence the required frequency

$$r = \frac{3}{12} = \frac{1}{4}.$$

Let us show what are the properties of the frequency. Let us suppose that in 15 shots there were 5 hits. From this, the frequency of hits

$$r = \frac{5}{15} = \frac{1}{3}.$$

Let us now fire 3 more shots. If, in this case there will be 1 hit and 2 misses, then the frequency of the hits will not alter:

$$r = \frac{5+1}{15+3} = \frac{6}{18} = \frac{1}{3}.$$

If these same 3 shots produce 3 hits, then the frequency of the hits will increase:

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13. $$r = \frac{5+3}{15+3} = \frac{8}{18} = \frac{4}{9}.$$

If the additional 3 shots will give misses, then the frequency of the hits will decrease:

$$r = \frac{5+0}{15+3} = \frac{5}{18}.$$

From this we draw the conclusion, that the frequency of an event changes with the change of the number of the trials made, and with the change in the number of the occurrences of the event.

The event, in a given number of trials, may occur in all the trials, or only in part of the trials, or yet may not occur at all, i.e. the number of occurrences of an event may be equal to the number of all the trials, or to a part of the number of the trials, or may be equal to zero. Therefore the frequency of an event is expressed by a number contained within the limits of zero and unity, i.e. by a proper fraction:

$$0 \leqslant r \leqslant 1,$$

or as a percentage:

$$0\% \leqslant r \leqslant 100\%.$$

Let us pause to consider the properties of frequencies of opposite events.

14. Let us suppose, that in a certain trial, an event A or an event B can occur, these events being opposite to each other. Let us also suppose, that in N such trials, the event A occurred M_1 times, and the event B - M_2 times. Then, the frequency of the event A

$$r_A = \frac{M_1}{N},$$

whilst the frequency of the event B

$$r_B = \frac{M_2}{N}.$$

As the events A and B are opposite, i.e.

$$M_1 + M_2 = N,$$

the sum of the frequencies of these events

$$r_A + r_B = \frac{M_1}{N} + \frac{M_2}{N} = \frac{M_1+M_2}{N} = \frac{N}{N} = 1.$$

EXAMPLE. In 8 shots there occurred 3 undershoots and 5 overshoots. Then the sum of the frequencies of undershoots and overshoots

$$r_u + r_o = \frac{3}{8} + \frac{5}{8} = \frac{8}{8} = 1.$$

The sum of frequencies of opposite events is equal to unity.

From this conclusion, and having the frequency of one event, we can obtain the frequency of another event which is opposite to the first.

EXAMPLE. The frequency of hits is 1/4. What is the frequency of misses?

Solution.

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$$r_m = 1 - r_h = 1 - \frac{1}{4} = \frac{3}{4}.$$

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14. It is obvious, that also in the case of any number of mutually exclusive events, which have taken place in a given number of trials, the sum of the frequencies of these events must likewise be equal to unity.

EXAMPLE. In 10 throws of a playing die:
score 6 occurred 3 times;
score 4 occurred 1 times;
score 3 occurred 4 times;
score 1 occurred 2 times.

Then the frequency of occurrences of scores:

$$\text{score 6: } r_6 = \frac{3}{10};$$

$$\text{score 4: } r_4 = \frac{1}{10};$$

$$\text{score 3: } r_3 = \frac{4}{10};$$

$$\text{score 1: } r_1 = \frac{2}{10}.$$

The sum of these frequencies

$$r_6 + r_4 + r_3 + r_1 = \frac{3}{10} + \frac{1}{10} + \frac{4}{10} + \frac{2}{10} = 1.$$

15. 5. Relationship between probability and frequency of an event.

Between probability and frequency of an event there exists a difference which consists in the following:

1. The probability of an event is established before trial (test) and answers the question: how frequently (on the average) can a given event be expected to occur. In order to establish the probability of an event, it is necessary to know the conditions of the trial, but there is no necessity to carry out trials.

The frequency of an event is established only after the trial and answers the question: how frequently had a certain event occurred.

2. The probability of an event under unchanging conditions of a given trial (for independent events)* remains constant, i.e. it does not depend on the number of trials and their results. Thus, if from a closed box, which contains 5 white and 3 black balls, one ball is taken out on a number of occasions, and on each such occasion the withdrawn ball is returned to the box, then the probability of a white ball appearing will be 5/8 at the commencement of each trial, independent of the number of times such a trial has been previously conducted, and also of the number of times a ball of one or another colour had appeared. If, however, we change the relationship between white and black balls, the probability of a white ball appearing will change. Thus, if we take a box containing 4 white and 4 black balls, the probability of the white ball appearing will now be 4/8.

Thus, the probability of an event remains unchanged for a given occurrence until the conditions, at which this occurrence takes place, change.

The frequency of an event is not constant - it changes with every trial and depends on the number of trials and on the number of occurrences of the event.

Apart from the above mentioned differences, between frequency and probability of an event, it is possible to establish one relationship which is expressed by the so called law of high numbers.

The nature of this law consists in that when the number of the trials (tests), carried out in identical conditions, is increased, the value of the frequency of the event approaches the probability of this same event, and when a sufficiently high number of such trials is made, the frequency of the event will numerically differ little from the probability of this event.

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15. The significance of the law of high numbers is enormous, and it is quite justifiably considered as the basis of the theory of probability.*

In the example which we had discussed, the calculation of the probabilities of events has been simple only because in the conditions of these examples, the number of cases favouring the happening of an
16. event, and the number of all the possible cases, were known. In actuality, however, the conditions of the problem seldom determine beforehand the number of favourable and the number of all the possible cases, and when attempting to find the numerical value of the probability of a certain event, we have to employ a different approach based on the law of high numbers. Having determined the frequency of the event from the results of a sufficiently large number of trials, we can take this frequency to be the unknown probability of this event.

If the probability of the event is known, then from this probability it is possible to judge, without carrying out any trials, the expected frequency of the event.

In artillery fire, the law of high numbers finds the widest application.

For example, from the frequency of deviations of the shells, the probability of such deviations is determined - mean (probable) deviations; from the frequencies of errors, obtained within certain limits - the probabilities of errors within the same limits etc. From the known probability of hits, the expected frequency of hits is calculated, i.e. the expected frequency of hits.

6. Theorem of addition of probabilities.

When investigating one or other occurrence, we are frequently interested in the appearance not only of any one definite result (event), but in the appearance of one random result from several results (events), which are combined by some common sign or property.

Thus, when investigating the dispersion in direct firing, we are interested in the position of no one single definite point of impact, but the position of a combination (group) of points of impact, giving, for example, undershoots, overshoots, hits on the target etc.

From this, therefore, arises the necessity to determine the probability of one out of several events, without naming any one in particular, as the occurrence of any one of them, under the conditions of a given trial, is sufficient for solving the set problem. This problem is solved by means of application of the <u>theorem of addition of probabilities</u>. It must be stressed, that the theorem of addition of probabilities is applicable only to mutually exclusive events.

Let us show by means of an example, why in order to determine the probability of one of several mutually exclusive events, without naming any one in particular, it is necessary to add the probabilities of these events.

Let us assume that the firing is being made with shells along the line of wire obstacles, the firing being laid in elliptical dispersion, as is shown in the illustration (fig.5).

We see, that the hits on the target are given by the shells
17. which are in the zone E, as well as in the zone F. The probability of hits on to the zone E is 25%, and on to the zone F - 16%. It is apparent, that the required probability will be equal to the sum

$$P = p_E + p_F = 25\% + 16\% = 41\%.$$

Let us now turn to the derivation of the theorem of addition of probabilities, i.e., let us show that such a method of calculation of probability is true for any group of mutually exclusive events

* The law of high numbers is discussed in greater detail in section 13.

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17.

Fig.5.
(1) - Mean trajectory.

Let us make use, for this purpose, of the following particular example. There are 50 balls in a closed box, these balls differing from each other only in colour: 8 white, 10 green, 5 blue, 12 black and 15 red. The experiment consists in withdrawing blindfold one ball from the box. Let us determine the probability for the appearance of a coloured ball, i.e. not a white or a black ball.

The total number of cases for the appearance of a ball from the box is n = 50, these cases being singly possible, mutually exclusive and equally possible.

Out of these 50 cases, according to the conditions of the problem, we are interested in the appearance of either a green, a blue or a red ball. There are 30 such balls in the box: 10 green, 5 blue and 15 red; hence the number of cases favouring the appearance of a coloured ball is m = 30.

Introducing the obtained numerical values for m and n in the general formula, we find the probability for the appearance of a coloured ball

$$P_c = \frac{m}{n} = \frac{30}{50}.$$

In order to derive the theorem, let us present this probability in another form, remembering that m = 30 = 10+5+15:

$$P_c = \frac{30}{50} = \frac{10+5+15}{50} = \frac{10}{50}+\frac{5}{50}+\frac{15}{50},$$

and let us see what these fractions denote.

In a box containing 50 balls, there are 10 green balls, i.e. 10
18. cases out of 50 possible cases, which favour the appearance of a green ball. Therefore, the fraction 10/50 is the probability of the appearance of a green ball from the box:

$$P_{green} = \frac{10}{50}.$$

Pursuing the same reasoning, we conclude that the fraction 5/50 is the probability of the appearance of the blue ball, whilst the fraction 15/50 is the probability of the appearance of a red ball. Hence

$$P_c = \frac{10}{50}+\frac{5}{50}+\frac{15}{50} = P_{green}+P_{blue}+P_{red},$$

i.e. the probability of the appearance of a coloured ball was found to be equal to the sum of the probabilities of the appearance of green, blue and red balls.

Expanding this conclusion into chance (accidental) results of any of the occurrences and into any number of mutually exclusive events, we obtain the expression for the theorem of addition of probabilities in a general form:

$$P = p_1+p_2+p_3+\ldots+p_{n-1}+p_n.$$

Probability of the occurrence of one of several mutually exclusive events, without indicating any one in particular, is equal to the sum of probabilities of these events.

In determining the probability of the appearance of a coloured ball, we combined the balls of several colours into one group, and we studied the appearance of the coloured ball from this group as one event having several forms. Therefore, the theorem of addition of probabilities can also be formulated thus: if the event has several mutually exclusive forms, then its probability is equal

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18. to the sum of probabilities of these forms.

EXAMPLE 1. The mean trajectory passes 10 beyond the target. What is the probability for undershoot with 1 shot?

Note. In all problems, where the depth of the target is not given, it is assumed that the target has no depth - that is a line, boundary, point, i.e. it is assumed that only undershoots and overshoots will be obtained.

Solution. Undershoots will be obtained with those shells which will fall in the zones A, B and C (fig.6). From this, the probability of undershoot with one shot will be made up of hits in the zone A, zone B and zone C, i.e. it is

$$P = p_A + p_B + p_C = 0.02 + 0.07 + 0.16 = 0.25$$

Fig.6.
(1) - Mean trajectory.

19. EXAMPLE 2. In a lottery of 300 tickets, there are: 2 wins of 50 roubles, 6 wins of 20 roubles, 10 wins of 10 roubles, 50 wins of 5 roubles and 100 wins of 3 roubles. What is the probability in this lottery, of a win of not less than 10 roubles with one ticket?

Solution. The condition is satisfied by wins of 50, 20 and 10 roubles. The probabilities of these wins are:

$$p_{50} = \frac{2}{300}; \quad p_{20} = \frac{6}{300}; \quad p_{10} = \frac{10}{300}.$$

Adding these probabilities we obtain the probability of a win of not less than 10 roubles:

$$p = p_{50} + p_{20} + p_{10} = \frac{2}{300} + \frac{6}{300} + \frac{10}{300} = \frac{18}{300} = 0.06.$$

In the cases when the probabilities of mutually exclusive events (different forms) are equal to each other:

$$p_1 = p_2 = p_3 \ldots = p_{k-1} = p_k = p$$

the theorem of addition of probabilities takes the form:

$$P = kp,$$

where k - the number of mutually exclusive events;
p - probability of any one of them.

The theorem of addition of probabilities states: the sum of probabilities of all possible mutually exclusive events of a given occurrence is equal to unity:

$$p_1 + p_2 + p_3 + \ldots p_k + \ldots + p_{s-1} + p_s = 1.$$

Let us check this statement using the example of the balls discussed above, and for this purpose, adding the probabilities of the appearance of the balls of all the colours:

$$p_{white} + p_{black} + p_{green} + p_{red} + p_{blue} = \frac{8}{50} + \frac{12}{50} + \frac{10}{50} + \frac{15}{50} + \frac{5}{50} = \frac{50}{50} = 1.$$

The certain result in letter the truth of the event. This means, that if the ball be removed, then of necessity that ball must [illegible] colour of all the different colour balls contained in

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[illegible] of the statement consists in that by applying

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19. events of a given occurrence, i.e. if this occurrence is being investigated from all aspects. Adding the probabilities of all the possible mutually exclusive events of a given occurrence and having obtained a value for the sum of less than unity, we are justified to say, that some error has been admitted in the estimation of the probabilities of individual events, or that one or several of all the possible events has not been considered at all. Also, making use of the statement we can correct these errors.

20. EXAMPLE 1. When firing a pistol, the marksman makes a hit only on the black ring of the target. Estimate the probability to score with one shot not less than 8 points, if the probability of hitting ten is 0.3, eight - 0.2, seven - 0.1 and six - 0.05.

Solution. It is easy to notice that the conditions of the example do not give the probability of hitting nine. This is confirmed by the fact that the sum of given probabilities is not equal to unity:

$$p_{10}+p_8+p_7+p_6 = 0.3+0.2+0.1+0.05 = 0.65.$$

In order to consider all the possible cases of hitting the target and to obtain unity for the sum of their probabilities, the obtained result must have the probability of hitting nine added to it:

$$0.65+p_9 = 1,$$

whence

$$p_9 = 1-0.65 = 0.35.$$

After this we can commence to estimate the required probability of scoring not less than 8 points, i.e. the probabilities to hit either ten, nine or eight

$$P = p_{10}+p_9+p_8 = 0.3+0.35+0.2 = 0.85.$$

EXAMPLE 2. There are 10 identical balls bearing numbers 1 to 10 in a closed box. It is required to determine the probability of appearance of a ball bearing a number which is a multiple of two or three.

Solution. Out of 10 balls, there are 5 balls with numbers which are multiples of two: 2, 4, 6, 8 and 10. The probability of appearance of such a ball is

$$p_2 = \frac{5}{10}.$$

The number of balls bearing numbers which are multiples of three, are only 3, viz.: 3, 6 and 9. The probability of appearance of such a ball is

$$p_3 = \frac{3}{10}.$$

Adding these probabilities, we should apparently obtain the probability of appearance of a ball bearing a number which is a multiple of two or three:

$$P = p_2+p_3 = \frac{5}{10}+\frac{3}{10} = \frac{8}{10}.$$

But such a solution is incorrect. We can easily confirm this by making use of the result of the theorem of addition of probabilities. Apart from the balls with numbers which are multiples of two or three, there are in the box other balls with numbers which are not multiples of either two or three: 1, 5 and 7. The probability of appearance of such a ball is

$$p_n = \frac{3}{10}.$$

As a ball with some number must definitely appear, the sum of probabilities of the appearance of balls with [illegible] must be equal to unity, and the sum obtained is not equal to unity:

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20. $$p_2+p_3+p_n = \frac{5}{10}+\frac{2}{10}+\frac{2}{10} = \frac{11}{10}.$$

This result indicates, that some error has been admitted. This error is easily exposed. It occurred because we had not considered the property of the ball bearing the number 6. The number of this ball is a multiple of both two and three. We had not complied with the requirement that the events should be definitely mutually exclusive.

21. In order to correct this error, it is necessary to take, in the first place, the balls with numbers which are multiples of two only, - 2, 4, 8 and 10, and then numbers which are multiples of three only, - 3 and 9 and, finally, the ball with number 6, which is a multiple of two and three at the same time. Adding the probability of appearance of such balls, we then obtain the required probability:

$$P = p_2+p_3+p_{2.3} = \frac{4}{10}+\frac{2}{10}+\frac{1}{10} = \frac{7}{10}.$$

Adding to this result the probability of appearance of a ball bearing the number which is not a multiple of either two or three, we will obtain a unity as the sum:

$$\frac{7}{10}+\frac{3}{10} = 1,$$

which demonstrates the correctness of our solution.

The above example confirms sufficiently clearly that the theorem of addition of probabilities is applicable only to mutually exclusive events.

The statement of the theorem of addition of probabilities is important also in that by means of it, it is possible to check the correctness of mathematical calculations with the condition, that in the remainder no errors have been admitted.

A particular case of the statement of the theorem of addition of probabilities is: the sum of probabilities of opposite events is equal to unity:

$$p+q = 1,$$

where p - probability of one event;
q - probability of the event opposite to the first.

No proof is required here, as that which was true for several mutually exclusive events will be true also for two events.

Knowing the probability of one of two opposite events, it is possible, by subtracting this probability from unity, to find the probability of the other.

$$p = 1-q \qquad q = 1-p.$$

[illegible]. The probability of an undershoot with one shot is 0.75. What is the probability of an overshoot?

Solution. Denoting the probability of undershoot by p, and the probability of overshoot by q and applying the particular case of the statement, we obtain

$$q = 1-p = 1-0.75 = 0.25.$$

7. Theorem of multiplication of probabilities.

So far we have had to deal with the, so called, simple events, i.e. with individual, single events which constitute final accidental result of this or other occurrence: obtaining of one undershoot, withdrawing of one ball, winning with one lottery ticket etc.

22. Apart from simple events, very frequently compound events take place, consisting of several simple events which obtain as a result of simultaneous or consecutive appearance of several simple events.

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[illegible] with four shots: return of a definite sum scored with

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22. several throws ofdice; obtaining of a ricochet and at the same time an overshoot with one firing of a shell - all these events are compound.

To determine the probability of a compound event is made possible by the application of the theorem of multiplication of probabilities.

A. The case of independent events.

Simple events, forming some compound event, can be <u>independent</u> or <u>dependent</u>.

Events are termed independent when the appearance or non-appearance of one of them does not affect the probability of appearance of the others.

The events are termed dependent when the probability of appearance of one event depends on the appearance of the others.

For example, a win in one lottery and a win in another lottery are independent events, as the probability of a win in one lottery will not influence in any way a win or a lose in another. Wins on two tickets in the same lottery are dependent events, as a win with the first ticket reduces the probability of a win with the second, and a lose with the first ticket increases this probability.

Let us derive the theorem of multiplication of probabilities, in the first place for independent events. For this purpose we will make use of the following individual example.

Let us suppose that we have 2 closed boxes with balls differing from each other only in colour. In the first box there are 4 white and 5 red balls, in the second box - 3 white and 2 red balls. For the convenience of further discussion we will number the balls in both boxes (fig.7).

Fig.7.
(1) - First box;
(2) - Second box.

Let us set some compound event arising out of these conditions, and let us determine its probability. Let us find, for example, what will be the probability of withdrawing, blindfold, from the two boxes of one white ball.

23. Using the general rule we find the required probability from the formula

$$P = \frac{m}{n}.$$

Having determined m - the number of cases, favouring the appearance of the compound event, and n - the number of all the possible cases, and taking the relationship of the first to the second, we will solve the set problem.

The compound event in which we are interested consists in the coincidence of two simple independent events: the appearance of a white ball from the first box and the appearance of a white ball from the second box. But, apart from such coincidences, there can be coincidences of two red balls and coincidences of a white ball with a red ball, or a red ball with a white. No other coincidences are possible, and hence the total number of all the possible coincidences of two balls is independent of their colour and will be equal to the number of all the possible cases of the appearance of the compound event with which we are concerned. What is then the number of such cases?

The ball No.1 from the first box may coincide with any of the 5 balls in the second box, i.e. there can be 5 coincidences of pairs of balls: No.1 with No.10, No.1 with No.11, No.1 with No.12, No.1 with No.13 and No.1 with No.14. The same number of coincidences can also be for every one of the remaining balls

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23. in that box, there are altogether 9 balls, the total number of all the possible coincidences will be

$$n = 9x5 = 45,$$

and these coincidences will also be singly possible, mutually exclusive and equally possible.

In analising the coincidences of the balls from the second box with the balls from the first, we arrive at the same result, because for every one of the 5 balls from the second box, there can be 9 coincidences with the balls from the first box. In this case the

---

Fig.8.

24. total number of all the possible events will likewise be

$$n = 5x9 = 45.$$

To demonstrate this more fully we will show these coincidences in the form of a diagram (fig.8).

Let us now turn to finding the number of cases which favour the appearance of the compound event in question.

Any of the white balls in the first box can coincide with any of the white balls from the second box, and as there are 4 white balls in the first box, and 3 in the second box, the number of coincidences of 2 white balls, and hence also the number of cases favouring the appearance of such a coincidence,

$$m = 4x3 = 12;$$

in the diagram these coincidences can be seen as combinations: Nos.1 - 4, 10 - 13, and 19 - 22.

Substituting the obtained numerical values of m and n in the general formula, we will determine the required probability, i.e. the probability of coincidence of 2 white balls:

$$P = \frac{m}{n} = \frac{12}{45}.$$

In order to arrive at the rule for determining the probability of a compound event, we will present the right-hand part of this equation in a slightly different form, remembering that $m = 12 = 4x3$, and $n = 45 = 9x5$, after which we will analise this equation:

$$P = \frac{m}{n} = \frac{12}{45} = \frac{4x3}{9x5} = \frac{4}{9}x\frac{3}{5}$$

In the first box, out of 9 balls there are 4 white balls, and hence the fraction 4/9 is the probability of the appearance of a white ball from this box:

$$p_{w_1} = \frac{4}{9}$$

In the second box, out of 5 balls there are 3 white balls, - the fraction 3/5 is the probability of appearance of a white ball from the second box

$$p_{w_2} = \frac{3}{5}$$

Hence the probability of appearance of two white balls (one from each box) is equal to the product of the probability of appearance of a white ball from the first box and the probability of appearance of a white ball from the second box:

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24. $$p_{ww} = \frac{4}{9} x \frac{3}{5} = p_{w_1} x p_{w_2}.$$

In this way, in order to determine the probability of a compound event, we have had to <u>multiply</u> the probabilities of simple events which constitute it.

25. Let us check this deduction by taking another compound event under conditions of those existing for the first example - the appearance of 2 red balls, one from each box.

The probability of appearance of a red ball from the first box

$$p_{r_1} = \frac{5}{9}.$$

The probability of appearance of a red ball from the second box

$$p_{r_2} = \frac{2}{5}.$$

Multiplying these probabilities we must obtain the probability of appearance of 2 red balls from both boxes:

$$P_{rr} = p_{r_1} x\ p_{r_2} = \frac{5}{9} x \frac{2}{5} = \frac{10}{45}.$$

In the diagram (fig.8) it is seen that the number of coincidences of two red balls is 10 out of 45 possible (coincidences Nos.32 to 36 and 41 to 45). Consequently, our deduction is true also for this compound event.

We have determined the probability of a compound event consisting of two simple events. Let us examine what will be the probability of a compound event if it consists of three simple events.

Let us add to the first and the second boxes, a third box, with 2 white and 4 red balls, and let us determine the probability of appearance of 3 white balls from 3 boxes.

We have had 45 of all the possible coincidences of pairs of balls from the first and second boxes. Each one of these pairs can coincide with any of the 6 balls from the third box, as a result of which there will be 210 coincidences (45x6), but, in this case, of 3 balls; this is then the number of all the possible cases for the appearance of the new compound event.

Coincidences of pairs of white balls were 12. Each such pair can coincide with any of the 2 white balls from the third box, as a result of which there will be 24 coincidences of 3 white balls (12x2) or 24 cases favouring the appearance of the compound event in question. Hence its probability is

$$P_{www} = \frac{24}{210} = \frac{12x2}{45x6} = \frac{4x3x2}{9x5x6} = \frac{4}{9} x \frac{3}{5} x \frac{2}{6} = p_{w_1} x\ p_{w_2} x\ p_{w_3}.$$

Consequently, also in this case, the probability of the compound event is equal to the product of probabilities of simple events which form this compound event.

It is easy to note, that determination of numerical values of m and n follows some general rule:

m - the number of cases, favouring the appearance of a compound event, is always equal to the product of the numbers of cases favouring the appearance of simple events;

26. n - the number of all the possible cases for the appearance of a compound event is equal to the product of the number of cases of the appearance of simple events:

$$n = n_1 x\ n_2 x\ n_3 \ldots$$

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26. Therefore after substitution of m and n in the general formula we shall always have in the sum the product of probabilities of simple events which make up the given compound event:

$$P = \frac{m}{n} = \frac{m_1 x m_2 x m_3 \ldots m_S}{n_1 x n_2 x n_3 \ldots n_S} = \frac{m_1}{n_1} x \frac{m_2}{n_2} x \frac{m_3}{n_3} \ldots \frac{m_S}{n_S},$$

or

$$P = p_1 x\ p_2 x\ p_3 \ldots p_S.$$

The probability of a compound event, consisting of a coincidence or consecutive appearance of several simple independent events, is equal to the product of probabilities of these simple events.

It must be pointed out, that the probability of a compound event as a product of vulgar fractions will always be less than any of the probabilities of simple events of which it consists. If the probability of each simple event is unity, then the probability of the compound event will likewise be equal to unity.

EXAMPLE 1. In firing from a pistol at a round target, the probability of hitting ten with 1 shot is 0.2; hitting nine - 0.25; hitting eight - 0.3; seven - 0.15 and six - 0.1. What is the probability of scoring consecutively 8, 9 and 10 with 3 shots?

Solution. Consecutive hits on eight, nine and ten is a compound event consisting of 3 simple events: hitting eight with the first shot, nine with the second and ten with the third. According to the theorem of multiplication, the required probability must be equal to the product of probabilities of these simple events:

$$P = p_8 x\ p_9 x\ p_{10} = 0.3 x 0.25 x 0.2 = 0.015.$$

EXAMPLE 2. In the first lottery, out of 150 tickets there are 20 prizes and in the second lottery there are 200 tickets and 30 prizes. What is the probability of a coincidence of a win with 1 ticket in the first lottery, with a win with 1 ticket in the second lottery?

Solution. The probability of a win with 1 ticket in the first lottery is

$$p_1 = \frac{20}{150},$$

and with one ticket in the second lottery

$$p_2 = \frac{30}{200};$$

the probability of a win with both tickets, i.e. the probability of a coincidence of two winning tickets,

$$P = p_1 x\ p_2 = \frac{20}{150} x \frac{30}{200} = \frac{1}{50} = 0.02.$$

27. If the probabilities of simple independent events, forming some compound event, are equal to each other, i.e. if

$$p_1 = p_2 = p_3 = \ldots = p_S,$$

then the probability P of such a compound event is equal to the probability p of the simple event raised to the power equal to the number of simple events:

$$P = p^S.$$

EXAMPLE. What is the probabilty of obtaining 4 undershoots consecutively when firing on the same sight, if the probability of undershoot with 1 shot is 0.75?

Solution. As the sight does not change in firing, the probability of undershoot for all the 4 shots remains unchanged:

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$$p_1 = p_2 = p_3 = p_4 = 0.75.$$

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27. Hence the probability of 4 undershoots as the probability of a compound event, consisting of 4 simple events, the probabilities of which are equal to each other, will be

$$P = p^4 = 0.75^4 = 0.3164.$$

The compound event can, in its turn, consist of compound events. The application of the theorem of multiplication of probabilities, in such cases, does not present any doubt, as any compound event can be represented as one consisting only of simple events.

EXAMPLE. The mean trajectories, corresponding to sights 100 and 102, are as shown in fig.9 (ΔX = 50m, ΔS = 25m). What is the probability of encompassing the target in the cone of fire, the near limit of which must have 2 undershoots, and the far limit - 2 overshoots?

Fig.9.
(1) - Sight 100;
(2) - Sight 102;
(3) - Target.

Solution. The cone of fire, at the near limit of which are obtained 2 undershoots and at the far limit - 2 overshoots, is a compound event, consisting of two compound events, each one of which consists of two simple events.

Let us first find the probability of obtaining one which forms the compound event, i.e. the probability of obtaining 2 undershoots on the sight 100. From the figure it is seen, that the probability of one undershoot when firing on this sight, is

$$P_{100} = 0.02+0.07+0.16+0.25-0.25 = 0.75;$$

hence the probability of obtaining 2 undershoots on this sight

$$P_{100} = p^2_{100} = 0.75^2 = 0.5625.$$

28. Let us find the probability of the other one, forming the compound event, i.e. the probability of obtaining 2 overshoots on the sight 102. The probability of obtaining of one overshoot when firing on this sight, is

$$q_{102} = 0.02+0.07+0.16+0.25+0.25+0.16+0.07 = 0.98;$$

the probability of obtaining 2 overshoots on the same sight

$$P_{102} = q^2_{102} = 0.98^2 = 0.9604.$$

Knowing the probability of both the compound events, we will find the required probability, i.e. the probability of obtaining the cone of fire, each limit of which should have 2 signs:

$$P_{cone} = P_{100} \times P_{102} = 0.5625 \times 0.9604 = 0.5402.$$

We would have obtained the same result, if we multiplied, in the first place, the probabilities of simple events which form the compound event in question:

$$P_{cone} = p_{100} \times p_{100} \times q_{102} \times q_{102} = 0.75 \times 0.75 \times 0.98 \times 0.98 = 0.5402.$$

In all the above examples we have had to deal with compound events consisting of simple mutually exclusive events, but the theorem of multiplication of probabilities is also applicable to the independent events.

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28. EXAMPLE 1. A closed box contains 10 balls out of which 7 are white and 3 red. All the balls are numbered: white from No.1 to No.7 and red from No.8 to No.10. What is the probability of withdrawing, blindfold, the white ball marked No.4?

Solution. Let us first find the required probability as a ratio of the number of white even balls to the number of all the balls contained in the box. Out of the 10 balls, 3 white balls are even: No.2, No.4 and No.6. Consequently, the probability of appearance of a white even ball

$$P = \frac{3}{10}.$$

We will now apply the theorem of multiplication of probabilities, by investigating the appearance of a white even ball as a compound event, consisting of a coincidence of two simple independent events: a white colour and an even number, or an even number and a white colour.

Out of ten balls, 7 are white. Therefore, the probability of appearance of a white ball is independent of its number

$$p_w = \frac{7}{10}.$$

The condition is satisfied only by the appearance of a white ball with an even number. Out of 7 white balls there are 3 even. Therefore, the probability of appearance of an even ball out of the white balls

$$p_{even-w} = \frac{3}{7}.$$

Hence the probability of appearance of an even white ball out of all the white balls

$$P = \frac{7}{10} \times \frac{3}{7} = \frac{3}{10}.$$

Both solutions have produced the same result, because the theorem of multiplication of probabilities is true also for the cases when the compound event consists of independent events.

29. It must be remembered, that when determining the probability of withdrawing the white ball with the No.4, either the probability of coincidence of a white ball (irrespective of its number) with an even number from the white balls only, is taken into consideration, or the probability of the coincidence of an even number (irrespective of the colour) with a white ball from the even numbers only.

If these conditions are upset, the following incorrect solution is possible:- The probability of appearance of a ball with an even number irrespective of its colour

$$p_{even} = \frac{5}{10};$$

- probability of appearance of a white ball irrespective of its number

$$p_w = \frac{7}{10}.$$

In such a case the probability of appearance of a white even ball out of the total number of balls

$$P = \frac{5}{10} \times \frac{7}{10} = \frac{35}{100}$$

which contradicts the ratio of the number of even white balls to the number of all the balls contained in the box.

Let us investigate how this error has come about. The product of the fractions

$$\frac{5}{10} \times \frac{7}{10}$$

denotes, that here we have taken either 7/10 from the number of all the balls with even numbers, or 5/10 from the number of all the white balls. Actually, according to the condition we have obtained that there are 7/5 white balls to the even balls, and not 7/10, and the number of even balls to the white balls is not a half, but 3/7.

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29. EXAMPLE 2. The probability of locating the target in the area upon which the fire is directed, $p_t = 0.3$. The probability of a hit with 1 shot and under condition that the target is located in the area covered by fire, $p = 0.2$. Determine the probabilities: a) of obtaining a hit with 1 shot; and b) of obtaining 2 hits with 2 shots.

Solution. a) Here we have a compound event consisting of a coincidence of two simple independent events - that of locating the target in a given area and hitting that target. Hence the required probability

$$P_a = p_t x\ p = 0.3x0.2 = 0.06.$$

b) In this case the compound event consists of the coincidence of the following independent events: simple event (that of locating the target in the area under fire) and a compound event (that of obtaining 2 consecutive hits).

If the target is located in the given area, the probability of 2 consecutive hits is

$$p^2 = 0.2^2 = 0.04.$$

The probability of two hits taking into consideration the probability of locating the target

$$P_b = p_t x\ p^2 = 0.3x0.04 = 0.012.$$

30. It would have been incorrect to have made use of the result of the first solution by raising it to the 2nd power and to consider, that

$$P_b = P_a^2 = 0.06^2 = 0.0036.$$

The error becomes obvious, if we present this result in the form of the product of probabilities of simple events:

$$0.0036 = 0.06^2 = (0.3x0.2)^2 = 0.3x0.3x0.2x0.2 = p_t x\ p_t x\ p\ x\ p.$$

In the given case the probability of locating the target is considered twice, i.e. it is taken as depending on the number of shots made. Actually, the probability of finding the target is independent of the number of shots, as the former shows what part of the probability of hitting the target should be taken with any number of shots under condition, that the target is definitely located in the area under fire.

EXAMPLE 3. The probability of a hit with 1 shot is 0.4. What is the probability of obtaining 2 consecutive hits by exploding shells, if failures to explode constitute 6% of the number of all the shells fired?

Solution. In firing there can be a hit and a miss, an explosion and failure to explode of a shell. Therefore, our compound event must be regarded as one consisting of two compound events, each one of which consists of two simple independent events - a hit and an explosion of a shell.

The probability of obtaining of an explosion of a shell with 1 shot taken as a probability of an event opposite to non-explosion,

$$p_{expl.} = 1-0.06 = 0.94.$$

The probability of obtaining a hit by an exploding shell with 1 shot

$$P_1 = p\ x\ p_{expl.} = 0.4x0.94 = 0.376;$$

the probability of obtaining, consecutively, 2 such hits

$$P = P_1^2 = 0.376^2 = 0.1414.$$

These three examples show with what care should the analysis of compound events which comprise independent events. A superficial or routine approach to the solution of such problems leads, as a rule, to gross

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30. In the theory of probability and in artillery computations, in particular, it is often necessary to apply simultaneously both the theorem of addition and the theorem of multiplication of probabilities.

EXAMPLE 1. There are 2 closed boxes containing balls: in the first there are 3 white and 4 black balls, in the second - 7 white and 5 black balls. What is the probability, when withdrawing (blindfold) one ball from each box, of obtaining simultaneously 2 balls of the same colour?

Solution. The solution of this example will be satisfied by the appearance of either 2 white or 2 black balls.

The probability of appearance of 2 white balls, according to the theorem of multiplication

$$P_w = p_{w_1} \times p_{w_2} = \frac{3}{7} \times \frac{7}{12} = \frac{21}{84}.$$

The probability of appearance of 2 black balls according to the same theorem

$$P_b = p_{b_1} \times p_{b_2} = \frac{4}{7} \times \frac{5}{12} = \frac{20}{84}.$$

31. According to the theorem of addition of probabilities the appearance of either 2 white or 2 black balls

$$P = P_w + P_b = \frac{21}{84} + \frac{20}{84} = \frac{41}{84}.$$

EXAMPLE 2. Probability of an undershoot with 1 shot is $p = 1/3$. What is the probability of obtaining a marker group (observation of various signs) with 3 shots, made on the same sights?

Solution. If the probability of an undershoot with 1 shot is $1/3$, then the probability of an overshoot

$$q = 1-p = 1-\frac{1}{3} = \frac{2}{3}.$$

With 3 shots it is possible to obtain a marker group either with a predominance of undershoots (2 undershoots and 1 overshoot), or a predominance of overshoots (1 undershoot and 2 overshoots).

The marker group, consisting of 2 undershoots and 1 overshoot can be obtained in the form of one of the following three variants: either --+, or -+-, or +--.

The probability of any of these variants is, according to the theorem of multiplication, equal to

$$p^2q = (\frac{1}{3})^2 \times \frac{2}{3} = \frac{2}{27};$$

and as the order of obtaining of undershoots and overshoots is immaterial, then according to the theorem of addition, the probability for the marker group with predominance of undershoots, is

$$P_1 = 3p^2q = 3 \times \frac{2}{27} = \frac{6}{27}.$$

Following a similar reasoning, we determine the probability for the marker group with predominance of overshoots basing on that such a group can be obtained in the form of one of the following three variants: either ++-, or +-+, or -++:

$$P_2 = 3pq^2 = 3 \times \frac{1}{3} \times (\frac{2}{3})^2 = \frac{12}{27}.$$

Hence according to the theorem of addition, the probability of the marker group is independent of the correspondence of signs

$$P = P_1 + P_2 = \frac{6}{27} + \frac{12}{27} = \frac{18}{27} = \frac{2}{3}.$$

The same problem may be solved differently: apart from the marker groups which have been discussed, it is possible to have, with [illegible] all the shots as undershoots or overshoots.

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31. $P_3 = p^3 = (\frac{1}{3})^3 = \frac{1}{27}$.

The probability of obtaining three overshoots

$$P_4 = q^3 = (\frac{2}{3})^3 = \frac{8}{27}.$$

32. If we subtract these probabilities from unity, then on the basis of the statement of the theorem of addition, we will obtain the required probability of the marker group:

$$P = 1-(P_3+P_4) = 1-(\frac{1}{27}+\frac{8}{27}) = \frac{18}{27} = \frac{2}{3}.$$

Adding the probabilities of all the possible combinations made up of undershoots and overshoots with 3 shots, we obtain:

$$P_1+P_2+P_3+P_4 = \frac{6}{27}+\frac{12}{27}+\frac{1}{27}+\frac{8}{27} = 1,$$

which confirms the correctness of our solutions.

B. The case of dependent events.

For dependent events we will derive the theorem of multiplication of probabilities using the following example.

Let us suppose that there are in a closed box 5 balls: 3 red and 2 white. We will determine the probability of withdrawing, blindfold, twice, consecutively a red ball, without returning the first withdrawn ball.

Here we are dealing with a compound event; it consists in consecutive appearance of two simple events. These simple events are dependent - the probability of appearance of the second red ball depends on what ball had been withdrawn in the first instance.

If the red ball was withdrawn first, then the probability of withdrawing a red ball second time will be equal to 2/4, as the number of cases, favouring the appearance of a red ball, will decrease from 3 to 2, and the number of all the possible cases - from 5 to 4.

If the white ball was withdrawn first, then the probability (after this) of appearance of a red ball will be 3/4, as the number of cases, favouring the appearance of a red ball, will not change, and only the number of all the possible cases will have decreased.

Fig.10.

For the convenience of further discussion we will number the balls in the box (fig.10).

Applying the same reasoning as when deriving the theorem of multiplication for independent events, we will determine first the number of all the possible (singly possible, mutually exclusive and equally possible) coincidences of two balls.

After the ball No.1, there can appear: either ball No.2, ball No.3, ball No.4 or ball No.5. The same number of coincidences can take place with balls No.2, 3, 4 and 5. In this way, there can be altogether 5x4 = 20 coincidences. This is actually the number of all the possible cases of appearance of the compound event in question.

33. We will show these coincidences (cases) in the form of a diagram (fig.11).

The number of cases, favouring the appearance of the compound event in question, must be equal to the number of coincidences of two red balls; such coincidences according to our diagram are 6: No.1 and No.2, No.1 and No.3, No.2 and No.1, No.2 and No.3, No.3 and No.1, and No.3 and No.2.

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Fig.11.

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33. Hence the probability of withdrawing consecutively 2 red balls, without returning the first withdrawn ball, is, according to the general formula, equal to:

$$p = \frac{m}{n} = \frac{6}{20}.$$

The probability of appearance of a red ball for the first time - $p_1 = 3/5$ and for the second time - $p_2 = 2/4$, under condition that the withdrawn red ball is not returned into the box. We will represent the obtained probability 6/20 in the form of a product of two fractions:

$$p = \frac{6}{20} = \frac{3}{5} \times \frac{2}{4} = p_1 \times p_2,$$

i.e. the probability of withdrawing a red ball twice consecutively, under condition that the ball, withdrawn in the first place, is not returned back into the box, is equal to the probability of withdrawing a red ball the first time, multiplied by the probability of withdrawing a red ball the second time, the condition for this calculation being that the ball withdrawn first, is red.

In general, for any number of simple dependent events, which compose a compound event, the probability of a compound event which consists of several simple dependent events, is equal to the probability of the first event multiplied by the probability of the second event, calculated under the assumption that the first event has taken place, multiplied by the probability of the third event, calculated under the assumption that the first two events have taken place etc.

EXAMPLE 1. In a lottery of 10 tickets, there are 3 prizes. What is the probability for 3 tickets taken at random to be the winning ones?

Solution. The probability of a win with the first ticket

$$p_1 = \frac{3}{10}.$$

34. The probability of a win with the second ticket under the condition that the first ticket was a winning one,

$$p_2 = \frac{2}{9}.$$

The probability of a win with the third ticket under the condition that the first and the second tickets were winning ones

$$p_3 = \frac{1}{8}.$$

From this, according to the theorem of multiplication, it follows that the probability of a win with 3 first tickets

$$P = p_1 \times p_2 \times p_3 = \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} = \frac{6}{720} = \frac{1}{120}.$$

EXAMPLE 2. There are 4 tickets in a drum; one of these is a winning ticket. The tickets are distributed between 4 participants, who withdraw the tickets out of the drum in turn. Which of the participants has a greater chance of a win?

Solution. The probability of a win of the participant who withdraws the first ticket,

$$p_1 = \frac{1}{4}.$$

The second participant can win only when the first one loses; the probability of a lose by the first participant is 3/4; therefore the probability of a win by the second, is

$$p_2 = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}.$$

The third participant can win if the condition is that the first two lose, and finally, the fourth participant can win, if all the others lose. The probabilities of a win of the third and fourth participants are

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34. $p_3 = \frac{2}{4}x\frac{2}{3}x\frac{1}{2} = \frac{1}{4}$ and $p_4 = \frac{2}{4}x\frac{2}{3}x\frac{1}{2}x1 = \frac{1}{4}$.

Thus, the chances of a win for all the participants are the same.

8. Geometrical probability.

Frequently, when determining the probabilities of some events, a direct estimation of the number of cases favouring their appearance, and the number of all the possible cases, is difficult (existance of very many cases), or is completely impossible (infinite number of cases). The ratio of the number of favourable cases to the number of all the possible cases is then substituted by the ratio of values proportional to these numbers, if, however, such a proportionality exists.

Let us suppose, that 5kg of grain were taken and that 2kg of grain were dyed red and then thoroughly mixed with the undyed grain. If all the grain is identical, then obviously, that without reckoning the amount of the dyed and the undyed grain, it is possible to state that the probability of appearance of one dyed grain must be equal to 2/5, as there are 5kg of undyed grain to 2kg of the dyed.

35. Here, the ratio of the number of cases favouring the appearance of the event in question (the number of dyed grain) to the number of all the possible cases (the total number of the grain), we have substituted by the ratio of their weights, proportional to these numbers.

In the given example, the number of favourable cases and the number of all the possible cases is great (it is sufficient to say that 1kg holds about 150000 grain) but these numbers are finite and their reckoning, although difficult, is possible. We will now turn to an example, in the condition ofwhich the number of cases favouring the appearance of some event, and the number of all the possible cases, are infinitely great.

Let us take a thin rod AB of length 10cm and which has 10 cuts made - one per centimetre over its length. Let us also suppose, that the rod, in falling, breaks in 2 parts and that the break is equally possible at any one of these cuts. Let us estimate the probability of a break of the rod in the section CD of a length of 3cm (fig.12).

Fig.12.

There are 10 of all the possible (singly possible, mutually exclusive and equally possible) cases for the breaks of the rod. The number of cases favouring a break in the section CD, is 3. Hence the probability of a break in the section CD:

$$p = \frac{3}{10}.$$

Let us increase the number of cuts on the rod by 2, 5, 100 and 1000 times. It is obvious, that the number of all the possible cases of the break of the rod will increase in the same ratio, as will the number of cases favouring the break in the section CD, and therefore the required probability will not change:

$$p = \frac{3x2}{10x2} = \frac{3x5}{10x5} = \frac{3x100}{10x100} = \frac{3x1000}{10x1000} = \frac{3}{10},$$

because when the number of cuts is increased by any amount, the number of cases favouring the break of the rod in the section CD, remains proportional to the length of this section in centimetres, and the number of all the possible cases of the break is proportional to the length of the rod, likewise in centimetres, as the number of cuts [illegible] the rod.

[illegible] number of cuts is increased [illegible] that the rod may break in 2 parts

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36. The proportionality established above will undoubtedly hold good here also - the required probability will be equal to the ratio of the same values, i.e. to the ratio of the length of the section CD to the length of the whole rod, because when the number of break points is infinite, the sum (or merging) of these points in the section CD will constitute its length, and in the section AB - the length of the rod.

Consequently, the probability in question can be represented in the form of a ratio of the section CD to the section AB, i.e. in the form of a ratio of geometrical quantities. When the number of all the favourable cases and the number of all the possible cases can be represented by geometrical quantities (segments, planes, volumes etc), the probability is conventionally termed as geometrical probability.

EXAMPLE 1. An aircraft bombs a target occupying an area which is taken to be a circle of radius r = 10m. In this operation, the bombs do not fall outside the limit represented by a circle of radius R = 50m and are distributed evenly within that circle. What is the probability of hitting the target with the release of one bomb?

Solution. The number of cases favouring a hit on the target, can be considered as proportional to the area occupied by the target, and the number of all the possible cases - proportional to the area upon which a bomb can fall. We will therefore find the required probability from the ratio of these areas

$$P = \frac{\pi r^2}{\pi R^2} = \frac{\pi 10^2}{\pi 50^2} = \frac{1}{25} = 0.04.$$

EXAMPLE 2. In shell fire on a blindage occupying a rectangular area of 6x4m, the target is covered by a single ellipse of dispersion, having main axes 24m and 3m. What is the probability of hitting the blindage with one shot, if the probability of hitting the ellipse is 0.2, assuming that the shells fall uniformly over the surface of this ellipse?

Solution. Let us first determine the probability of hitting the blindage, without considering the probability of hitting the ellipse, as the ratio of the areas of the blindage and the ellipse

$$p = \frac{6x4}{\pi x24x3} = \frac{1}{3x\pi}$$

The probability of hitting the blindage, taking into consideration the probability of hitting the ellipse, is, according to the theorem of multiplication

$$P = \frac{1}{3x\pi} \times 0.2 \approx 0.02.$$

In practice, when making artillery calculations, we have to employ constantly, ratios of geometrical quantities in estimating the probabilities. It is sufficient to state, that such probabilities, as probabilities of effecting a hit, obtaining an overshoot, locating the target, - all these are probabilities of geometrical order.

37. ## 9. Problems on probability and frequency of an event.

PROBLEM 1. In a recess holding 50 shells, which are not graded by weight, 10 are marked by two minuses, 25 by one plus and 15 by three pluses. What is the probability that a shell taken at random will have three pluses?

Answer. 0.3.

PROBLEM 2. In a closed box, there are 30 identical balls, differing only in colour, viz. blue and red. How many red balls are there in the box, if the probability of selecting, blindfold, one blue ball is equal to 0.4?

Answer. 18.

PROBLEM 3. In a closed box, there are white and black balls, differing only in colour. What is the total number of the balls contained in the box, if the probability of selecting, blindfold, one white ball, is 1/3, and the black balls 10?

Answer. 15.

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PROBLEM 4. Out of 28 thoroughly mixed dominoes, one domino is selected. What is the probability that the selected domino will bear the score of 6?

Answer. 1/7.

PROBLEM 5. There are 10 balls in a drum, 7 being white. We select 3 balls simultaneously. What is the probability of drawing 3 white balls.

Answer. 7/24.

PROBLEM 6. With 12 shots, 2 are hits. What is the frequency of the hits?

Answer. 1/6.

PROBLEM 7. The frequency of an overshoot is 0.3. What is the number of overshoots if the total number of shots is 20?

Answer. 6.

PROBLEM 8. In firing of shells, the frequency of a ricochet is 80%. What is the frequency of the non-ricochet?

Answer. 20%.

PROBLEM 9. From a box containing 8 white and 6 black balls, 4 white balls were taken consecutively. What is the frequency of appearance of the white ball?

PROBLEM 10. A box contains white, red and black balls. The probability of withdrawing, blindfold, 1 white ball is 1/2, 1 red ball - 1/3 and 1 black ball - 1/6. What is the probability of withdrawing a white or a black ball?

Answer. 2/3.

PROBLEM 11. Fire is directed on a target so that it is covered completely by the dispersion ellipse. What is the probability of obtaining an overshoot with 1 shot, if the mean trajectory passes in front of the target at 2P$?

Answer. 0.09.

PROBLEM 12. In a lottery of 500 tickets there are: 3 prizes of 100 roubles, 12 prizes of 50 roubles, 25 prizes of 20 roubles, 60 prizes of 10 roubles, and 100 prizes of 5 roubles. What is the probability of winning not more than 20 roubles with one ticket in this lottery?

Answer. 0.37.

PROBLEM 13. In firing on an area consisting of trenches made up of three lines, the probabilities of hitting I, II and III lines are 0.08, 0.15 and 0.1 respectively. What is the probability of hitting one of the trench lines with 1 shot?

Answer. 0.33.

PROBLEM 14. The probability of a hit with 1 shot is 0.12. What is the probability of a miss.

Answer. 0.88.

PROBLEM 15. In firing from a rifle onto a round target, the probability of a hit on ten is 0.15, on nine - 0.25, on seven - 0.2 and on six - 0.1. What is the probability of scoring with 1 shot not less than seven points, if the marksman places his bullets onto black circle?

Answer. 0.9.

PROBLEM 16. Probability of an undershoot, with 1 shot, is 0.3. What is the probability of obtaining 2 consecutive overshoots?

Answer. 0.49.

PROBLEM 17. The mean trajectories, corresponding to sights 80 and 82, occupying positions relative to the target, such that the probability of an overshoot in firing with the sight 80 is 0.15, and the probability of an undershoot when firing with the sight 82 is 0.2. What is the probability, after having made 2 shots on each one of these sights, of obtaining a double zone providing 2 markers (signs) to each limit?

Answer. .4684.

PROBLEM 18. The probability of locating the target in the section upon which the fire is directed, is 0.3. The probability of [illegible] hit with 1 shot under the condition that the target is located in the given section, is 0.2. Determine the probability [illegible]

[illegible]

Answer. .8536.

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38. PROBLEM 19. The probability of striking the target by ricochet shell splinters is 0.25. What is the probability of striking the target with 1 shot, if the ricochet are obtained on the average from 80% of the shells fired?

Answer. 0.2.

PROBLEM 20. In a closed box there are 5 white and 3 black balls. What is the probability of withdrawing, blindfold, consecutively 3 white balls, if the withdrawn balls are not returned into the box and differ only in colour?

Answer. 5/28.

PROBLEM 21. Out of 28 thoroughly mixed dominoes, 2 are taken at random. What is the probability that the first will bear the score of 5, and the second - 10?

Answer. 1/126.

PROBLEM 22. The mean trajectory passes at 1[illegible] in front of the target. What is the probability of obtaining an observation of the same sign when 2 shots are made?

Answer. 5/8.

PROBLEM 23. There are 3 boxes with balls differing only in colour. In the first box there are 3 white, 5 red and 2 black balls, in the second box, 6 white, 4 red and 5 black and in the third box, 1 white, 2 red and 3 black balls. Determine the probability of withdrawing from these boxes, simultaneously, 3 balls of the same colour. The balls are withdrawn blindfold, one from each box.

Answer. 22/225.

PROBLEM 24. The probability of finding the target on a sector covered by fire, is 0.4. The probability of making a hit with 1 shot, under the condition that the target is located in the sector upon which the fire is directed, is 0.1. What is the probability of destroying the target with 2 shots, if for the destruction of the target 1 hit is required?

Answer. 0.076.

39. PROBLEM 25. During firing from a rifle along one line on a round target, all the bullets fall in the black circle. Determine the probability of scoring with 2 shots, not less than 18 points, if the probabilities of hitting with one shot are: a ten - 0.15, nine - 0.25, eight - 0.2, seven - 0.2 and six - 0.1.

Answer. 0.09.

PROBLEM 26. What is the probability when throwing 2 dice simultaneously, of obtaining with the first throw, a total of points of not less than 10, and with the second throw, a total of points of not less than 9?

Answer. 5/108.

PROBLEM 27. In the first box there are 2 white and 3 black balls, and in the second - 4 white and 5 black balls. The balls differ only in colour. One ball is transferred, blindfold, from the first box into the second. Determine the probability of withdrawing, after this, a white ball from the second box.

Answer. 0.47.

PROBLEM 28. What is the probability of blowing up a tank which [illegible] mines, if the distances between mines are 5m, the [illegible] 10m, and the tank moves perpendicularly [illegible]?

Answer. [illegible].

PROBLEM 29. A right-angle triangle is drawn inside a circle. [illegible] that a point made at random inside the circle, [illegible].

Answer. [illegible].

PROBLEM 30. [illegible] consisting of 2 bridges: [illegible] target, falling evenly, within [illegible] which is a rectangle 100x20m. [illegible], to increase the use of this [illegible].

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Chapter 2.

RECURRENCE OF TRIALS.

10. Probability of a combination.

Let us suppose, that at some trial (for example, a shot) one of two opposite events must take place - an event A or an event B (for example, an undershoot or an overshoot), the probabilities of which remain unchanged when such a trial is repeated under identical conditions (for example, when a shot is repeated from on the same sight, with the same charge and the same type of shell etc).

If such a trial be repeated several times, we will then obtain some connection or, as we will term, some kind of combination made up of events A and B (for example, a combination of undershoots and overshoots).

In conducting artillery fire, it is often necessary to deal with combinations of undershoots and overshoots, hits and misses, failuresto explode and aerial explosions etc. Therefore, the knowledge of probabilities of such combinations has an extremely important practical and theoretical significance.

When determining the probabilities of combinations as probabilities of compound events, it might have been sufficient to make use of the theorems of addition and multiplication of probabilities, with which we are already acquainted, had not the combinations, taken as compound events, differed in one essential particular.

This particular consists in that a combination with a given ratio of opposite events has a series of variants, differing in succession (order) of appearance of these events. These variants are equally possible - any one of them may be obtained when the trial is repeated, and therefore, in order to determine the probability of a combination, it is necessary to know the number of its variants.

41. We will show this by an example. Let us assume, that we are interested in the probability of a combination consisting of two undershoots and two overshoots, and the order of their appearance is immaterial.

This combination has 6 variants, differing in order (succession) of occurrence of undershoots and overshoots.

Variant	Number of shots.			
	1	2	3	4
1	Undershoot	Undershoot	Overshoot	Overshoot
2	Undershoot	Overshoot	Undershoot	Overshoot
3	Overshoot	Undershoot	Overshoot	Undershoot
4	Overshoot	Overshoot	Overshoot	Undershoot
5	Overshoot	Undershoot	Undershoot	Overshoot
6	Overshoot	Overshoot	Undershoot	Undershoot

We will denote the probability of an undershoot with [illegible] by p, and the probability of an overshoot - by q. Then the probability of one definite (any one of the 6) variant, according to the theorem of multiplication of probabilities, will be [illegible]. Since all the 6 variants are equally possible, the probability [illegible], according to theorem of addition of probabilities, will be [illegible].

But the special feature of the combination, [illegible] can not be worked out this way - with the increase in the number of trials, the number of variants increases rapidly. Thus, for a combination in the form of equal [illegible] with 4 shots has 6 variants, then with [illegible] will have 20 variants, and with [illegible] becomes 70.

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41. It is quite obvious, that to determine the number of possible variants of a given combination using a method of sorting, as we had done above, is possible only for a very small number of repetitions of a trial. With a large number of repeated trials, such a method is very complex and, in the main does not guarantee that all the possible variants will be considered by us.

Therefore, it is necessary to find such a general formula, which will be true for the probability of any combination with any number of trials.

In order to derive such a formula, let us calculate the probabilities of all the possible combinations obtained with two, three and four trials, and draw a generalisation for the results of these calculations. For better demonstration we will make use of combinations consisting of undershoots and overshoots when a shot is repeated, and we will denote an undershoot by a "minus" sign and an overshoot by a "plus" sign.

42. When making calculations of probabilities of individual combinations we will make use of both the theorem of multiplication (probability of one variant), and the theorem of addition (probability of a combination), in the same way as was done earlier when determining the probabilities of combinations made up of 2 undershoots and two overshoots.

We will group these results in a general table.

Number of shots	Combinations	No. of a shot 1 2 3 4	Probabilities of variants	Probabilities of combinations
2	2 undershoots. 1 undershoot and 1 overshoot. 2 overshoots.			
3	3 undershoots. 2 undershoots and 1 overshoot. 1 undershoot and 2 overshoots. 3 overshoots.			
4	4 undershoots. 3 undershoots and 1 overshoot. 2 undershoots and 2 overshoots. 1 undershoot and 3 overshoots. 4 overshoots.			

We will add the probabilities found, for every number of shots, separately, remembering that (p+q) is the sum of probabilities of opposite events and is equal to unity.

43. It is easy to establish here, that the sum of probabilities of all the possible combinations with two shots represents a square of the sum of probabilities of an undershoot and an overshoot with 1 shot:

$$p^2+2pq+q^2 = (p+q)^2 = 1,$$

with three shots - a cube of the sum:

$$p^3+3p^2q+3pq^2+q^3 = (p+q)^3 = 1,$$

and with four shots - this sum to the fourth power:

$$p^4+4p^3q+6p^2q^2+4pq^3+q^4 = (p+q)^4 = 1.$$

We will note in the first place, that for every number of shots [illegible] undershoots and overshoots and their sum is unity. This confirms that we have definitely taken into

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43. consideration all the possible combinations and their variants for a given number of shots.

Now let us turn to the generalisation of the obtained results. Comparing these results we can assume, that with further increase in the number of repetitions of a trial, the probabilities of combinations will obey the same mathematical law, i.e. with any number (S) of repetitions of a trial under completely identical conditions, the sum of all the possible combinations made up of two opposite events, can be expressed by the sum of all the terms in the binomial expansion of Newton, in the power equal to the number of repetitions of the trial, the first power of this binomial having the form: $(p+q) = 1$, where p - probability of an event A, and q - probability of an event B, which is opposite to the first

$$p^S + Sp^{S-1}q + \frac{S(S-1)}{1 \cdot 2}p^{S-2}q^2 + \frac{S(S-1)(S-2)}{1 \cdot 2 \cdot 3}p^{S-3}q^3 + \ldots +$$

$$+ \frac{S(S-1)(S-2)\ldots(S-n+1)}{1 \cdot 2 \cdot 3 \ldots (n-1) \cdot n}p^{S-n}q^n + \ldots + \frac{S(S-1)(S-2)}{1 \cdot 2 \cdot 3}p^3q^{S-3} +$$

$$+ \frac{S(S-1)}{1 \cdot 2}p^2q^{S-2} + Spq^{S-1} + q^S = (p+q)^S = 1,$$

or, briefly,

$$P_0 + P_1 + P_2 + \ldots + P_n + \ldots + P_{S-1} + P_S = 1.$$

Here the letter n denotes the order of the term of this series if taken from zero.

The term $P_0 = p^S$ expresses the probability of occurrence of the event A, S times, the term $P_1 = Sp^{S-1}q$ expresses the probability of occurrence of the event A, (S-1) times and the event B, once; the term $P_2 = S(S-1)/1 \cdot 2\ p^{S-2}q^2$ expresses the probability of occurrence of the event A, (S-2) times and the event B, twice etc.

44. Let us note, that the indeces with p, correspond to the power integer with q, i.e. to the number of occurrences of the event B, from which this designation of probabilities of combinations virtually originates, commencing with P_0 and ending with P_S. These probabilities can be represented in the form:

$$P_0 = p^S q^0 \quad \text{and} \quad P_S = p^0 q^S,$$

which corresponds to the occurrence of the event B, zero times in the first case and S in the second.

It is obvious, that the term

$$P_n = \frac{S(S-1)(S-2)\ldots(S-n+1)}{1 \cdot 2 \cdot 3 \ldots (n-1) \cdot n}p^{S-n}q^n,$$

expresses the required probability of any combination of the (S-n)th occurrence of the event A, and of the (n)th occurrence of the event B; the coefficient $S(S-1)(S-2)\ldots(S-n)/1 \cdot 2 \cdot 3 \ldots (n-1) \cdot n$ being the number of variants of the given combination, and the product $p^{S-n}q^n$ - the probability of one variant.

To facilitate the employment of this formula we will effect some transformations: we will multiply the numerator and the denominator of the coefficient by the product of all the natural numbers, i.e. whole and positive numbers from (S-n) to unity, and we will substitute (S-n) by m, starting from the equation: $m = S-n$, whence $m+n = S$. Then

$$P_n = \frac{S(S-1)(S-2)\ldots(S-n+1)(S-n)(S-n-1)\ldots 3 \cdot 2 \cdot 1}{n(n-1)(n-2)\ldots 3 \cdot 2 \cdot 1(S-n)(S-n-1)\ldots 3 \cdot 2 \cdot 1}p^m q^n.$$

Now the numerator of the coefficient represents the product of all the natural numbers from S to 1. Such a product is usually termed a factorial (and is denoted by the symbol S!). In the denominator, however, we have a product of two factorials: n! and (S-n)!, but as S-n = m, then also (S-n)! = m! and we have the following final form:

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44.

$$P_n = \frac{S!}{m!n!}p^m q^n.$$

Let us note that 0! is taken as unity: which is derived from the following: we express S! by the equation $S! = S(S-1)(S-2)...3 \cdot 2 \cdot 1$. Substituting $(S-1)(S-2)...3 \cdot 2 \cdot 1$ by $(S-1)!$, we have:

$$S! = S(S-1)!, \quad \text{whence} \quad (S-1)! = \frac{S!}{S};$$

taking $S = 1$, we have:

$$(1-1)! = 0! = \frac{1!}{1} = 1.$$

The equation $0! = 1$ means that the general formula is true also for the probabilities P_0 and P_S which can be represented in the form:

$$P_0 = p^S = \frac{S!}{S!0!}p^S q^0 \quad \text{and} \quad P_S = q^S = \frac{S!}{0!S!}p^0 q^S \ *$$

45. If with S repetitions of the trial, the event A occurred m times, and the event B, n times, then this signifies that the frequency of the event A is m/S, and the frequency of the event B is n/S. Therefore, in determining the probability of a combination with a given ratio of opposite events which make up this combination, we thereby determine the probability of the frequency of these events. Thus, for example, in determining the probability of a combination consisting of 2 undershoots and 6 overshoots with 8 shots, we thereby determine the probability of the frequency of the undershoot, which is 2/8. Hence the expression "probability of a combination" will in future be often referred to as "probability of frequency".

EXAMPLE 1. Probability of an undershoot with 1 shot is $p = 1/3$. What is the probability of a combination of 2 undershoots and 4 overshoots with 6 shots?

Solution. According to the condition, the number of repetitions of the trial (shots) $S = 6$, the number of occurrences of one event (undershoot) $m = 2$, the number of occurrences of the event, opposite to the first (overshoot), $n = 4$.

If the probability of 1 undershoot is $p = 1/3$, then the probability of the overshoot is $q = 1-1/3 = 2/3$. Substituting these numerical values in the general formula we obtain:

$$P_n = \frac{S!}{m!n!}p^m q^n = P_4 = \frac{6!}{2!4!}\left(\frac{1}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^4 = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{1}{9} \cdot \frac{16}{81} =$$

$$= 15 \cdot \frac{16}{729} = \frac{80}{243}.$$

From analysis of this solution it is possible to establish, that:

1. The probability of one variant, i.e. of one definite succession of occurrence of undershoots and overshoots,

$$p^m q^n = \frac{16}{729}.$$

2. The number of all the possible variants is 15; these variants are given in the table below.

No. of a variant.	No. of a shot. 1 2 3 4 5 6	No. of a variant.	No. of a shot. 1 2 3 4 5 6	No. of a variant.	No. of a shot. 1 2 3 4 5 6
1		6		11	
2		7		12	
3		8		13	
4		9		14	
5		10		15	

* It is [illegible] symbolic notation and does not [illegible] would have otherwise [illegible].

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46. 3. The probability of this combination in the binomial series occupies the fifth place, as the index n for P is 4, and the numbering commences from zero.

Note. The condition for this example can be stated in a different form: what is the probability of obtaining the frequency of the undershoot equal to 1/3 (or the frequency of the overshoot, equal to 2/3), with 6 shots?

EXAMPLE 2. The probability of an explosion in air of an HE shell, in 1 shot q = 0.8. What is the probability of obtaining one third of failures to explode (probability of the frequency of a failure to explode equal to 1/3) with 24 shots?

Solution. We will denote: S = 24; m = 8; n = 16 and p = 1-0.8 = 0.2. Then

$$P_{16} = \frac{24!}{8!16!} 0.2^8 \times 0.8^{16}.$$

In this case it is necessary to use logarithms. From the logarithmic tables we find the logarithms of the factorials, and then

$$\begin{aligned} \log P_{16} = \quad & +\log 24! = 23.79271 \\ & -\log 8! = \bar{5}.39448 \\ & -\log 16! = \overline{14}.67938 \\ & +8 \log 0.2 = \bar{6}.40824 \\ & +16 \log 0.8 = \bar{2}.44944 \\ & \log P_{16} = \bar{2}.72425, \end{aligned}$$

whence $P_{16} = 0.053$. It is of interest to note, that the combination corresponding to this probability has 735471 variants.

If in the same problem we denote: S = 24; m = 16 and n = 8, then p = 0.8 and q = 0.2.

Hence

$$P_8 = \frac{24!}{16!\,8!} 0.8^{16} \times 0.2^8.$$

It is obvious, that the answer will be the same, as it does not matter which of the two opposite events will occupy the first place in the general formula and which - the second. It is important only that the indeces with p and q should be equal to the number of occurrences of corresponding events.

11. The most probable combination.

In order to visualise clearly the grouping of the probabilities of all the possible combinations, which obtain in recurrence of trials, let us calculate such probabilities, by taking different numerical notations for n, p and q:

$$\left(\tfrac{1}{3}+\tfrac{2}{3}\right)^6 = \text{[illegible]}\,14+0.0164+\text{[illegible]}\,08[illegible]3+0.2195+0.3292+0.2634+0.0878 = 1.0000;$$

$$\left(\tfrac{1}{3}+\tfrac{2}{3}\right)^{1[illegible]} = \text{[illegible]}\,0000+\text{[illegible]}\,0001+0.0005+0.0033+0.0149+0.0477+0.1113+0.1908+$$

$$+\text{[illegible]}\,0.1784+\text{[illegible]}\,7119+0.1277+0.0466+0.0077 = 1.0000;$$

47. $(0.6+0.4)^{1[illegible]} = \text{[illegible]}\,0000+0.0174+0.0638+\text{[illegible]}\,1419+0.2128+0.2270+0.1766+$

$$+\text{[illegible]}\,1134+\text{[illegible]}\,0424+\text{[illegible]}\,0085+0.0003+0.0000 = 1.0000.$$

For better comparison, we will express these probabilities, graphically, by setting off to an arbitrary scale, a number of even divisions along the abscissa to represent the numbers of combinations, and along the ordinate, the corresponding probabilities. In this way, the probability of each combination will be expressed by a corresponding ordinate, and the sum of the probabilities of all the possible combinations of given values - by the area of the graph (figs.13,

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[illegible], the latter are constructed to the same scales.

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47.

Fig.13.

After comparing the results of calculations and the plotted graphs, it is possible to make the following conclusions.

1. The smallest numerical values of the probability of combinations are at the beginning and the end, of each of these three binomial series, these values becoming negligibly small as the value of S increases.

Fig.14.

2. From the beginning of each series, the probabilities of combinations increase, reaching at some place in the series the greatest value, which corresponds to the so called most probable combination (most probable frequency).

48. It must be noted, that in some individual cases, the probability of the most probable combination may be either at the beginning or at the end of the binomial series:

$$(0.8+0.2)^3 = 0.512+0.384+0.096+0.008 = 1.0;$$

$$(0.1+0.9)^4 = 0.0001+0.0036+0.0486+0.2916+0.6561 = 1.0.$$

Fig.15.

The most probable combination and its probability are of great interest to us. The method of determining (selection) of the most probable combination from the series of probabilities of all the possible combinations is unsatisfactory due to being too cumbersome.

The place of the probability of the most probable combination in the binomial series is determined from the numerical vaues of S, p and q, and therefore it is necessary to find such a mathematical relationship between these values which will allow us to determine the place of the probability of the most probable combination and its value more simply.

We will denote the probability of the most probable combination by

$$P_n = \frac{S!}{m!n!}p^m q^n.$$

If P_n has a maximum numerical value, then the following inequalities are true:

$$P_n > P_{n-1} \quad \text{and} \quad P_n > P_{n+1},$$

or

$$\frac{S!}{m!n!}p^m q^n > \frac{S!}{(m+1)!\,(n-1)!}p^{m+1}q^{n-1}$$

$$\frac{S!}{m!n!}p^m q^n > \frac{S!}{(m-1)!\,(n+1)!}p^{m-1}q^{n+1}.$$

49. Carrying out possible simplifications in these two inequalities, and substituting m by S-n, we solve them with respect to n:

I) $\frac{1}{n}q > \frac{1}{S-n+1}p$:

$q(S-n+1) > pn;$
$qS-qn+q > pn;$
$qS+q > pn+qn;$
$qS+q > n(p+q);$
$n < qS+q.$

II) $\frac{1}{S-n}p > \frac{1}{n+1}q;$

$p(n+1) > q(S-n);$
$pn+p > qS-qn;$
$pn+qn > qS-p;$
$n(p+q) > qS-p;$
$n > Sq-p.$

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49. Substituting n by S-m in the two obtained inequalities, we can solve them with respect to m:

I) $S-m < Sq+q;$
$S-Sq-q < m;$
$m > S(1-q)-q;$
$m > Sp-q.$

II) $S-m > qS-p;$
$S-qS+p > m;$
$m < S(1-q)+p;$
$m < Sp+p.$

In this way, we obtain 2 pairs of inequalities which determine the limits within which are contained the numerical values of n and m.

$n > Sq-p;$
$n < Sq+q.$

$m > Sp-q;$
$m < Sp+p.$

Employing these inequalities, it is possible from the numerical values of S, p and q, to find the place of the most probable combination, and hence from the general formula its probability. One pair of inequalities only is sufficient, the other pair being used for checking.

EXAMPLE 1. Find the most probable combination of overshoots and undershoots with 6 shots and its probability, if the probability of the undershoot with 1 shot $p = 1/3$.

Solution. According to the condition, $S = 6$. Denoting the probability of the overshoot with 1 shot by $q = 1-1/3 = 2/3$ and substituting numerical values of S, p and q in the pair of inequalities, which determine the limits of n within which is contained the number of overshoots, we obtain:

$$n > 6 \cdot \frac{2}{3}-\frac{1}{3} \quad \text{and} \quad n < 6 \cdot \frac{2}{3}+\frac{2}{3},$$

from which

$$n > 3\frac{2}{3} \quad \text{and} \quad n < 4\frac{2}{3},$$

or

$$3\frac{2}{3} < n < 4\frac{2}{3}.$$

50. But as n can only be a whole number, so $n = 4$; then $m = 6-4 = 2$, i.e. the most probable combination consists of 2 undershoots and 4 overshoots, and its probability

$$P_n = \frac{S!}{m!n!}p^m q^n = \frac{6!}{2!4!}\left(\frac{1}{3}\right)^2 \cdot \left(\frac{2}{3}\right)^4 = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{1}{9} \cdot \frac{16}{81} =$$

$$= \frac{80}{243} = 0.3292.$$

Let us check the solution by determining m in the inequalities:

$$m > 6 \cdot \frac{1}{3}-\frac{2}{3} \quad \text{and} \quad m < 6 \cdot \frac{1}{3}+\frac{1}{3},$$

from which

$$1\frac{1}{3} < m < 2\frac{1}{3},$$

or

$$m = 2;$$

then

$$n = 6-2 = 4.$$

EXAMPLE 2. Find the most probable combination of overshoots and undershoots with 15 shots, if the probability of the overshoot with 1 shot is $q = 0.75$.

Solution. $S = 15$; $p = 0.25$; $q = 0.75$. We will determine m:

$$m > 15 \times 0.25-0.75 \quad \text{and} \quad m < 15 \times 0.25+0.25,$$

whence

$$m > 3 \quad \text{and} \quad m < 4.$$

As m is a whole number and can not be simultaneously greater than 3 and less than 4, it is obvious, that we have two most probable

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50. combinations: one consists of 3 undershoots and 12 overshoots, and the second - of 4 undershoots and 11 overshoots. In this example n and m have two numerical values which differ from each other by one:

$$m_1 = Sp-q = 3; \qquad n_1 = Sq+q = 12;$$
$$m_2 = Sp+p = 4. \qquad n_2 = Sq-p = 11.$$

Probabilities of two such combinations must be:

$$P_{n_1} = \frac{15!}{3!12!} x0.25^3 x0.75^{12} \approx 0.2252;$$

$$P_{n_2} = \frac{15!}{4!11!} x0.25^4 x0.75^{11} \approx 0.2252.$$

An indication that there are two most probable combinations is that the answers obtained are in whole numbers when solving the inequalities which give the numerical values of m and n.

It is necessary to stress, once again, that the probability of the most probable combination, consisting of two opposite events, is at the same time also the probability of the most probable frequencies of these events. Thus, in example 1, the most probable combination consisting of 2 undershoots and 4 overshoots, corresponds to the most probable frequencies of the undershoot 2/6 and of the overshoot 4/6,
51. and in the example 2 the most probable combinations consisting of 3 undershoots and 12 overshoots, and of 4 undershoots and 11 overshoots, correspond to the most probable frequencies of undershoot 3/15 and 4/15, and of overshoot 12/15 and 11/15.

From this the inequalities which determine the limits of numerical values m and n, allow us to make one special deduction.

We will represent these inequalities in the form:

$$Sq-p < n < Sq+q \qquad \text{and} \qquad Sp-q < m < Sp+p.$$

Dividing the inequalities by S and simplifying, we obtain:

$$q-\frac{p}{S} < \frac{n}{S} < q+\frac{q}{S} \qquad \text{and} \qquad p-\frac{q}{S} < \frac{m}{S} < p+\frac{p}{S}.$$

As p and q are always less than unity, then as S increases, the fractions p/S and q/S become smaller and smaller, and the most probable frequencies n/S and m/S tend, in value, to approach their corresponding probabilities q and p:

$$\frac{n}{S} \to q \qquad \text{and} \qquad \frac{m}{S} \to p,$$

and with infinitely large number of S (with $S \to \infty$):

$$\frac{n}{S} = q \qquad \text{and} \qquad \frac{m}{S} = p.$$

This means, that if some event A has a probability p, and an event B opposite to the first has a probability q, then with sufficiently large number of S repetitions of a trial, the greatest probability will possess the their frequencies m/S and n/S, which in value will be very close to the probabilities p and q to which they correspond.

This deduction confirms the well known law of high numbers which we had accepted earlier only on the basis of experimental data.

16. Curve of distribution of the probabilities of combinations.

For further deductions we must know in what way the probabilities (or the number of frequencies) change as the number of repetitions of a trial increases. We will investigate the nature of these changes on the following example.

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Let there be a box containing balls which differ in that 1 ball is taken out blindfold and

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51. Let us calculate the probabilities of all the possible combinations consisting of white and red balls with 10, 20 and 100 repetitions of the trial. We will limit our calculations to the fifth decimal point, ignoring probabilities of less than 0.000005. We will enter the calculations in the tables (tables 1-3).

52. S = 10.

Table 1.

Number of appearances of balls.		Probability P_i	Number of appearances of balls.		Probability P_i	Number of appearances of balls.		Probability P_i
White	Red		White	Red		White	Red	
0			4			8		
1			5			9		
2			6			10		
3			7					

S = 20 Table 2 *

S = 100 Table 3 *

53. For better demonstration we will express the calculated probabilities, graphically.

We commence with the graph when S = 10. In the rectangular system of coordinates (fig.16) we set off along the abscissa the numbers of divisions, of 1cm each, equal to the number of all the possible combinations, and along the ordinate - their corresponding probabilities to the scale of 0.1 for 1cm and we will write against every division the frequencies of that of the two opposite events, the probability of which is equal to q (in our example - the frequencies of appearance of the white ball, the probability of which is 3/4 = 0.75).

Fig.16.

This graph will give us a good representation of the relationship or, as we will term, distribution of probabilities of combinations (of frequencies) with a 10-fold repetition of the trial.

Let us turn to the graph of distribution of probabilities with S = 20. If we had preserved the same scales, this graph would have occupied 21cm along the abscissa axis, and along the ordinate axis, as a result of decrease in probabilities of individual combinations, the accuracy with which the graph would have been drawn would have been less, and the distribution of probabilities would have been less apparent. As the sum of probabilities of all the possible combinations at any value of S, is unity, let us reduce the size of the division along the abscissa, corresponding to the increase in S, by 2, i.e. to 0.5cm, and let us increase the scale along the ordinate also by 2, i.e. to 0.05 per 1cm (fig.17).

When constructing the graph with S = 100 we will reduce the size of a division along the abscissa and increase again the scale along the ordinate by 5 times (fig.18).

54.

Fig.17.

Fig.18.

55. In this way, the base of all graphs remains the same, the division along the abscissa in all cases is 1/2 of the base, and the value of the probability of individual combination (frequency) is expressed by the area of the rectangle with the base 1/2 and the height equal to that probability.

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* Translator's note. Headings in table 2 are the same as in table 1.

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55. Comparing the given tables 1, 2 and 3, and the graphs of the distribution of probabilities of combinations (frequencies), we see that with the increase in the number of repetitions of the trial:

- the areas of rectangles, representing the probabilities of combinations, decrease unevenly - the greater the value of S, the steeper is the decrease in the values of probabilities with respect to the greatest, and the less are the limits within which the probabilities have a significant value;
- the rectangle of the probability of the most probable combination (frequency), as we progress along the abscissa axis, approaches the position corresponding to the probability q = 3/4, i.e. confirming the assumption derived above, that when the number of S increases, the most probable frequency of the event tends to approach in value the probability of this event;
- the asymmetry of the graph becomes gradually less and its broken line begins to assume the shape of a curve which is symmetrical with respect to the ordinate corresponding to the probability of the most probable combination (frequency).

All this gives us grounds to conclude, that at infinite increase in the number of repetitions of the trial, i.e. at $S \to \infty$, the broken line of the graph of distribution of probabilities of combinations (frequencies), must change into a curve which is symmetrical about the ordinate, and the abscissa of which is q (fig.19). This curve is termed the curve of normal distribution of probabilities, or the Gauss curve. We will often have occasion to deal with this curve in connection with another part of the theory of probability - the theory of errors.

Fig.19.

56.

13. Law of high numbers.

The law of high numbers consists in that, with a sufficiently large number of repetitions of a trial, the frequency of an event differs very little in value from the probability of this event.

Two questions arise: what should be considered as a sufficiently large number of repetitions of a trial, and by how much can the frequency differ in value from its corresponding probability?

To answer these questions we will take some definite deviation (a total of deviations within definite limits) of frequency from the probability, and we will see in what way will the probability of such a deviation change when the number of repetitions of the trial is increased.

We will use for this purpose the distributions of probabilities of frequencies, already worked out by us, for different numerical values of S (see preceding section 12).

We will take the total of deviations of the frequency of appearance of a white ball, from its probability, within the limits of minus 0.1 and plus 0.1 with S = 20.

Of the frequencies, which can be obtained with such a number of repetitions of the trial, one of the following frequencies corresponds to the deviation we have selected: 13/20, 14/20, 15/20, 16/20 and 17/20, because the differences are

$$\frac{13}{20}-\frac{15}{20} = -0.1 \quad \text{and} \quad \frac{17}{20}-\frac{15}{20} = +0.1.$$

According to the theorem of addition, the probability of obtaining one of these frequencies (see table 2 in section 12)

$$P_{13-17} = 0.11240+0.16862+0.20233+0.18978+0.13278 = 0.80702.$$

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56. This means that with a sufficiently great number of repetitions of the trial, we will on the average, for every 20 such repetitions, have, approximately in 81 cases out of 100, a deviation in frequency of the appearance of a white ball from its probability, which (frequency) does not come outside the limits of ±0.1. In the remaining 19 cases, the frequency will have deviations greater than ±0.1.

Now let us determine the probability of this deviation when $S = 100$. It is obvious, that this probability will be equal to the probability of obtaining one of the frequencies within the limits of 65/100 and 85/100 (65/100-75/100 = -0.1 and 85/100-75/100 = +0.1). Adding these probabilities (see table 3 in section 12) we obtain:

$$P_{65-85} = 0.00702+0.01117+\ldots+0.01003+0.00566 = 0.9852,$$

57. i.e. in this case one of the frequencies within the indicated limits will be obtained approximately in 99 cases out of 100 and only 1 case will give a deviation exceeding ±0.1.

Let us increase the number of deviations of the trial to three hundred ($S = 300$), then the probability of the deviation of frequency within the limits of ±0.1 or the obtaining of one of the frequencies withing the limits of 195/300 and 255/300 will be:

$$P_{195-255} = 0.99993,$$

i.e. with $S = 300$ the frequency will practically remain within the set limits.

We had taken comparitively a large deviation of frequency from the probability. We will take another, smaller deviation: ±0.02, and we will work out the probabilities of this deviation for various values of S.

With $S = 100$, the frequencies within the limits of 0.73 and 0.77 correspond to the above deviation; the probability of this deviation

$$P_{73-77} = 0.436.$$

With $S = 300$, the probability of the same deviation

$$P_{219-231} = 0.568.$$

With $S = 1500$, the probability of the deviation

$$P_{1095-1155} = 0.92634.$$

And, finally, with $S = 6000$, the probability of the deviation

$$P_{4380-4620} = 0.99965.$$

In this way, when the limits of deviation of frequency from the probability are decreased, and the number of repetitions of the trial gradually increased, it is possible to arrive at a stage when the probability of such a deviation differs from unity by any small amount desired. It is obvious, that it is possible also to set out to obtain whatever small values of deviation we require.

This assumption is formulated in the form of the theorem of James Bernoulli.

<u>With a sufficiently large number of repetitions of the trial with the probability at any desired proximity (indefinitely close) to unity, it is possible to assert, that the frequency of the event will differ less from the probability of the event than any desired (indefinitely) small value previously taken.</u>

It is certain, that the assertion converse to the first will also be true, this assertion stating that with a sufficiently large number of repetitions of the trial, the frequency, obtained experimentally, can be taken as the probability of this event.

58. The theorem which is converse to the theorem of James Bernoulli,

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58. With a sufficiently large number of repetitions of the trial with the probability, at any desired proximity (indefinitely close) to unity, it is possible to assert, that the unknown probability of the event will differ from the frequency of this event obtained experimentally by any desired (indefinitely) small amount.

14. Probability of the event occurring once, twice, three times, etc.

In some cases, when the trials are repeated, the problem is set in such a way, that in order to solve it, it is sufficient for one of the two opposite events to take place, at least once, twice, three times or, which is the same, not less than once, twice, three times, etc.

For example, in artillery fire, depending on the number of hits, required to destroy a given target, the problem is solved when at least one or two or three etc. hits are obtained.

From this originates the necessity to determine the probabilities of the occurrence of the event in question, at least once, twice, three times, etc.

Let us show by an example, the method of calculating these probabilities, commencing with the probability of an occurrence of the event at least once.

Let us suppose, that we require to know the probability of obtaining at least one hit with 10 shots, if the probability of a hit with 1 shot, is $p = 0.1$.

Let us denote the probabilities of combinations made up of hits and misses by P_0, P_1, P_2, etc. and let us make up a table.

Sequence No.	Combination		Probability of combinations.
	Hits	Misses	

59. We see, from the table, that the conditions of our problem are satisfied by the first 10 combinations, as each one of them contains not less than one hit. The obtaining of any of these 10 combinations allows of a solution of the given firing problem.

Having denoted the probability of a hit at least once, by P_I, we obtain according to the theorem of addition of probabilities:

$$P_I = P_0 + P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9.$$

Substituting numerical values of probability of a hit and a miss with 1 shot, we determine the values P_0, P_1, P_2 etc. up to P_9 and we find these probabilities.

This problem can be solved very much simpler.

It is known, that the sum of probabilities of all the possible combinations when the trial is repeated, is equal to unity.

Consequently,

$$P_0 + P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 + P_{10} = P_I + P_{10} = 1,$$

whence

$$P_I = 1 - P_{10},$$

where P_{10} - probability of all misses.

Denoting the probability of a miss with 1 shot, by q, we obtain, according to the theorem of multiplication of probabilities:

$$P_{10} = q^{10}.$$

Consequently, the probability of at least once with 10 shots will,

$$P_I = 1 - q^{10}.$$

In general if we investigate the [illegible], the probability of all the [illegible] misses

$$= 1,$$

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59. whence

$$P_I = 1-q^{10}.$$

Knowing that $q = 1-p = 1-0.1 = 0.9$, we can conclude the solution of the problem:

$$P_I = 1-0.9^{10} = 1-0.3487 = 0.6513 \approx 0.65 = 65\%.$$

This means, that, in carrying out a large number of similar firings we will, on the average, in 65 cases out of 100, solve the given firing problem by using 10 shells. For the other 35 cases, a larger number of shells will be required for the solution of the problem.

Expanding this deduction, for any event and for any number of repetitions of the trial, we obtain:

$$P_I = 1-q^S = 1-(1-p)^S.$$

60. The probability of occurrence of one opposite event, at least once with S repetitions of the trial, is unity without the probability of occurrence of S times of the other opposite event.

EXAMPLE 1. Probability of an undershoot with 1 shot is 1/2. What is the probability of obtaining at least 1 undershoot with 4 shots?

Solution.

$$P_I = 1-(1-p)^S = 1-(1-\tfrac{1}{2})^4 = 1-\frac{1}{16} = \frac{15}{16} = 0.9375 \approx 0.94.$$

EXAMPLE 2. Determine the probability of a hit, at least once, with 24 shots, if the probability of a miss with 1 shot is 0.94.

Solution.

$$P_I = 1-0.94^{24};$$

$$24 \log 0.94 = \bar{1}.35512;$$

$$0.94^{24} = 0.2265;$$

$$P_I = 1-0.2265 = 0.7735 \approx 0.77.$$

EXAMPLE 3. Determine the probability of obtaining a hit, at least once, with 16 shots, if the probability of location of the target in the sector under fire, is $P_t = 0.25$, and the probability of obtaining a hit with 1 shot also with the condition that the target is located in the sector upon which the fire is directed, is $p = 0.15$.

Solution. If the target were in the sector under fire, then

$$P_I = 1-(1-0.15)^{16} = 1-0.85^{16}.$$

Using logarithms, we obtain:

$$16 \log 0.85 = \bar{1}.87772;$$

$$0.85^{16} = 0.0743,$$

Then

$$P_I = 1-0.0743 = 0.9257;$$

taking into consideration the probability of [illegible],
we obtain:

$$P_{It} = P_I \times P_t = 0.9257 \times 0.25 \approx 0.231 \approx 23\%.$$

In order to determine the probability of occurrence [illegible] of two opposite events, with [illegible] twice, we must subtract from unity, [illegible] combinations in which the event in question occurs twice, i.e. the prob[illegible]

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60. opposite to the first (probability of all misses), and the probability of occurrence of the first event only once (probability of only 1 hit).

Knowing that the probability of occurrence of the first opposite event takes place only once

$$P_{S-1} = Spq^{S-1},$$

we obtain:

$$P_{II} = 1-(P_{S-1}+P_S);$$

$$P_{II} = 1-(Spq^{S-1}+q^S).$$

61. Substituting in this formula the numerical values: S = 10; p = 0.1 and q = 0.9, we obtain the probability of a hit, at least twice, with 10 shots:

$$P_{II} = 1-(10\times0.1\times0.9^9+0.9^{10}) = 1-(0.3874+0.3487) =$$

$$= 1-0.7361 = 0.2639 \approx 26\%.$$

Arguing in the same way, we conclude, that the probability of occurrence of one out of two opposite events, at least 3 times, must be equal to unity minus the probabilities of combinations which contain this event less than 3 times, i.e.

$$P_{III} = 1-(P_{S-2}+P_{S-1}+P_S),$$

or

$$P_{III} = 1-\left[\frac{S(S-1)}{1\cdot 2}p^2q^{S-2}+Spq^{S-1}+q^S\right].$$

For any given value, say K times of the occurrence of the event

$$P_K = 1-(P_{S-K+1}+P_{S-K}+P_{S-K-1}+\ldots+P_{S-1}+P_S).$$

EXAMPLE 1. The probability of a hit with 1 shot, is p = 0.05. What is the probability of a hit, at least twice, with 40 shots?

Solution.

$$P_{II} = 1-(40\times0.05\times0.95^{39}+0.95^{40});$$

$$40 \log 0.95 = \bar{1}.1088;$$

$$P_{40} = 0.1285;$$

$$\log 40+\log 0.05+39 \log 0.95 = 1.60206+\bar{2}.69897+\bar{1}.13108 = \bar{1}.43211;$$

$$P_{39} = 0.2705;$$

consequently

$$P_{II} = 1-(0.2705+0.1285) = 1-0.399 = 0.601 \approx 60\%.$$

EXAMPLE 2. The probability of finding the target in a given sector, is $P_t = 1/2$. In the condition that the fire is directed on this sector, the probability of the undershoot with 1 shot, is 1/4. What is the probability of obtaining not less than 4 overshoots with 8 shots?

Solution. In the condition that the target is located in the given sector, the probability of obtaining not less than 4 overshoots

$$P_{IV} = 1-\left[\frac{8\cdot7\cdot6}{1\cdot2\cdot3}\cdot\left(\frac{3}{4}\right)^3\cdot\left(\frac{1}{4}\right)^5+\frac{8\cdot7}{1\cdot2}\cdot\left(\frac{3}{4}\right)^2\cdot\left(\frac{1}{4}\right)^6+8\left(\frac{3}{4}\right)\cdot\left(\frac{1}{4}\right)^7+\left(\frac{1}{4}\right)^8\right] = 1-\frac{1512+252+24+1}{65536} = 1-\frac{1789}{65536} = \frac{63747}{65536} \approx 0.9727.$$

But taking into consideration the probability of location of the target

$$P_{IV_t} = 0.9727\cdot\frac{1}{2} = 0.4864 \approx 49\%.$$

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62. 15. Reliability of firing.

All firing must be reliable, and as the ultimate problem of firing is the destruction of the target, this reliability is estimated by the probability of the destruction of the target.

Under the conditions of modern combat, when all types of defence installations and armoured vehicles are employed, many of the targets may be put out of action only by direct hits. Direct hits with best results are obtained also when firing at such targets, for the destruction (suppression) of which no direct hits are required (openly situated firing points etc.).

It is therefore usual to assess the reliability of firing by the probability of obtaining at least one, two, three, etc. hits, depending on the required number of hits for the destruction of the target.

In order to have a clear conception of what influences the reliability of firing, we will analise the formula of the probability of obtaining a hit, at least once, as one (formula) most frequently employed in artillery calculations.

This formula:

$$P_I = 1-(1-p)^S,$$

shows that the reliability of firing increases with the increase in the probability of obtaining a hit with 1 round and with the increase in the number of rounds used for the destruction of the target. It can be stated that:

- the greater the probability p of obtaining a hit with 1 round, the less, is (1-p);
- the greater the number of rounds expended S, the less is $(1-p)^S$, which, as a proper fraction, decreases with the increase in the power index.

Hence with the decrease of $(1-p)^S$ the value of $1-(1-p)^S$ naturally increases.

Having established this relationship, let us see in what way will the probabilities of at least one hit differ with the increase in numerical values of p and S. We will tabulate these calculations.

S	Probability of obtaining a hit, at least once.	Increase in probability of obtaining a hit, at least once.	
		When S increases.	When p increases.

63. We see that the increase in the probability of a hit with 1 round, by two, gives a greater increase in reliability of firing in comparison with the increase in the number of the rounds expended, also by two. Thus, with the increase in p from 0.05 to 0.1, with S = 10, the reliability of firing increases by 62%, and with the increase in S from 10 to 20 the reliability increases by 58% with p = 0.05. With the increase in p from 0.05 to 0.1, with S = 20, the reliability increases to 37%, and with the increase in S from 20 to 40, with the same p = 0.05, the reliability rises to 36% etc.

From this it is possible to make a deduction, that the increase in the reliability of firing obtained, is approximately the same with the increase in p, as with the increase in S by the same number of times. But this is an arithmatical and not an artillery deduction.

It is known, that the probability of a hit increases with the approach of the mean trajectory to the target, and reaches the maximum value when the mean trajectory coincides with the centre of the target; this is due to the uneven dispersion of rounds. In order to move the mean trajectory nearer to the target, fire for adjustment is made, and as such fire in comparison with the fire for the destruction of the target, requires considerably smaller expenditure of rounds, it is quite obvious, that the reliability of fire must be increased, in the first place, by raising the accuracy of the fire for adjustment, i.e. by means of increasing the probability of a hit.

This is why the rules for Firing with Increased Accuracy indicate, that the effectiveness of fire for the purpose of destruction

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63. of the target, is ensured by the accuracy of the fire for adjustment" (page 175) and that "the essential condition for a successful execution of the task with the minimum number of rounds, the following are required:

- the fire for adjustment direct on the target for the purpose of estimation of the sight settings for the destruction, must be as accurate as possible;
- continuous observation of explosions and timed corrections with the object of leading the mean trajectory to the selected point of the target, and the holding of this trajectory on that point" (page 195).

Artillerymen require to know, not so much the reliability of fire, as the expenditure of the rounds which ensures this reliability.

When estimating the reliability of fire, we have been taking numerical values for the quantity of rounds, and for the probability of obtaining a hit with 1 round, and we have been substituting these in the formula for the probability of obtaining at least 1, 2, 3 etc. hits. In order to find the quantity of rounds required for the destruction of the target, it is necessary to substitute, in this formula, the numerical values for the probability of obtaining a hit with 1 round, and the reliability of the fire.

What should be this reliability? It is usual to consider that the fire is sufficiently reliable, if the probability of the destruction of the target is close to 90%, and completely reliable, if the probability for the destruction is of the order of 96-98%. It is not expedient to take probabilities for the destruction of the target, very close to unity, as such an increase in reliability of fire will

64. call for a very great increase in the expenditure of rounds.

Let us commence with the determination of the number of rounds required for obtaining of at least one hit. For this, we write the formula, for the probability of obtaining at least 1 hit, in the form:

$$1-P_I = (1-p)^S,$$

and using logarithms we calculate with respect to S:

$$\log (1-P_I) = S \log (1-p);$$

$$S = \frac{\log (1-P_I)}{\log (1-p)}.$$

We will show, by examples, the manner in which this formula can be used.

EXAMPLE 1. What must be the expenditure of rounds for the reliability to be 0.95, if the probability of a hit with 1 round is 0.12, and that for the destruction of the target, one direct hit is required?

Solution.

$$S = \frac{\log (1-0.95)}{\log (1-0.12)} = \frac{\log 0.05}{\log 0.88} = \frac{\bar{2}.69897}{\bar{1}.94448} = \frac{-1.30103}{-0.05552} \approx 24 \text{ rounds.}$$

EXAMPLE 2. Determine the quantity of rounds required to obtain at least 1 hit, if the probability of a hit with 1 round is 0.07, and the reliability of fire must be equal to 98%.

Solution.

Substituting the numerical values of p and P_I:

$$S = \frac{\log (1-0.98)}{\log (1-0.07)} = \frac{\log 0.02}{\log 0.93} = \frac{\bar{2}.30103}{\bar{1}.96848} = \frac{-1.69897}{-0.03152} \approx 54 \text{ rounds.}$$

EXAMPLE 3. Calculate for the conditions given in example 2 the expenditure of rounds for reliability of fire of 0.99 and 0.999.

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$$S = \frac{\log 0.01}{\log 0.93} = \frac{\bar{2}.00000}{\bar{1}.96848} = \frac{-2.00000}{-0.03152} \approx 64 \text{ rounds;}$$

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64. $$S_2 = \frac{\log 0.001}{\log 0.93} = \frac{\overline{3}.00000}{1.96848} = \frac{-3.00000}{-0.03152} \approx 96 \text{ rounds.}$$

Comparing these results in the calculations of examples 2 and 3, we see, that with $P_I = 0.98$ the average expenditure of rounds for every 100 similar firings, is 5400, and also, if the rounds are not made up, in 2 instances the firing problem will not be satisfied. With $P_I = 0.99$ the average expenditure of rounds is 6400, i.e. it is increased by 1000 rounds, at the same time that the number of omitted firings is reduced only from two to one (with a given release of the rounds for one firing). Even a greater contrast will be apparent when the reliability of fire is increased - with $P_I = 0.999$: the expenditure of rounds increases further by 3200 rounds. All this, sufficiently clearly confirms, that a very great reliability of fire should not be undertaken..

65. The determination of the expenditure of rounds becomes considerably more complex, if for the destruction of the target, 2 or more direct hits are required; this is because it is not possible to set out the probability formula for at least 1, 2, 3 etc hits, in the form convenient for logarithmic calculation. In such cases, it is necessary to resort to a rather complex method of selection.

16. Problems on probability with recurrence of trials.

PROBLEM 31. What is the probability of obtaining a combination of 8 undershoots and 4 overshoots with 12 shots, if the mean trajectory passes 1Bδ beyond the target?

Answer. 0.0024.

PROBLEM 32. The probability of a hit with 1 shot, is 0.05. What is the probability of obtaining 2 hits with 30 shots?

Answer. 0.1886.

PROBLEM 33. The probability of explosion in air of a high explosive shell with 1 shot, is 3/4. Determine the probability of obtaining 1/3 failures to explode with 24 shots.

Answer. 0.1125.

PROBLEM 34. The probability of an undershoot with 1 shot, is 0.4. What is the probability of obtaining a ratio of signs of 1:3 with 8 shots?

Answer. 0.25.

PROBLEM 35. The probability of locating the target in the sector, upon which the fire is directed, is 0.6. What is the probability of obtaining 5 hits with 24 shots, if the probability of obtaining 1 hit is 0.15?

Answer. 0.08631.

PROBLEM 36. The probability of an undershoot with 1 shot is 0.7. Determine the most probable combination of signs with 60 shots.

Answer. 5 undershoots and 11 overshoots.

PROBLEM 37. The probability of a hit with 1 shot is 0.7. Determine the most probable frequency of obtaining a hit with 10 shots, and its probability.

Answer. Zero hits; 0.5596.

PROBLEM 38. The probability of an undershoot with 1 shot is 1/3. What is the probability of obtaining at least 1 undershoot with 6 shots?

Answer. 6560/6561.

PROBLEM 39. Determine the probability of obtaining at least 1 hit with 12 shots, if the probability of a hit with 1 shot is 0.3.

Answer. 0.9313.

PROBLEM 40. The probability of explosion in air of a high explosive shell with 1 shot is 0.75. What is the probability of obtaining at least 1 failure to explode with 4 shots?

Answer. 0.9960.

PROBLEM 41. The probability of [illegible] probability of obtaining [illegible] probability of [illegible]

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65. is directed, is 0.35?

Answer. 0.3229.

PROBLEM 42. The probability of a miss with 1 shot is 0.87. What is the probability of obtaining at least 2 hits with 8 shots?

Answer. 0.28.

66. PROBLEM 43. What is the probability of obtaining at least 2 hits with 30 shots, if the probability of a hit with 1 shot is 0.06?

Answer. 0.5445.

PROBLEM 44. The probability of a hit with 1 shot is 0.1. What is the probability of obtaining at least 3 hits with 20 shots?

Answer. 0.3232.

PROBLEM 45. Determine the number of rounds required to obtain at least 1 hit, if the reliability of fire must be 90% and the probability of a hit, with 1 shot, 0.3.

Answer. 6-7 rounds.

PROBLEM 46. What must be the expenditure of rounds for the reliability of fire to be 0.93, if it is necessary, for the destruction of the target, to obtain 1 direct hit, and if the probability of a hit with 1 shot is 0.15?

Answer. 16-17 rounds.

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Chapter 3.

MATHEMATICAL EXPECTATION.

17. Mathematical expectation of a variable quantity. Mathematical expectation of the sum and product.

In artillery practice we have frequently to deal with **variable** quantities, which, in repetition of analagous trial, assume particular, chance values.

Thus, for example, when the fire is carried out (in repeated fire) under identical conditions (system, sights, shell, charge etc.) the expenditure of the shells for the destruction of the target is the variable quantity, as in some cases the destruction will be obtained with the first lot of shells, and in others - the destruction will be obtained only after a considerable expenditure of shells. Here, in each individual case, the number of shells expended for the destruction of the target will be a chance quantity.

As the expenditure of shells, required for the destruction of the target, will be different for different cases, then in order to make an approximation of the number of shells required for one firing under given conditions, it is necessary to take some **average** (mean) expenditure of shells.

The question of an average expected value of the variable quantity is dealt with by the mathematical expectation of this quantity.

The mathematical expectation of the expenditure of shells answers the question relating to the average expected expenditure of shells for one firing when a large quantity of firings under identical conditions have been made; the mathematical expectation of the number of hits answers the question relating to the average expected number of hits for one firing when a large number of similar firings have been made; the mathematical expectation of the expenditure of time answers the question relating to the average expected expenditure of time for one firing etc.

By the mathematical expectation of a quantity is termed the sum of paired products consisting of all the possible individual values of a given quantity and their corresponding probabilities.

68. The mathematical expectation is denoted by the letter "a" or by the letters M.E. Thus the mathematical expectation of the quantity x will be expressed, in general form, by the expression

$$M.E.(x) = a = x_1p_1 + x_2p_2 + x_3p_3 + \ldots + x_ip_i + \ldots + x_Sp_S = \sum_1^S x_ip_i,$$

where $x_1, x_2, x_3, \ldots, x_S$ - individual values of the quantity x;
$p_1, p_2, p_3, \ldots, p_S$ - probabilities corresponding to these individual values.

Before commencing to prove that this expression is true, it is necessary to consider the mean value of the variable quantity obtained by trial, - the trial mean.

The derivation of the mathematical expression of the mean quantity obtained from the trial, will be shown in the following example.

Let us suppose, that with 22 firings, carried out under identical conditions, the following results were obtained:

4 firings with 5 hits;
7 firings with 4 hits;
5 firings with 3 hits;
3 firings with 2 hits;
1 firing with 1 hit;
2 firings with 0 hits.

Employing a simple arithmetical method of calculation, we find

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68. the mean number of hits for one firing, by dividing the total number of hits by the number of all the firings:

$$\frac{5x4+4x7+3x5+2x3+1x1+0x2}{22} = \frac{70}{22} = 3\frac{4}{22} = 3\frac{2}{11} \text{ hits.}$$

Let us put down the obtained result in a slightly different form, by carrying out all the calculations in the reverse order:

$$3\frac{2}{11} = 3\frac{4}{22} = \frac{70}{22} = \frac{5x4+4x7+3x5+2x3+1x1+0x2}{22} =$$

$$= 5x\frac{4}{22}+4x\frac{7}{22}+3x\frac{5}{22}+2x\frac{3}{22}+1x\frac{1}{22}+0x\frac{2}{22}.$$

In this expression the numbers 5, 4, 3, 2, 1 and 0 show the number of hits obtained in individual firing, i.e. they are individual (separate) values of the variable quantity we are discussing. The fractions however: 4/22, 7/22, 5/22, 3/22, 1/22 and 2/22 are frequencies of individual values of the same quantity.

69. The product 5x4/22 shows, that the firings with 5 hits in each, occur in 4 cases out of 22; the product 3x5/22 - that the firings with 3 hits in each, occur in 5 cases out of 22; the product, however, 0x2/22 - that only misses were obtained in 2 cases out of 22, etc.

Denoting the trial mean by X_{mean}, the individual values of the variable quantity by $x_1, x_2, x_3, \ldots, x_i, \ldots, x_S$, and the frequencies corresponding to these values by $r_1, r_2, r_3, \ldots, r_i, \ldots, r_S$, we obtain:

$$X_{mean} = x_1r_1+x_2r_2+x_3r_3+\ldots+x_ir_i+\ldots+x_Sr_S.$$

The mean value of the variable quantity (the trial mean) is equal to the sum of paired products of individual values of this quantity, and the frequencies which correspond to them.

EXAMPLE. Find the mean number of the score obtained with 30 throws of a playing die if it is known that:

with	7	throws	the	score	obtained	was 6
"	3	"	"	"	"	" 5
"	8	"	"	"	"	" 4
"	4	"	"	"	"	" 3
"	6	"	"	"	"	" 2
"	2	"	"	"	"	" 1.

Solution. The values of the score: 6, 5, 4, 3, 2 and 1 are individual values of this variable quantity, and 7/30, 3/30, 8/30, 4/30, 6/30 and 2/30 the frequencies corresponding to them. Then according to the general formula

$$\text{mean} = 6x\frac{7}{30}+5x\frac{3}{30}+4x\frac{8}{30}+3x\frac{4}{30}+2x\frac{6}{30}+1x\frac{2}{30} = \frac{115}{30} = 3\frac{5}{6} \text{ points.}$$

The arithmetical calculation provides the same result:

$$\text{mean} = \frac{6x7+5x3+4x8+3x4+2x6+1x2}{30} = \frac{115}{30} = 3\frac{5}{6} \text{ points.}$$

[illegible] the law of high numbers, it follows that as the number of [illegible] the trial increases, the frequency of the event tends, in value, to approach the probability of the event, and with sufficiently high number of repetitions of the trial, the frequencies [illegible] their corresponding probabilities.

[illegible] expression for the trial mean, it is possible to [illegible] individual values of the variable quantity, [illegible] probabilities. Then, denoting these probabilities [illegible] respectively, we obtain the expression for

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[illegible]

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70. If, when determining the mathematical expectation, all the possible individual values of the variable quantities are taken into account, then according to the statement of the theorem of addition of probabilities,

$$p_1+p_2+p_3+\ldots+p_i+\ldots+p_S = 1.$$

Comparing the expressions for the trial mean and the mathematical expectation, we can conclude that there exists, between the trial mean of any variable quantity and its mathematical expectation, the same relationship as between the frequency and the probability of an event, i.e. that as the number of trials increases, the mean value of the variable quantity will approach its mathematical expectation.

With a sufficiently high number of repetitions of the trial, the unknown mathematical expectation of the variable quantity will differ very little from the mean value of that quantity, obtained by trial. And conversely, if the mathematical expectation of the variable quantity is known, then with a high number of repetitions of the trial, the mean value of this quantity obtained, will differ very little from its mathematical expectation. Thus, the foregoing is actually the law of high numbers as applied to the trial mean and the mathematical expectation.

EXAMPLE 1. During firing from a pistol, at a circular target, the probability of hitting ten is 0.3, nine - 0.35, eight - 0.2, seven - 0.1 and six - 0.05.* Determine the mathematical expectation of the number of points scored with 1 shot.

Solution. In this example, the variable quantity is the number of points scored which is obtained with 1 shot. Individual values of this quantity are: 10, 9, 8, 7 and 6 points. The probabilities of these individual values are respectively: 0.3, 0.35, 0.2, 0.05 and 0.1.

Before commencing to determine the mathematical expectation of the number of points scored, let us check if in the conditions of the example all the possible individual values of the variable quantity in question, have been considered, and for this purpose we will add the probabilities of these values:

$$0.3+0.35+0.2+0.05+0.1 = 1.$$

This shows that all the individual values, according to the conditions of the example, have been taken into consideration and that, in firing, the possibility of a bullet falling outside the black circle of the target is precluded.

Having made certain of the above, we substitute the numerical values of x_i and p_i in the general formula:

$$M.E.(x) = a = 10\times0.3+9\times0.35+8\times0.2+7\times0.05+6\times0.1 = 8.7 \text{ points.}$$

This means that when a large number of similar firings are made, we can expect to obtain on the average 8.7 points for every shot.

EXAMPLE 2. The target is located within a region divided into sections A, B, C, D and E. For finding the target in the section A, 13 shells are expended on fire for adjustment; in section B - 11 shells; in section C - 8 shells; in section D - 10 shells; in section E - 14. Find the mathematical expectation of the expenditure of shells for the fire for adjustment, if the probabilities of finding the target in sections A, B, C, D and E are respectively: 0.08, .15, .45, .25 and 0.1 (fig.20).

71.

Fig.20.
(1) - Region of possible positions of the target.

* Translators note. According to calculations the probabilities

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71. Solution. Here the variable quantity is the quantity of shells expended on the fire for adjustment. Individual values of this quantity are 13, 11, 8, 10 and 12. According to the condition, the expenditure of shells on the fire for adjustment is determined by finding the target in one or other of the sections of the region of possible positions of the target. Hence the probabilities of finding the target in the different sections of this region, are at the same time also the probabilities of the corresponding individual values of the expenditure of shells; the sum of these probabilities, if all the possible positions of the target have been considered, must be equal to unity. And actually:

$$P_A+P_B+P_C+P_D+P_E = 0.05+0.15+0.45+0.25+0.1 = 1.$$

Then,

$$M.E.(x) = a = 13\times0.05+11\times0.15+8\times0.45+10\times0.25+12\times0.1 = 9.6 \text{ shells.}$$

EXAMPLE 3. Find the mathematical expectation of the sum of the points scored, which can be obtained when 2 playing dice are thrown once simultaneously.

Solution. The variable quantity is the sum of points scored. Individual values of this sum x_i and the probabilities P_i corresponding to these individual values, are given in the table.

Table.

The sum of the points scored, which is equal to 3, can be obtained as (2+1) or as (1+2), and the probability of this sum is independent of which die will have 2 points and which 1 point:

$$P_3 = \frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6} = \frac{2}{36}.$$

The sum of the points scored, which is equal to 7, can be obtained as (6+1) or as (5+2) or (4+3) or (3+4) or (2+5) or (1+6). The probability of this sum

$$P_7 = \frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6} = \frac{6}{36}$$

The sum of probabilities of all the possible individual values of the sum of the points scored

$$P_i = \frac{1}{36}+\frac{2}{36}+\frac{3}{36}+\frac{4}{36}+\frac{5}{36}+\frac{6}{36}+\frac{5}{36}+\frac{4}{36}+\frac{3}{36}+\frac{2}{36}+\frac{1}{36} = \frac{36}{36} = 1.$$

then

$$M.E.(x) = a = 2\times\frac{1}{36}+3\times\frac{2}{36}+4\times\frac{3}{36}+5\times\frac{4}{36}+6\times\frac{5}{36}+7\times\frac{6}{36}+8\times\frac{5}{36}+9\times\frac{4}{36}+$$

$$+10\times\frac{3}{36}+11\times\frac{2}{36}+12\times\frac{1}{36} = 7 \text{ points.}$$

72. The examples worked out show that the mathematical expectation of the variable quantity is expressed by any fractional or whole concrete number.

In these examples, the mathematical probabilities were expressed by positive numbers, but they can also be expressed by negative numbers, if the individual values of the variable quantity are negative.

We have discussed the case when, in the conditions of a given trial, the individual values were those of one variable quantity. The [illegible] of a trial is often determined by the addition of individual values of several such quantities. For example, in having tickets for several lotteries, we obtain, upon the completion of all [illegible], some total win, which is made up of wins on the tickets of different lotteries, in which a win in one lottery does not depend on [illegible].

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The question of the mathematical expectation of such a total win

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72. Approved For Release 1999/09/08 : CIA-RDP78-04861A000100010001-1 mathematical expectation of the sum of several independent variable quantities.

Let us derive the mathematical expectation in the first place of the sum of only two such variable quantities.

Let us suppose, that there are two independent variable quantities x and y, to the individual values of which: $x_1, x_2, x_3, \ldots, x_S$ correspond the following probabilities: $p_1, p_2, p_3, \ldots, p_S$; and in the same way to $y_1, y_2, y_3, \ldots, y_S$, correspond the probabilities $q_1, q_2, q_3, \ldots, q_S$.

Any individual value of the quantity x can coincide with any individual value of the quantity y. Such coincidences will therefore give the individual values of the sum:

$$\begin{array}{cccc} x_1+y_1 & x_1+y_2 & \ldots. & x_1+y_S \\ x_2+y_1 & x_2+y_2 & \ldots. & x_2+y_S \\ \ldots & \ldots & \ldots. & \ldots \\ \ldots & \ldots & \ldots. & \ldots \\ x_S+y_1 & x_S+y_2 & \ldots. & x_S+y_S \end{array}$$

The sum (x_1+y_1) can be obtained only when the quantity x assumes the value x_1, and the quantity y - the value y_1, and as the probability of the value x_1 according to the condition is equal to p_1, and the probability of the value y_1 is equal to q_1, then from the theorem of multiplication, the probability of such a sum will be p_1q_1. In the same way the probability of the sum (x_2+y_2) will be p_2q_2, and of the sum (x_3+y_4) - p_3q_4 etc.

Substituting the individual values of the sum and their corresponding probabilities in the general formula for mathematical expectation, we obtain:

$$M.E.\ (x+y) = (x_1+y_1)p_1q_1+(x_1+y_2)p_1q_2+\ldots+(x_S+y_S)p_Sq_S.$$

73. Opening brackets, we make comparitively simple transformations, using equations

$$x_1p_1+x_2p_2+\ldots+x_Sp_S = M.E.\ (x) = a,$$
$$x_1q_1+y_2q_2+\ldots+y_Sq_S = M.E.\ (y) = b$$

and remembering that

$$p_1+p_2+\ldots+p_S = 1,$$
$$q_1+q_2+\ldots+q_S = 1,$$

after which we obtain

$$M.E.(x+y) = a+b = M.E.(x)+M.E.(y),$$

i.e. the mathematical expectation of the sum of two independent variable quantities is equal to the sum of mathematical expectations of these quantities.

This deduction will be true also for any number of independent variable quantities, as it is easy to take the sum of several quantities, first in the form of the sum of only two quantities and then to solve the problem successively.

For example, if there are three independent variable quantities x, y and z, the mathematical expectations of which are respectively equal to a, b and c, then

$$M.E.(x+y+z) = M.E.(x+y)+M.E.(z) = M.E.(x)+M.E.(y)+M.E.(z) = a+b+c.$$

<u>The mathematical expectation of the sum of several independent variable quantities is equal to the sum of mathematical expectations of these quantities.</u>

EXAMPLE. Firing is conducted on three settings of the sight: 80, 81 and 82. Determine the mathematical expectation of the number of hits, if the probabilities of a hit with one shot corresponding to the sights 80, 81 and 82 are: 0.05, 0.15 and 0.1 and if the instructions are to fire 8 shots on sights 80 and 82, and 12 shots on the sight 81.

<u>Solution</u>. The mathematical expectations of the number of hits in firing, are

on the sight 80 - $a_1 = 8 \times 0.05 = 0.4$ hits,
on the sight 81 - $a_2 = 12 \times 0.15 = 1.8$ hits,
on the sight 82 - $a_3 = 8 \times 0.1 = 0.8$ hits.

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73. Thus the sum of these mathematical expectations, will be

$$a_1+a_2+a_3 = 0.4+1.8+0.8 = 3 \text{ hits},$$

which answers the question on the mathematical expectation of the number of hits of the firing.

If the individual values of several independent variable quantities are not added, but are multiplied, it is possible to refer to this as the mathematical expectation of the product of these quantities.

The mathematical expectation of the product of several independent variable quantities is equal to the product of the mathematical expectations of these quantities.

$$M.E.(x \times y \times z) = M.E.(x) \times M.E.(y) \times M.E.(z).$$

74. ## 18. A particular case of mathematical expectation.

In the case when the variable quantity at one trial can have only two particular values:

$$x_1 = 1 \quad \text{and} \quad x_2 = 0,$$

these two values correspond to the opposite events (for example, undershoot and overshoot, hit and miss). The expression for the mathematical expectation for such a variable quantity can be considerably simplified.

Let us derive this simplified expression as applied to the more important case for the artilleryman - the mathematical expectation of the number of hits. Let us note, that we will be discussing only direct hits and only with impact firing.

With one shot can be obtained either a hit or a miss, i.e. the particular values of the number of hits: $x_1 = 1$ (a hit) and $x_2 = 0$ (a miss).

Let us denote the probability of a hit with 1 shot by p and the probability of a miss by q, i.e. the probabilities of the above particular values, are: $p_1 = p$ and $p_2 = q$.

Then according to the general formula

$$M.E.(x) = a_1 = x_1p_1+x_2p_2 = 1 \times p+0 \times q = p \text{ of the hits.}$$

Consequently, with 1 shot, the mathematical expectation of the number of hits is numerically equal to the probability of a hit with 1 shot. This equality is only numerical because the probability of a hit is an abstract quantity, but the mathematical expectation is a concrete quantity.

Problem. Find the mathematical expectation of the number of hits with 1 shot if the probability of a hit with 1 shot is p = 0.15.

Solution. p = 0.15; a = 0.15 hits.

Such a numerical equality is correct only for 1 shot. For 2 and more shots this equality disappears. Actually, the probability of obtaining all hits, for example, with 3 shots, is $pxpxp = p^3$, i.e. less than with one hit ($p^3 < p$), and the mathematical expectation of the number of hits with 3 shots will doubtlessly be greater than with one. In order to determine by what amount this will be, we will calculate the mathematical expectation of the number of hits with 2 and 3 shots.

With two shots, the particular values of the number of hits (x_i) and the corresponding probabilities (p_i) will be as follows:

Combination of hits and misses	x_i	p_i
2 hits.	2	p^2
1 hit and 1 miss.	1	$2pq$

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. . . putting the data in this table into the general formula for

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75. $M.E.(x) = a_2 = x_1p_1+x_2p_2+x_3p_3 = 2\times p^2+1\times 2pq+0\times q^2 = 2p^2+2pq =$
$= 2p(p+q) = 2p\times 1 = 2p$ hits,

i.e. with 2 shots, the mathematical expectation of the number of hits is numerically equal to twice (doubled) reliability of a hit with 1 shot.

With three shots, the particular vaues of the number of hits and their corresponding probabilities, are:

Sequence No.	Combinations of hits and misses.	x_i	p_i
1	3 hits.	3	p^3
2	2 hits and 1 miss	2	$3p^2q$
3	1 hit and 2 misses.	1	$3pq^2$
4	3 misses.	0	q^3

Then according to the general formula

$$M.E.(x) = a_3 = p_1x_1+p_2x_2+p_3x_3+p_4x_4 = 3\times p^3+2\times 3p^2q+1\times 3pq^2+0\times q^3 =$$

$$= 3p^3+6p^2q+3pq^2 = 3p(p^2+2pq+q^2) = 3p(p+q)^2 = 3p\times 1 = 3p \text{ hits},$$

i.e. the mathematical expectation of the number of hits with 3 shots is numerically equal to three times the probability of a hit with 1 shot.

Comparing the results of these calculations:

$a_1 = p$ hits;
$a_2 = 2p$ hits;
$a_3 = 3p$ hits,

we can assume that for any number of shots S, the mathematical expectation of the number of hits, is

$$M.E.(x) = a_S = Sp \text{ hits}.$$

EXAMPLE 1. Find the mathematical expectation of the number of hits with 16 shots, if the probability of a hit with 1 shot is 0.15.

Solution. $M.E.(x) = a_{16} = Sxp = 16\times 0.15 = 2.4$ hits.

This means, that with a large number of firings, in the conditions given in our example, the mean expected number of hits per firing is 2.4 hits, although individual firings give different numbers of hits - from 0 to 16.

EXAMPLE 2. The probability of finding the target in the sector under fire, is 0.3. Determine the mathematical expectation of the number of hits with 12 shots if the probability of a hit with 1 shot, in the condition that the target is in the sector upon which the fire is directed, is 0.08.

76. Solution. The probability of a hit with 1 shot taking into consideration the probability of finding the target

$$P = P_t \times p = 0.3\times 0.08 = 0.024.$$

Then

$$M.E.(x) = a_{12} = 12\times 0.024 = 0.288 \text{ hits}.$$

Taking a definite mathematical expectation of the number of hits, it is possible to calculate the mean expenditure of shells which is required for the solution of the firing problem.

$$a = S \times p,$$

then

$$S = \frac{a}{p}.$$

EXAMPLE. Determine the mean expenditure of shells, if for the destruction of the target, 3 direct hits are required, and if the probability of a hit with 1 shot is 0.05.

Solution.

$$S = \frac{a}{p} = \frac{3}{0.05} = 60 \text{ shells}.$$

This means, that under [illegible] conditions given in our example, in order to obtain 3 hits it is necessary to expend an average of 60 shells per 1 firing.

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76. **19. Efficiency of the firing.**

Every firing must be sufficiently reliable, but that alone does not meet all the requirements of the firing; the firing must be efficient, i.e. the firing problem must be resolved with a minimum expenditure of rounds and time.

If in two different methods of firing, the mathematical expectation of the number of hits in the first method is greater than in the second, then the first method of firing is more advantageous, more efficient than the second.

Let us suppose, that in carrying out a firing with the first method, the mathematical expectation of the number of hits is 2, and in the second method the number of hits is 1, and also that the requirements of the firing task call for 1 direct hit. It is obvious, that with the same expenditure of rounds, it is possible, using the first method of firing to perform 2 such firing tasks, as for each firing the expenditure of rounds will be a half.

Consequently, the mathematical expectation of the number of hits characterises the efficiency of artillery fire.

In setting out rules for artillery fire, and in ascertaining the most advantageous general methods of conducting the firing, it is
77. necessary to consider, together with the reliability of firing, the efficiency of the latter. Only by comparing the reliability and the efficiency of the firing, is it possible to arrive at the correct deductions.

EXAMPLE 1. Find the reliability and efficiency of firing with 40 shots, if the probability of a hit with 1 shot is 0.1 and if for the destruction of the target, 2 direct hits are required.

Solution. We will determine the reliability of firing as the probability of at least two hits.

$$P_{II} = 1-(Spq^{S-1}+q^{S}) = 1-(40x0.1x0.9^{39}+0.9^{40}) = 1-0.0805 \approx 92\%,$$

i.e. with a given expenditure of shells for 1 firing, the problem is solved in 92 cases out of 100; in the remaining 8 cases the expenditure of shells will require to be increased.

We will find the efficiency of firing as a mathematical expectation of the number of hits:

$$a = S \times p = 40x0.1 = 4 \text{ hits},$$

i.e. for every firing we can expect to obtain an average of 4 hits.

Here we seem to have come up on an obvious contradiction - it is required to obtain 2 hits, and yet in 1 firing we can expect to obtain 4 hits.

If, for the equation of the mathematical expectation of the number of hits with the required (according to the condition) quantity of hits, we decrease the expenditure of shells to a half, i.e. we expend on the average only 20 shells for the firing, then the reliability of the firing will drop. Actually,

$$P_{II} = (1-20x0.1x0.9^{19}-0.9^{20}) = 1-0.3917 = 0.6093 \approx 61\%,$$

which is obviously not enough.

EXAMPLE 2. Determine the reliability and efficiency of firing with 30 shots, if the probability of a hit with 1 shot, is 0.2, and for the destruction of the target 1 direct hit is required.

Solution.

The reliability of firing $P_1 = 1-0.8^{30} = 0.9988$.

The efficiency of firing a = 30x0.2 = 6 hits.

It is obvious, that the reliability of firing is too high and can be reduced; reduction of reliability will, in its turn, lead to the reduction in the expenditure of shells during firing. For example, with reliability of firing of 96%,

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17. $s = \frac{\log(1-0.967)}{\log(1-0.2)} = \frac{\log 0.04}{\log 0.8} = \frac{2.60206}{1.90309} = \frac{-1.39794}{-0.09691} = 15$ shells,

and the efficiency of firing

$$a = 15 \times 0.2 = 3 \text{ hits.}$$

If, the conditions of firing allow a lower reliability, for example 90%, then

$$s = \frac{\log 0.1}{\log 0.8} = \frac{-1.00000}{-0.09691} = 11 \text{ shells;}$$

$$a = 11 \times 0.2 = 2.2 \text{ hits.}$$

In this way, to perform the set firing task it is possible to confine the expenditure to an average of 15 shells for 1 firing. Further reduction in expenditure of shells for the firing is impracticable, as the economy in shells obtained is quite small (15-11 = 4 shells), whilst the reliability of firing drops steeply by 6%.

20. Problems on mathematical expectation.

PROBLEM 47. 40 shots were fired on a group of targets, resulting in the following destruction:

- with 6 shots, 3 targets;
- with 13 shots, 4 targets;
- with 12 shots, 5 targets;
- with 9 shots, 6 targets.

Determine the average number of targets destroyed with 1 shot.

Answer. 4.6 targets.

PROBLEM 48. In firing with shrapnel shells, the probability of obtaining craters, 30cm deep is 0.2; 40cm deep - 0.4; 50cm deep - 0.3; 60cm deep - 0.1. What is the mathematical expectation of the number of destroyed targets with 1 shot, if it is known, that when craters of depth of 30, 40, 50 and 60cm are obtained, an average of 8, 7, 5 and 3 targets are destroyed respectively?

Answer. 6.2 targets.

PROBLEM 49. Determine the mathematical expectation of the total number of points on one domino taken at random.

Answer. 6 points.

PROBLEM 50. What is the mathematical expectation of the number of direct hits with 36 shots, if the probability of a hit with 1 shot is 0.07?

Answer. 2.52 hits.

PROBLEM 51. The probability of finding the target in the zone upon which the fire is directed, is 0.6. Determine the mathematical expectation of the number of hits with 16 shots, if the probability of a hit with 1 shot, with the condition that the target is located in the zone of fire, is 0.15.

Answer. 1.5 hits.

PROBLEM 52. Determine the average expenditure of shells in firing, if the probability of a hit with 1 shot is 0.04, and for the destruction of the target 2 direct hits are required.

Answer. 50 shells.

PROBLEM 53. In firing from a pistol upon a circular target, the probability of a hit on ten is 0.2, on nine - 0.3, on eight - 0.25, on seven - 0.15 and on six - 0.1. What is the mathematical expectation of the number of points scored with 5 shots?

Answer. 41.75 points.

PROBLEM 54. Determine the reliability and efficiency of firing with 16 shots, if the probability of a hit with 1 shot is 0.12, and for the destruction of the target 1 direct hit is required.

Answer. 0.871; 1.92 hits.

PROBLEM 55. Determine the average expenditure of shells on one firing and the reliability of firing with such an average expenditure of shells, if the probability of a hit with 1 shot is 0.25 and for the destruction of the target 2 direct hits are required.

Answer. 8 shells; 0.633

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Chapter 4.

PROBABILITY OF HYPOTHESES.

21. On probabilities of hypotheses.

When investigating some occurrences it is often necessary to deal with events, the probabilities of which can assume various numerical values depending on the conditions (circumstances) in which these events take place. In other words, depending on these conditions (circumstances), different probabilities can correspond to one and the same event.

As a very discriptive and at the same time a frequently encountered example in artillery, we can refer to the probabilities of an undershoot and an overshoot in impact firing. These probabilities, as we know, have various numerical values depending on the mean trajectory with respect to the target. Thus, if the target is located at the point A (fig.21), the probability of the undershoot is 0, and the probability of the overshoot, 1.0; when the target is situated at the point B, the probability of the undershoot 0.02 and the probability of the overshoot 0.98; with the target situated at the point C, the probabilities of the undershoot and overshoot are respectively 0.09 and 0.91 etc.

Fig.21.
(1) - Mean trajectory;
(2) - Probability of undershoot;
(3) - Probability of overshoot.

Knowing the condition in which the event with which we are concerned takes place, we can find the particular value of the probability of this event, i.e. if the position of the target with
80. respect to the mean trajectory is known, we can find the particular value of the probability of the undershoot or overshoot. But the question can be put in a different way: according to the particular value of the probability of an event, we can determine the concrete conditions in which this event takes place; thus, from the particular value of the probability of the undershoot or overshoot we can judge the actual position of the target with respect to the mean trajectory. The question set in this way has a specially important significance, because if the position of the target with respect to the mean trajectory is known, we can calculate the correction necessary to superpose the mean trajectory on the target and thereby solve the basic problem of artillery fire.

If it were possible to carry out, under the same circumstances, a sufficiently large number of shots - 100, 200, 300, the solution of this problem would have been very simple. Having calculated from the result of such a firing, the frequency of the undershoot or the overshoot and having equated, according to the law of high numbers, (using the theorem converse to the theorem of James Bernoulli) the numerical value of the frequency obtained, to its corresponding probability, to determine from this probability the position of the target, in which we are concerned, with respect to the mean trajectory. In practice there can be no question of such a number of shots in one firing. As a rule, in firing, it is necessary to judge the position of the target with respect to the mean trajectory and to make the solution for the correction of the sights, from [illegible] observations, frequently even from one observation. Owing to the considerable dispersion in firing, different [illegible] with respect to the target can [illegible] position, sign, of an individual random explosion

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undershoot and three overshoots have been obtained, then the mean trajectory can in such a case pass beyond the target, in front of the target and even through the target (fig.22).

Fig.22.
(1), (2), (3) - Mean trajectory;
(4), (5), (6) - Target.

Before making a decision to reduce, to increase or to leave unaltered the sight setting, a natural question suggests itself: what is then the probability of each one of these positions, and which of these positions is the most probable? This question is answered by the theorem of hypotheses.

The basis of this theorem, which is applicable to firing from the observation of signs of explosions, consists in the following.

Knowing that the target can occupy different positions with respect to the mean trajectory, we take, even prior to the shot, a series of suppositions (hypotheses) regarding these positions of the target. Each such hypothesis is estimated in terms of its probability. Numerical values of these probabilities are derived from the manner of working out the original data.

Having fired a shot and having obtained one or other observation (sign or signs), we now find, from the probabilities of hypotheses on the position of the target before the shot, and from the probabilities of obtaining of the observed sign (or signs) at different positions of the target, new probabilities of hypotheses on the position of the target after the shot. Further, taking as the basis, the most probable, after-the-shot hypothesis, we take the appropriate decision regarding the correction of the sight setting.

To show, how the result of the trial alters the reasoning on the probabilities of hypotheses, we make use of the following example.

In 2 closed boxes there are balls differing in colour: in the first box - 3 white and 2 black and in the second - only white balls. One ball is transferred, blindfold, from the first box into the second.

Regarding which ball has been transferred, we can make two suppositions, two hypotheses:

- a white ball has been transferred - probability of this hypothesis is 3/5;

- a black ball has been transferred - probability of this hypothesis is 2/5.

Let us suppose, that after this, one ball is withdrawn blindfold, out of the second box, and that this ball is found to be black.

It is quite obvious, that this could have occurred only in the case when a black ball had been transferred from the first box into the second.

The probability of the second hypothesis, prior to the trial, was 2/5, and after the trial it became unity. The probability of the first hypothesis has changed from 3/5 to zero. The result of the trial has fundamentally changed the reasoning on the probabilities of hypotheses.

Let us return to the probabilities of hypotheses regarding the position of the target. Having calculated the probabilities of all the hypotheses, which we have made, about the position of the target with respect to the mean trajectory, we will obtain a series of suppositions of the posible positions of the target, to every one of which
82. corresponds its own probability, i.e. we will obtain, as it is termed in the theory of firing, - disposition of the target.

The essential difference of the theorem of hypotheses in comparison with other theorems, which have been applied up to now, consists in that the theorem of hypotheses does not fully provide an unambiguous answer to the question on the probability of an event. It gives a series of answers, a series of probabilities, and only the totality of these answers, [illegible] hypotheses, we arrive at a more practicable [illegible]

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82. Appropriate to make a special emphasis, viz, that this solution will be true only in conditions governing a given circumstance, only for a given result of the trial in firing - only for a given observation (sign).

Having obtained the first observation and having changed the sight settings and continuing the firing, we will be obtaining new observations. Each such new observation will change the disposition of the target. With each new observation, the number of hypotheses will be changing (disposition of the target will be narrowing) and at the same time the probabilities of the most probable hypotheses will be increasing. In the final reckoning, with comparitively small expenditure of shells, we determine the position of the target with respect to the mean trajectory, with an accuracy which ensures both the reliability and the efficiency of the firing for destruction of the target.

In contrast to the law of high numbers, the theorem of hypotheses gives a basis for the judgement of probability of an event we are investigating, from the results of a small number of trials. It is due to this, that this theorem finds a very wide application in the theory of artillery firing.

22. Theorem of hypotheses.

To derive the numerical expression (general formula) of the probability of a hypothesis, we will make use of the following particular example.

Let us suppose, that there are 3 groups of closed boxes which are completely similar in appearance, weight and shape. These boxes contain balls which differ from each other only in colour.

The number of boxes in the first group is 3, with 2 white and 4 black balls in each box.

The number of boxes in the second group is 2, with 4 white and 3 black balls in each box.

The number of boxes in the third group is 4, with 3 white and 5 black balls in each box.

The boxes of different groups are distributed in a haphazard manner; one box is taken at random and a ball is withdrawn, blindfold, from that box, and it is found that this ball is white.

Let us determine what is the probability that the withdrawn ball came from the first group of boxes?

83. Before trial there were white balls in all the boxes. Consequently, we could have withdrawn a white ball from a box of the first, second or third group. Hence, it is possible to make three suppositions, to form three hypotheses regarding to which group belonged the box from which the white ball was withdrawn.

The appearance of a white ball from the box of the first group under conditions in which the trial was carried out, is a compound event, as it consists of the coincidence of selection of a box in the first group, and the appearance of a white ball from that box.

There are altogether 9 boxes, 3 of which belong to the first group. Therefore, the probability of selection of the box of the first group, i.e. the probability of the first hypothesis prior to the trial,

$$p_1 = \frac{3}{9}$$

In each box of the first group there are 2 white and 4 black balls. From this, the probability of appearance of a white ball from the box of the first group, considering that the box of this particular group has already been selected, i.e. the probability of the event from the first hypothesis

$$p_1 = \frac{2}{6}$$

According to the theorem of multiplication, the probability of appearance of a white ball from the box of the first group, taken as the probability of coincidence of appearance of the box of the first group, with the appearance of a white ball from the box, viz. of

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83. $$P_1 p_1 = \frac{3}{9} \cdot \frac{2}{6}.$$

By similar reasoning we find that the probability of appearance of a white ball from the box of the second group, is

$$P_2 p_2 = \frac{2}{9} \cdot \frac{4}{7},$$

and the probability of appearance of a white ball from the box of the third group, is

$$P_3 p_3 = \frac{4}{9} \cdot \frac{3}{8}.$$

Whence from the theorem of addition, the probability of appearance of a white ball from a box of any group

$$\Pi = P_1 p_1 + P_2 p_2 + P_3 p_3 = \frac{3}{9} \cdot \frac{2}{6} + \frac{2}{9} \cdot \frac{4}{7} + \frac{4}{9} \cdot \frac{3}{8}.$$

We will bring the right hand part of this equation under a common denominator without, intentionally, making any contractions, and we will complete the calculation:

$$\Pi = \frac{3 \cdot 2 \cdot 7 \cdot 8 + 2 \cdot 4 \cdot 6 \cdot 8 + 4 \cdot 3 \cdot 6 \cdot 7}{9 \cdot 6 \cdot 7 \cdot 8} = \frac{336+384+504}{3024} = \frac{1224}{3024}.$$

The obtained result allows us to draw the conclusion that the appearance of a white ball from the box of any group before trial had 1224 favourable cases out of 3024 of all the equally possible cases.

84. It must be emphasised, that 1224/3024 is not the ratio of the number of all the white balls to the total quantity of balls, but the ratio of cases favouring the appearance of a white ball to the number of all the possible cases.

According to the condition of the example, the 9 boxes contain altogether 26 white balls and the total number of balls (both white and black) in the boxes, is 64. If in determining the probability of appearance of a white ball from any group of boxes, we took the ratio 26/64 (which is not equal to 1224/3024), we would have made an error, as the boxes contain different numbers of balls.

Let us now suppose, that the ratio of white balls to black, in the boxes, remains the same, and the number of balls in each box is the same, namely, 6 . 7 . 8, - altogether 336 balls per box.

Then in each box of the first group, there must be 2 . 7 . 8 = 112 white balls, in each box of the second group, 4 . 6 . 8 = 192 white balls and in each box of the third group 3 . 6 . 7 = 126 white balls. This gives us the same numerical values of the probabilities of appearance of a white ball:

$$p_1 = \frac{112}{336} = \frac{2}{6}; \qquad p_2 = \frac{192}{336} = \frac{4}{7}; \qquad p_3 = \frac{126}{336} = \frac{3}{8}.$$

Taking into consideration the total number of boxes and the number of boxes in each group, we obtain the number of all the equally possible cases for the appearance of the ball:

$$n = 336 \cdot 9 = 3024,$$

and the number of cases favouring the appearance of a white ball,

$$m = 112 \cdot 3 + 192 \cdot 2 + 126 \cdot 4 = 336+384+504 = 1224,$$

i.e. we obtain the numerical value of the probability of appearance of a white ball before trial, which we obtained earlier.

The probability of appearance of a white ball before trial from any row of boxes (without indicating any one in particular) is termed full or mean probability of appearance of a white ball. In its

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formula,

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84. $$\Pi = P_1p_1+P_2p_2+P_3p_3+\ldots+P_ip_i+\ldots+P_sp_s = \sum_1^s P_ip_i,$$

where P_i - probabilities of hypotheses before trial;
p_i - probabilities of events according to these hypotheses.

The trial was made - a white ball appeared.

Obviously, one of the 1224 cases, favouring the appearance of a white ball before trial, has taken place.

85. We had established above, that out of these 1224 cases, 336 favoured the appearance of a white ball from a box of the first group, 384 cases - from a box of the second group and 504 cases from a box of the third group.

Let us determine the probability of appearance of a white ball specifically from a box of the first group after the white ball had already been removed, i.e. the probability of the first hypothesis after trial.

As the appearance of a white ball from a box of the first group had 336 favourable cases out of all the 1224 equally possible cases for the appearance of a white ball, then the probability of appearance of a white ball from a box specifically from the first group or the probability that, after the appearance of the white ball, it is the first of our hypotheses which takes place,

$$Q_1 = \frac{336}{1224}$$

In order to turn from the particular solution to a general deduction, let us recall how the numbers 336, 384, 504 and 1224 were obtained:

$336 = 3 \cdot 2 \cdot 7 \cdot 8$; $384 = 2 \cdot 4 \cdot 6 \cdot 8$; $504 = 4 \cdot 3 \cdot 6 \cdot 7$;

$1224 = 336+384+504 = 3 \cdot 2 \cdot 7 \cdot 8+2 \cdot 4 \cdot 6 \cdot 8+4 \cdot 3 \cdot 6 \cdot 7$,

then $$Q_1 = \frac{3 \cdot 2 \cdot 7 \cdot 8}{3 \cdot 2 \cdot 7 \cdot 8+2 \cdot 4 \cdot 6 \cdot 8+4 \cdot 3 \cdot 6 \cdot 7}.$$

In the right hand part of this equation we have only the cases which favour the appearance of a white ball. For a general derivation we must also take into consideration all the equally possible cases.

For this purpose, we divide the numerator and the denominator on the right hand side of the equation by the product $9 \cdot 6 \cdot 7 \cdot 8 = 3024$, i.e. by the product of all the equally possible cases, after which we make the essential cancellations:

$$Q_1 = \frac{\frac{3 \cdot 2 \cdot 7 \cdot 8}{9 \cdot 6 \cdot 7 \cdot 8}}{\frac{3 \cdot 2 \cdot 7 \cdot 8}{9 \cdot 6 \cdot 7 \cdot 8}+\frac{2 \cdot 4 \cdot 6 \cdot 8}{9 \cdot 6 \cdot 7 \cdot 8}+\frac{4 \cdot 3 \cdot 6 \cdot 7}{9 \cdot 6 \cdot 7 \cdot 8}} = \frac{\frac{2}{9} \cdot \frac{2}{6}}{\frac{3}{9} \cdot \frac{2}{6}+\frac{2}{9} \cdot \frac{4}{7}+\frac{4}{9} \cdot \frac{3}{7}}.$$

Let us analize the obtained result and let us find what it represented by these fractions.

The fractions 3/9, 2/9 and 4/9, as proved above, are the probabilities of selection of a box of one or other of the groups, i.e. they are the probabilities of hypotheses before trial (before the appearance of a white ball): P_1, P_2 and P_3.

The fractions, however, 2/6, 4/7 and 3/7 are the probabilities of appearance of a white ball from a box of a definite group, in other words, the probabilities of an event according to the above mentioned hypotheses: p_1, p_2 and p_3.

86. Then, retaining these symbols, we obtain:

$$Q_1 = \frac{P_1p_1}{P_1p_1+P_2p_2+P_3p_3}.$$

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Thus, the probability that [illegible] group of boxes, i.e. the probability that [illegible] hypothesis takes [illegible]

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86. the selection of a box of the first group before the appearance of a white ball (probabilities of the first hypothesis before trial), and the probability of appearance of a white ball specifically from such a box (probability of an event according to the given hypothesis), divided by the sum of such products for all groups (for all hypotheses).

Developing this deduction for any event and any number of hypotheses, we can define the theorem of hypotheses.

The probability of a hypothesis after trial is equal to the product of the probability of this hypothesis before trial, and the probability of the event according to this hypothesis, similar to the one (event) which took place during trial, divided by the sum of such products for all the possible hypotheses.

Or in a general form, for S hypotheses

$$q_i = \frac{p_i p_i}{p_1 p_1 + p_2 p_2 + p_3 p_3 + \ldots + p_i p_i + \ldots + p_S p_S} = \frac{p_i p_i}{\sum_1^S p_i p_i}$$

Here, the sum

$$p_1 + p_2 + p_3 + \ldots + p_i + \ldots + p_S = 1,$$

i.e. when determining the probabilities of hypotheses after trial, it is necessary to take into consideration all hypotheses which hold before trial. This is actually confirmed by the conditions of our example, where

$$p_1 + p_2 + p_3 = \frac{3}{9} + \frac{2}{9} + \frac{4}{9} = 1.$$

When applied to artillery fire, the theorem of hypotheses may be defined thus.

The probability of a given position of the target with respect to the mean trajectory, after the shot which has provided an observation of a given sign, is equal to the product of the probability of this position of the target before the shot, and the probability of the obtained observation with the given position of the target, divided by the sum of such products for all the possible positions of the target.

We could have derived the theorem of hypotheses without making use of a particular numerical example.

Let us suppose, that it is possible to make 4 hypotheses, with respect to the appearance of the event A, the probabilities of which before trial, are

$$p_1 = \frac{S_1}{S}; \quad p_2 = \frac{S_2}{S}; \quad p_3 = \frac{S_3}{S}; \quad p_4 = \frac{S_4}{S}.$$

87. The probabilities of appearance of the event A under condition that these hypotheses specifically, hold, i.e. the probabilities of events according to the hypotheses, are

$$p_1 = \frac{m_1}{n_1}; \quad p_2 = \frac{m_2}{n_2}; \quad p_3 = \frac{m_3}{n_3}; \quad p_4 = \frac{m_4}{n_4}.$$

Then, the full probability of the appearance of the event A before trial (according to all hypotheses)

$$p = p_1 p_1 + p_2 p_2 + p_3 p_3 + p_4 p_4 = \frac{S_1}{S} \cdot \frac{m_1}{n_1} + \frac{S_2}{S} \cdot \frac{m_2}{n_2} + \frac{S_3}{S} \cdot \frac{m_3}{n_3} + \frac{S_4}{S} \cdot \frac{m_4}{n_4},$$

or, after reducing to common denominator

$$p = \frac{S_1 \cdot m_1 \cdot n_2 \cdot n_3 \cdot n_4 + S_2 \cdot m_2 \cdot n_1 \cdot n_3 \cdot n_4 + S_3 \cdot m_3 \cdot n_1 \cdot n_2 \cdot n_4 + S_4 \cdot m_4 \cdot n_1 \cdot n_2 \cdot n_3}{S \cdot n_1 \cdot n_2 \cdot n_3 \cdot n_4}.$$

Trial made - event A occurred.

The remaining items before when solving the particular example. The number of all the equally possible cases before trial, was

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87. The number of all the equally possible cases after trial, became

$$S_1.m_1.n_2.n_3.n_4+S_2.m_2.n_1.n_3.n_4+S_3.m_3.n_1.n_2.n_4+S_4.m_4.n_1.n_2.n_3,$$

where out of all of these equally possible cases, according to the first hypothesis, those favouring the occurrence of the event A after trial, are $S_1.m_1.n_2.n_3.n_4$ cases, those according to the second hypothesis - $S_2.m_2.n_1.n_3.n_4$ cases, according to the third hypothesis - $S_3.m_3.n_1.n_2.n_4$ cases and, finally, according to the fourth hypothesis - $S_4.m_4.n_1.n_2.n_3$ cases.

Hence the probability of the first hypothesis after trial

$$q_1 = \frac{S_1.m_1.n_2.n_3.n_4}{S_1.m_1.n_2.n_3.n_4+S_2.m_2.n_1.n_3.n_4+S_3.m_3.n_1.n_2.n_4+S_4.m_4.n_1.n_2.n_3}.$$

Dividing the numerator and the denominator of the right hand side of the equation by the number of all the equally possible cases before trial, i.e. by the product $S.n_1.n_2.n_3.n_4$ and cancelling, we obtain

$$q_1 = \frac{\frac{S_1}{S}.\frac{m_1}{n_1}}{\frac{S_1}{S}.\frac{m_1}{n_1}+\frac{S_2}{S}.\frac{m_2}{n_2}+\frac{S_3}{S}.\frac{m_3}{n_3}+\frac{S_4}{S}.\frac{m_4}{n_4}} = \frac{P_1p_1}{P_1p_1+P_2p_2+P_3p_3+P_4p_4}.$$

88. In this way we have arrived at the same deduction as that obtained in the numerical example.

We have shown above, that the theorem of hypotheses allows us to compare the probabilities of hypotheses after trial.

Let us show this, using our example, by determining, from the theorem of hypotheses, the probabilities of appearance of a white ball from a box of the second group and from a box of the third group, i.e. the probabilities of the second and third hypotheses after trial.

Substituting in the general formula, derived by us, corresponding numerical values of probabilities, we obtain:

$$q_2 = \frac{\frac{2}{9}.\frac{4}{7}}{\frac{3}{9}.\frac{2}{6}+\frac{2}{9}.\frac{4}{7}+\frac{4}{9}.\frac{3}{8}} = \frac{384}{336+384+504} = \frac{384}{1224};$$

$$q_3 = \frac{\frac{4}{9}.\frac{3}{8}}{\frac{3}{9}.\frac{2}{6}+\frac{2}{9}.\frac{4}{7}+\frac{4}{9}.\frac{3}{8}} = \frac{504}{336+384+504} = \frac{504}{1224}.$$

It is easy to note, that it is sufficient to calculate the probability of only one of all the hypotheses made. The probabilities of the remaining hypotheses will differ only by the numerator, which represents the number of favourable cases for the occurrence of a given event, according to each one of all the remaining hypotheses.

Comparing q_1, q_2 and q_3, we see, that the most probable hypothesis is that a white ball be withdrawn from a box of the third group, as after the trial, the third hypothesis was seen to be the most probable.

And here,

$$q_1+q_2+q_3 = \frac{336}{1224}+\frac{384}{1224}+\frac{504}{1224} = 1.0,$$

which confirms the correctness of the reasoning and of all the calculations made.

EXAMPLE 1. It is established that in the region of possible positions of the target, the probabilities of finding the target in separate zones of this region, are as follows:

Zones in mm. (region of the possible positions of the target).	A	B	C	D	E	F	G	H	I
Probabilities of finding the target.	0.02	0.10	0.[illegible]	0.[illegible]	0.[illegible]	0.[illegible]	0.[illegible]	0.[illegible]	0.[illegible]

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89. Each zone of the region of the possible positions of the target (RPrT) has a corresponding sight setting. If on the sight setting, corresponding to the zone E, 1 shot be fired, then the probabilities of obtaining an undershoot (or an overshoot), with the condition of finding the target in various zones of RPrT, will be as follows:

Zones of RPrT.	A	B	C	D	E	F	G	H	I
Probability of undershoot.	0	0.02	0.09	0.25	0.50	0.75	0.91	0.98	1
Probability of overshoot.	1	0.98	0.91	0.75	0.50	0.25	0.09	0.02	0

Let us find the probability of finding the target in the zone C after 2 shots have been fired on the sight corresponding to the zone E, when one undershoot and one overshoot were obtained.

Solution. Let us first check if the conditions of the example take into account all the possible positions of the target. For this purpose, we add the given probabilities of finding the target:

$$0+0.02+0.10+0.23+0.30+0.23+0.10+0.02+0 = 1.0.$$

The sum of the probabilities is unity, which confirms that all the possible positions of the target have been considered.

As, according to the condition of the example, the target can occupy in RPrT one of the 9 positions, we can form 9 hypotheses about the position of the target. The probabilities of each of the nine hypotheses before trial are given in the first table of this example.

For each position of the target, i.e. to each hypothesis with one shot fired on the sight for the zone E, corresponds a definite probability of obtaining an undershoot (or overshoot), and hence also the probability of obtaining an undershoot and an overshoot with 2 shots, i.e. the probability of an event from the hypothesis of a similar one to that occurring during trial.

Denoting the probability of the undershoot by p and the probability of the overshoot by q, we write the probability of an event according to the hypothesis, in a general form:

$$P_i = p_i q_i.$$

Substituting the numerical values for p and q, we determine the probabilities of obtaining of an undershoot and an overshoot with various positions of the target. Thus, for example, with the condition of finding the target in the zone C and also with the condition that the shots are fired on the sight corresponding to the zone E, the probability of the event according to the given hypothesis

$$P_C = pq = 0.09 \times 0.91 = 0.0519.*$$

90. Making similar calculations for all the possible positions of the target, we can commence to solve from the general formula

$$q_C = \frac{P_C p_C}{P_A p_A + P_B p_B + P_C p_C + P_D p_D + P_E p_E + P_F p_F + P_G p_G + P_H p_H + P_I p_I} =$$

$$= \frac{0.10 \times 0.91 \times 0.09}{0+0.2 \times 0.98 \times 0.02+0.10 \times 0.91 \times 0.09+0.23 \times 0.75 \times 0.25+0.30 \times 0.5 \times 0.5 + 0.23 \times 0.25 \times 0.75+0.10 \times 0.9 \times 0.91+0.02 \times 0.02 \times 0.98+0} =$$

* When calculating the probabilities of events from the hypotheses (in example - $p_i q_i$), the binomial coefficient (in example - 2) is not taken into consideration. This is explained by the fact that here we are calculating the probabilities of the event similar to the one which takes place during trial, and during trial (in shooting), the events occur in a kind of a single sequence (in the example - at first an undershoot, then an overshoot). The binomial coefficient does not apply in this case.

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90. $$= \frac{0.008190}{0+0.000392+0.008190+0.043125+0.075000+0.043125+0.008190+0.000392} =$$

$$= \frac{0.008190}{0.178414} = 0.045903.$$

The solution of problems on the application of the theorem of hypotheses is made easier by means of the following table.

Hypotheses.		Probability of hypotheses before trial P_i	Probability of event from the hypothesis p_i	$P_i p_i$	Probability of hypothesis after trial q_i
Target in zone	A	0	1x0	0	0
- do -	B	0.02	0.98x0.02	0.000392	0.002197
- do -	C	0.10	0.91x0.09	0.008190	0.045903
- do -	D	0.23	0.75x0.25	0.043125	0.241713
- do -	E	0.30	0.50x0.50	0.075000	0.420374
- do -	F	0.23	0.25x0.75	0.043125	0.241713
- do -	G	0.10	0.09x0.91	0.008190	0.045903
- do -	H	0.02	0.02x0.98	0.000392	0.002197
- do -	I	0	0x1	0	0
Totals		1.00		0.178414	1.000000

The sum of probabilities of hypotheses after trial came to unity, which actually should be the case if the reasoning and the calculations are correct.

It was pointed out above, that each following trial changes the probabilities of finding the target, i.e. it changes the disposition of the target.

Let us compare the dispositions of the target before the shots and after the shots, taking all the probabilities correct to the second decimal place.

Actually, the number of hypotheses has decreased; the depth of RPPT has decreased because the probabilities of finding the target in the zones B and H became, after the shots, sufficiently small to be ignored. The probability of the most probable hypothesis - the most probable position of the target (in zone E) has risen sharply: before trial it was 0.30, but after trial it became 0.42.

91.

Zones of RPPT.	A	B	C	D	E	F	G	H	I
Probability of finding the target before shooting.	0	0.02	0.10	0.23	0.30	0.23	0.10	0.02	0
Probability of finding the target after shooting.	0	0	0.05	0.24	0.42	0.24	0.05	0	0

The theorem of hypotheses is applied in all the cases when it is possible to have a series of hypotheses regarding the occurrence of the event with which we are concerned.

EXAMPLE 2. There are three groups of cartridges:

No. of the group.	Quantity of batches.	Probability of misfire in %.
8	20	0.2
11	25	0.3
16	15	0.5

A cartridge taken from a batch at random, misfired. What is the most probable group to which the batch, from which the cartridge was taken, belonged?

Solution.

No. of hypotheses.	Hypotheses.	P_i	p_i	$P_i p_i$	q_i
I	From the 8th group	20/60	0.002	40/6000	8/190
II	" " 11th group	25/60	0.003	75/6000	75/190
III	" " 16th "	15/60	0.005	75/6000	75/190

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91. After the trial, two most probable hypotheses are obtained: II and III. The probabilities that the cartridge was taken from the 11th group and from the 16th group are identical.

92. A particular case of the theorem of hypotheses.

Quite frequently we have no grounds to consider that, before trial, one of the hypotheses is more or less probable than another. In such cases we would consider that all the hypotheses, before trial, are equally probable, i.e.

$$P_1 = P_2 = P_3 = \ldots = P_i = \ldots = P_S = P.$$

Returning to the example discussed at the beginning of this section, let us allow that the number of boxes in each group is the same; then the probabilities of all the hypotheses before trial, regarding the group of the box from which a white ball appeared, will be equal to each other.

Making corresponding substitutions in the general formula for the probability of hypothesis after trial, and simplifying, we obtain:

$$q_i = \frac{Pp_i}{Pp_1+Pp_2+Pp_3+\ldots+Pp_i+\ldots+Pp_S} = \frac{p_i}{p_1+p_2+p_3+\ldots+p_i+\ldots+p_S}.$$

If, before trial, all the hypotheses are equally probable, then the probability of the hypothesis after trial is equal to the probability of the event in this hypothesis, divided by the sum of such probabilities for all the possible hypotheses.

It is easy to note, that the probability of the hypothesis after trial is proportional to the probability of the event in the given hypothesis.

EXAMPLE 1. The target may be located in one of the following zones of KLMNT: K, L, M, N and O. The probabilities of finding the target in these zones are equal to each other.

In firing a shot with some sight setting, the probability of hitting the target, when the condition that the target is located in the zone K, is 0.05; that in zone L - 0.07; in zone M - 0.10; in zone N - 0.08; and in zone O - 0.06.

Let us determine what are the probabilities of finding the target in the mentioned zones, if with the given sight setting a hit on the target was obtained with the first shot?

Solution. According to the condition, there are altogether five hypotheses regarding the position of the target. Their probabilities before trial are the same. Using the formula of the theorem of hypotheses, for a particular case, we obtain:

$$q_K = \frac{0.05}{0.05+0.07+0.10+0.08+0.06} = \frac{0.05}{0.36} = \frac{5}{36}.$$

For the remaining hypotheses, their probabilities after trial will be found by substitution of the numerator by the corresponding probabilities of the event, and then

$$q_L = \frac{7}{36}; \quad q_M = \frac{10}{36}; \quad q_N = \frac{8}{36} \quad \text{and} \quad q_O = \frac{6}{36}.$$

$$\Sigma q_i = \frac{5}{36}+\frac{7}{36}+\frac{10}{36}+\frac{8}{36}+\frac{6}{36} = 1$$

93. EXAMPLE 2. In a closed box there are 6 balls, which can be only white or black. The ratio of the white to the black balls is not known. What is the probability that there is an equal number of white and black balls, if 3 balls withdrawn consecutively were found to be white, [illegible] (the first ball was not returned to the box)?

Solution. It is possible to make 7 hypotheses on the composition of the balls in the box before trial:

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93. 1. The box contains only white balls.
2. The box contains 5 white and 1 black balls.
3. The box contains 4 white and 2 black balls.
4. The box contains 3 white and 3 black balls.
5. The box contains 2 white and 4 black balls.
6. The box contains 1 white and 5 black balls.
7. The box contains only black balls.

All these hypotheses are equally probable before trial.

The probability of appearance of two white balls, from these hypotheses:

$$p_1 = \frac{6}{6} \cdot \frac{5}{5} = \frac{30}{30}; \qquad p_4 = \frac{3}{6} \cdot \frac{2}{5} = \frac{6}{30};$$

$$p_2 = \frac{5}{6} \cdot \frac{4}{5} = \frac{20}{30}; \qquad p_5 = \frac{2}{6} \cdot \frac{1}{5} = \frac{2}{30};$$

$$p_3 = \frac{4}{6} \cdot \frac{3}{5} = \frac{12}{30}; \qquad p_6 = \frac{1}{6} \cdot \frac{0}{5} = 0;$$

$$p_7 = 0.$$

Then the required probability

$$q_4 = \frac{\frac{6}{30}}{\frac{30}{30}+\frac{20}{30}+\frac{12}{30}+\frac{6}{30}+\frac{2}{30}+0+0} = \frac{6}{30+20+12+6+2} = \frac{6}{70} = 0.0857.$$

EXAMPLE 3. Conditions for this example are the same as in example 2, but the white ball, withdrawn first, was returned back into the box.

Solution. The hypotheses regarding the quantity of white and black balls before trial, are the same, but the probabilities of the event from these hypotheses are different, because the conditions of the trial have changed:

$$p_1 = \frac{6}{6} \cdot \frac{6}{6} = \frac{36}{36}; \qquad p_4 = \frac{3}{6} \cdot \frac{3}{6} = \frac{9}{36};$$

$$p_2 = \frac{5}{6} \cdot \frac{5}{6} = \frac{25}{36}; \qquad p_5 = \frac{2}{6} \cdot \frac{2}{6} = \frac{4}{36};$$

$$p_3 = \frac{4}{6} \cdot \frac{4}{6} = \frac{16}{36}; \qquad p_6 = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36};$$

$$p_7 = 0.$$

Hence

$$q_4 = \frac{9}{36+25+16+9+4+1} = \frac{9}{91} \approx 0.0989.$$

The conditions of the trial having changed, causes a change in probability of the hypothesis after trial. This, once again, confirms that the judgement of the probability of the event from the results of a trial is true only for the conditions of a given circumstance.

94. 23. Theorem of future events.

Having made the trial, we can from its results, using the theorem of hypotheses, determine the probabilities of all the hypotheses taken by us, and take an appropriate decision. Thus, in firing we determine the probabilities of location of the target in various zones of the region of possible positions of the target (RPT), in order to select the sight setting for consecutive fire command.

Continuing the trials under the same conditions, we naturally [illegible]

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94. arises on the probability of the so called future event - an event expected after trial which has given a definite result. In firing, for example, a question can arise on the probability of obtaining an undershoot or an overshoot with the next consecutive shot. This question is answered by the theorem of future events.

To derive a general formula for the theorem of future events, we will use the same particular example as we used in proving the theorem of hypotheses.

We have 3 groups of completely identical boxes: there are 3 boxes in the first group, 2 in the second and 4 in the third. In each box of the first group there are 2 white and 4 black balls; of the boxes of the second group, each contains 4 white and 3 black balls, and of these of the third group - 3 white and 5 black balls. The balls differ only in colour.

From a box, selected at random, one ball has been withdrawn at random and this ball proved to be white. What is the probability that the next ball to be withdrawn from the same box will be black, if the white ball has been returned into the box?

We will pay a special attention to the manner in which this question is put - the consecutive ball is withdrawn from the same box from which a white ball has been previously withdrawn, as only in such a construction of the question, are we right in making use of the result of a preceding trial, for determination of the probability of the result of the next (future) trial. To what group this box belongs, we do not know, as this box had not been opened and its content of balls is not known.

If this box were of the first group, then the probability of appearance of a black ball,

$$q_1 = \frac{4}{6}.$$

The probability that this box belongs to the first group of boxes is, as we know (section 22),

$$\varphi_1 = \frac{336}{1224} = \frac{14}{51}.$$

95. Hence, the probability of appearance of a black ball from a given box, being a box of the first group, i.e. the probability of a coincidence that the given box belongs to the first group, and that a black ball appears from that box, expressed as a probability of a compound event

$$\varphi_1 \cdot q_1 = \frac{14}{51} \cdot \frac{4}{6}.$$

The probability of appearance of a black ball from a given box, the box being of the second group,

$$\varphi_2 \cdot q_2 = \frac{384}{1224} \cdot \frac{3}{7} = \frac{16}{51} \cdot \frac{3}{8}.$$

And, finally, the probability of appearance of a black ball from a given box, the box being of the third group,

$$\varphi_3 \cdot q_3 = \frac{504}{1224} \cdot \frac{5}{8} = \frac{21}{51} \cdot \frac{5}{8}$$

It is immaterial to us to what group of boxes belongs the one we have chosen. Therefore the probability of appearance of a black ball is independent to what group of boxes a given box may belong, i.e. the probability of our future event, from the theorem of addition

$$\Pi_b = \varphi_1 q_1 + \varphi_2 q_2 + \varphi_3 q_3 = \frac{14}{51} \cdot \frac{4}{6} + \frac{16}{51} \cdot \frac{3}{7} + \frac{21}{51} \cdot \frac{5}{8} = \frac{4925}{8568} = 0.575.$$

In order to check our calculations, we find the probability of appearance of a white ball after trial, the probability of the second appearance of a white ball from the same box:

$$\Pi_w = \frac{14}{51} \cdot \frac{2}{6} + \frac{16}{51} \cdot \frac{4}{7} + \frac{21}{51} \cdot \frac{3}{8} = \frac{3643}{8568} = 0.4251.$$

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95. [illegible] events, we obtain

$$\frac{4925.3643}{8568\ 8568} = \frac{8568}{8568} = 1,$$

which actually should be the case, because the appearances after trial of white and of black balls are opposite events.

Thus the probability of appearance of a black ball, after a white ball has appeared, is equal to the sum of paired products consisting of the probabilities of hypotheses after trial multiplied by the probabilities of appearance of a black ball for all the hypotheses.

96. Expanding this deduction for any future event with any number of hypotheses, we can enunciate the theorem of future events, as follows.

The probability of a future event after trial, which has produced a definite result, is equal to the sum of paired products consisting of the probabilities of hypotheses after trial multiplied by the probabilities of the future event, with which we are concerned, the latter probabilities being for all the hypotheses.

Or in a general form:

$$\Pi = q_1 q_1' + q_2 q_2' + \ldots + q_i q_i' + \ldots + q_s q_s' = \sum_1^s q_i q_i'.$$

This formula can also be given in expanded form derived from the general formula of the theorem of hypotheses:

$$q_i = \frac{P_i p_i}{P_1 p_1' + P_2 p_2' + \ldots + P_i p_i' + \ldots + P_s p_s'}.$$

Then the probability of a future event

$$\Pi = \frac{P_1 p_1 q_1' + P_2 p_2 q_2' + \ldots + P_i p_i q_i' + \ldots + P_s p_s q_s'}{P_1 p_1' + P_2 p_2' + \ldots + P_i p_i' + \ldots + P_s p_s'}.$$

As can be seen, the theorem of future events originates directly from the theorem of hypotheses and is its logical outcome. At the same time there exists a principal difference between these two theorems. The theorem of hypotheses gives a series of probabilities - a distribution of probabilities; the theorem of hypotheses assists us in taking a more practical decision in given circumstances. The theorem of future events gives a quite definite probability - probability of a future event; the theorem of future events answers the question, what can be expected of a decision taken according to the theorem of hypotheses.

Let us return to our example and see in what way the probability of an expected event changes with respect to the result of the trial.

Before trial the probability of appearance of a black ball independent of the group to which the box, from which the ball was taken, belonged, is equal to

$$P_b = \frac{1800}{3024} = \frac{25}{42} \approx 0.595;$$

after trial the probability of appearance of a black ball became

$$\Pi_b \approx 0.575.$$

As can be seen, the emergence of a white ball, during trial, has reduced the probability of appearance of a black ball. It is logical that this should be so. It is obvious, that after an undershoot has been obtained with the first shot, the probability of obtaining an overshoot with the second shot on the same sight setting, must be less.

97. At the same time, the probability of appearance of a white ball, which before trial was equal to

$$P_w = \frac{1224}{3024} = \frac{17}{42} \approx 0.405,$$

[illegible]

$$\Pi_w \approx 0.425.$$

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17. The probability of an expected event after trial increases with respect to that event which has taken place during that trial. Therefore, every new trial will give new numerical values for the probability of a future event.

EXAMPLE 1. There are 2 closed boxes containing balls which differ only in colour: in the first box - 2 white and 3 black balls and in the second box - 4 white and 2 black balls. From the first box into the second 1 ball is transferred blindfold, and after this, one ball is withdrawn from the second box. This latter ball is found to be black. What is the probability that a white ball will be withdrawn from the second box if the black ball previously withdrawn has been returned back into the box?

Solution. In this example, the future event is the appearance of a white ball from the second box after a black ball has been withdrawn from the box. Before the appearance of a black ball from the second box, one ball, of unknown colour, was transferred from the first box to the second.

On the question what ball was transferred from the first box to the second, we can have two hypotheses:

1. A white ball was transferred; the probability of such an assumption, i.e. the probability of the first hypothesis before trial,

$$p_1 = \frac{2}{5}.$$

2. A black ball was transferred; the probability of such an assumption, i.e. the probability of the second hypothesis before trial,

$$p_2 = \frac{3}{5}.$$

If a white ball was transferred from the first box into the second, then the second box will contain 5 white and 2 black balls, and the probability of appearance of a black ball from the second box, in this case, i.e. the probability of the event from the first hypothesis, is

$$p_1 = \frac{2}{7},$$

and the probability of appearance from this box of a white ball, i.e. the probability of a future event,

$$q_1 = \frac{5}{7}.$$

If from the first box into the second, a black ball was transferred, then the second box will contain 4 white and 3 black balls, and then the probability of appearance of a black ball from the second box, i.e. the probability of an event from the second hypothesis,

$$p_2 = \frac{3}{7},$$

and the probability of appearance of a white ball

$$q_2 = \frac{4}{7}.$$

18. Trial made - black ball appeared. Then the probability of the first hypothesis after trial according to the theorem of hypotheses

$$q_1 = \frac{\frac{2}{5} \cdot \frac{2}{7}}{\frac{2}{5} \cdot \frac{2}{7} + \frac{3}{5} \cdot \frac{3}{7}} = \frac{4}{13}$$

and the probability of the second hypothesis after trial

$$q_2 = \frac{\frac{3}{5} \cdot \frac{3}{7}}{\frac{2}{5} \cdot \frac{2}{7} + \frac{3}{5} \cdot \frac{3}{7}} = \frac{9}{13}$$

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98. Hence, the probability of the future event, i.e. the probability of appearance of a white ball from the second box after a black ball has been withdrawn from that box, and independent of which ball was previously transferred, from the general formula of the theorem of future events

$$\Pi_b = q_1 p_1 + q_2 p_2 = \frac{4}{13} \cdot \frac{5}{7} + \frac{9}{13} \cdot \frac{4}{7} = \frac{20}{91} + \frac{36}{91} = \frac{56}{91} = \frac{8}{13}.$$

For check purposes, we calculate the probability of the opposite future event - the probability of appearance of a black ball:

$$\Pi_b = \frac{4}{13} \cdot \frac{2}{7} + \frac{9}{13} \cdot \frac{3}{7} = \frac{8}{91} + \frac{27}{91} = \frac{35}{91} = \frac{5}{13}.$$

The sum of probabilities of these opposite events comes to unity, which confirms the correctness of our procedure:

$$\Pi_w + \Pi_b = \frac{8}{13} + \frac{5}{13} = 1.$$

As in the case of problems on the application of the theorem of hypotheses, the solution of problems on the application on the theorem of future events, can be carried out more conveniently according to the method which is a further development of that used by us for the theorem of hypotheses.

No. of hypothesis	Summary of hypothesis.	Probability of hypothesis before trial P_i	Probability of event from hypothesis p_i	$P_i p_i$	Probability of hypothesis after trial q_i	Probability of future event after trial q_i	$q_i q_i$
1.	From first box into second, white ball was transferred.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.	From first box into second, black ball was transferred.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	Total	[illegible]		[illegible]	[illegible]		[illegible]

$$\Pi_w = \frac{56}{91} = \frac{8}{13}$$

99. EXAMPLE 2. With the same conditions as in the above example, we will determine the probability of appearance of a black ball if a previously withdrawn black ball was not returned into its box.

Solution.

No. of hypothesis.	Summary of hypothesis.	P_i	p_i	$P_i p_i$	q_i	q_i	$q_i q_i$
1.	White ball was transferred.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2.	Black ball was transferred.	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	Total	[illegible]		[illegible]	[illegible]		[illegible]

$$\Pi_b = \frac{22}{78} = \frac{11}{39}.$$

EXAMPLE 3. A target is located within the limits of one of eight given zones of fire. The probabilities of finding the target in the zones of fire are given in fig 23. On the sight setting 80, which corresponds to the boundary between zones B and B, 1 undershoot and 1 overshoot were obtained. What is the probability that with the next three shots on the same sight, 2 undershoots and 1 overshoot will be obtained, if the probabilities of obtaining an undershoot and an overshoot with 1 shot on the sight 80, with the condition that the

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...target is located in the indicated zones of MRT, are as given in Fig.237.

Solution.

No. of hypo-theses	Summary of hypothesis	p_i	$p_i(p_q)$	p_1p_i	q_i	$q_i(3p^2q)$	q_1q_i
1	Target in zone A	0.02	0.01x0.99	0.000198	0.00119	$3x0.01^2x0.99$	0
2	Target in zone B	0.07	0.05x0.95	0.003325	0.01995	$3x0.05^2x0.95$	0.00014
3	Target in zone C	0.16	0.16x0.84	0.021504	0.12916	$3x0.16^2x0.84$	0.00883
4	Target in zone D	0.23	0.37x0.63	0.058275	0.34970	$3x0.37^2x0.63$	0.09046
5	Target in zone E	0.23	0.63x0.37	0.058275	0.34970	$3x0.63^2x0.37$	0.15400
6	Target in zone F	0.16	0.84x0.16	0.021504	0.12916	$3x0.84^2x0.16$	0.04374
7	Target in zone G	0.07	0.95x0.05	0.003325	0.01995	$3x0.95^2x0.05$	0.00273
8	Target in zone H	0.02	0.99x0.01	0.000198	0.00119	$3x0.99^2x0.01$	0
	Total	1.00		0.166804	1.00000		0.29390

$$\Pi = 0.2994.$$

Fig.23.

(1) - flight SC;
(2) - Mean trajectory;
(3) - Zones of MRT;
(4) - Probability of locating the target;
(5) - Probability of obtaining an undershoot;
(6) - Probability of obtaining an overshoot.

If the hypothesis before trial were equally-probable, then the expression of the general formula of the theorem of future events will be:

$$\Pi = \frac{p_1q_1' + p_2q_2' + \ldots + p_iq_i' + \ldots + p_sq_s}{p_1 + p_2 + \ldots + p_i + \ldots + p_s}.$$

EXAMPLE 4. There is a box with 5 identical balls. Some of these balls are painted white and others - black. The number of balls of either colour is not known. One ball is taken out of this box; this ball proving to be white. What is the probability that the next ball to be removed will be black, if the first removed ball has been returned into the box?

Solution. In respect of the composition of the balls in the box, we can form 4 hypotheses, these hypotheses before trial being equally probable. We can therefore somewhat curtail the method of the solution.

No. of hypothesis	Summary of hypothesis	p_i	q_i	q_i	q_iq_i
1	4 white and 1 black balls	4/5	4/10	1/5	4/50
2	3 " " 2 " "	3/5	3/10	2/5	6/50
3	2 " " 3 " "	2/5	2/10	3/5	6/50
4	1 " " 4 " "	1/5	1/10	4/5	4/50
	Total	10/5	10/10		20/50

$$\Pi_1 = \frac{20}{50} = 0.4.$$

Example 5. With the same conditions as in example 4, we will determine the probability of appearance of two black balls if the ball

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[illegible] white ball from the first box into the second box.

Solution.

No. of hypotheses	Summary of hypothesis	p_i	q_1	q_i	$q_{1i}q_i$
1	4 white and 1 black balls	4/5	4/10	1/25	4/250
2	3 " " 2 " "	3/5	3/10	4/25	12/250
3	2 " " 3 " "	2/5	2/10	9/25	18/250
4	1 " " 4 " "	1/5	1/10	16/25	16/250
	Total	10/5	10/10		50/250

$$\Pi_2 = \frac{50}{250} = 0.2.$$

Comparing the solutions of examples 4 and 5, we see, that Π_1^2 is not equal to Π_2. This is explained by the fact that the probability of appearance of the second black ball depends on which ball was removed first - black or white.

EXAMPLE 6. The probabilities of finding the target in zones K, L, M, N and O of the region of possible positions of the target, differ by such a small value that they may be considered as equal.

With the condition of finding the target in the zone K, the probability of an undershoot with one shot on the sight setting 100, is equal to Q_1 in the zone L - 0.09, in the zone M - 0.5, in the zone N - 0.91 and in the zone O - 1. On the sight 100, one shot was fired and an overshoot was obtained. What is the probability that with the next two shots, on the same sight, one undershoot and one overshoot will be obtained?

Solution.

No. of hypotheses	Summary of hypothesis	p_i	q_i	$q_i(2p_iq_i)$	$q_{1i}q_i$
1	Target is located in zone K	1.00	100/250	0	0
2	" " " " " L	0.91	91/250	0.1638	0.0596
3	" " " " " M	0.50	50/250	0.5000	0.1000
4	" " " " " N	0.09	9/250	0.1638	0.0059
5	" " " " " O	0	0	0	0
	Total	2.50	250/250		0.1655

$$\Pi = 0.1655.$$

24. Problems on probabilities of hypotheses.

PROBLEM 56. A box contains 2 black and 3 white balls. From another box containing 3 black and 2 white balls, one ball is transferred into the first box and the balls in the latter box are then thoroughly mixed. What is the probability that a black ball was transferred, if in transferring back from the first box into the second, a white ball appears?

Answer. 13/23.

PROBLEM 57. There are 10 revolvers on a table, one of which is not sight-adjusted. The probability of hitting the target with an adjusted revolver is 0.73, and with unadjusted - 0.23. With a shot fired from a revolver selected at random, a miss was obtained. What is the probability that the shot was fired from the unadjusted revolver?

Answer. 77/320.

PROBLEM 58. There are three groups of completely identical boxes. In the first group there are 3 boxes, in the second - 5 and in the third - 2. In each box of the first group there are 2 white and 1 black balls, in each box of the second group - 1 white and 1 black balls and in each box of the third group - 1 white and 2 black balls. From a box selected at random, one ball is withdrawn and is found to be white. What is the probability that this ball was withdrawn from a box of the second group?

Answer. 15/31.

PROBLEM 59. The [illegible] K, L, M, N and O of the region of possible positions of the target, are

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102. respectively: 0.1, 0.2, 0.4, 0.2 and 0.1. With the condition that the target is located in zone A, the probability of an undershoot with one shot on some sight setting is 1/4, in zone B - 1/3, in zone C - 1/2, in zone D - 2/3 and in zone E - 3/4. With two shots fired on a given setting of the sight, two undershoots were obtained. Find the most probable position of the target after these two shots.

Answer. In zone C; Qc = 0.366.

PROBLEM 60. Let us suppose that an enemy tank can move at the following course angles against our anti-tank gun: 0°, 45°, 90°, 135° and 180°. The probability that the tank will have a course angle of 0° is 0.1, that of angle 45° - 0.3, angle 90° - 0.3, angle 135° - 0.2 and angle 180° - 0.1. The probability of the hit on the enemy tank from our gun when the tank course angle is 0° is 0.35, with course angle of 45° - 0.27, with course angle of 90° - 0.24, with course angle of 135° - 0.29 and with course angle of 180° - 0.35.

With the first shot from our gun, a direct hit was obtained on the enemy tank. What was the probability of obtaining a hit on the tank with the first shot, and what was the most probable course angle the tank was following?

Answer. P = 0.277; angle of 45°.

PROBLEM 61. A closed box contains 6 identical balls differing only in colour. From this box 1 ball was withdrawn, which proved to be black. What is the probability that out of the 6 balls contained in the box, only 1 ball was black?

Answer. 1/21.

103. PROBLEM 62. There are three cartridge cases. In the first case there are 25 cartridges, 2 of which are faulty. In the second case there are 20 cartridges, 1 of which is faulty. The third case contains 30 cartridges, none of which is faulty. All the cases are opened. A cartridge taken at random from the case which happens to be nearest at hand, proves to be good. What is the probability that the cartridge was taken out of the second case?

Answer. 285/861.

PROBLEM 63. Out of 10 candidates, 5 had prepared all the 20 questions of the examination paper, 3 prepared 15 questions, and 2 prepared 10 questions. In the examination each candidate is required to answer two questions. The first candidate to be called passes in both questions. What is the probability that this candidate belongs to the group of those who had prepared all the questions of the examination paper?

Answer. 0.7.

PROBLEM 64. One of the four guns of a battery has a sight 72 and the remaining three 73. With one shot on sight 72 the probability of an undershoot is 0.7 and the probability of an overshoot - 0.3. With one shot on the sight 73 the probability of an undershoot is 0.3 and the probability of an overshoot - 0.7. The firing is conducted from one gun. With the first two shots, an undershoot and an overshoot were obtained. What is the probability that the next shot will give an undershoot?

Answer. 0.4.

PROBLEM 65. The probabilities that the target is located in zones I, II, III, IV and V of the region of possible positions of the target, are respectively: 0.05, 0.25, 0.40, 0.20 and 0.10. The probability of an overshoot with one shot on some sight setting and with the condition that the target is located in the zone I, is 0.75; that the target is located in the zone II - 0.65; in the zone III - 0.50; in the zone IV - 0.35 and in the zone V - 0.25. With three shots on a given sight setting were obtained: an undershoot, an overshoot and an undershoot. What is the probability that with the next two shots on the same sight setting, an undershoot and an overshoot will be obtained?

Answer. 0.4631.

PROBLEM 66. There are 10 completely identical closed boxes containing balls. Three of these boxes are marked No.1, and contain 2 white and 2 black balls each. One box is marked No.2 and contains [illegible] boxes are marked No.3 and contain 3 white and 3 black balls each. From a box selected at random,

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a ball was removed and replaced 3 times consecutively. The balls removed and replaced were respectively: white, white and black. What is the probability that the next balls to be withdrawn from the same box will be 2 white and 1 black?

Answer. 0.42.

PROBLEM 67. A closed box contains 3 balls: white and black. A ball taken out of this box was white. What is the probability that a white ball will appear another 3 times consecutively, if the withdrawn ball is returned into the box on each occasion?

Answer. 49/81.

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104.

Part II.

THEORY OF ERRORS.

Chapter 5.

LINEAR ERRORS.

25. Errors of measurement.

In order to determine the numerical value of a quantity (distance, weight, volume, temperature, etc) with which we are concerned, we measure that quantity.

Daily experience confirms that any measurement, independent of the method by which it is made, is invariably accompanied by some discrepancy, by some error.

It is not difficult to confirm this if we measure the same quantity (for example, a distance between the same two points) several times by the same method using the same instrument. However thorough our measurements, the individual results obtained will differ from each other, even if this difference be small. As a rule, with each new measurement we will be obtaining new results differing from those obtained previously. The measured quantity during the process of measurement remains the same, and yet we obtain several different results of our measurement. It is apparent that none of our results gives a true (precise) value of the measured quantity, and each of these results gives only an approximation of its value.

Hence we conclude that errors during measurements are unavoidable.

The difference between an approximate and a true value of a measured quantity is termed an error or a discrepancy of measurement.

$$x_i - x_o = \Delta_i,$$

where x_o - true value of the measured quantity;
x_i - result of individual measurement;
Δ_i - error of a given result of measurement.

105. The smaller the absolute magnitude of the error, i.e. the nearer the individual result of the measurement to the true value of the measured quantity, the more accurate is the given measurement. For example, if during measurement of one and the same distance, the first result is obtained with an error of 10m, and the second - of 20m, then the first measurement was made more accurately than the second. The accuracy of the measurement in the first case is twice that of the measurement in the second case.

The absolute magnitude of an error characterises the accuracy of the given result of the measurement*.

The errors of individual results of a measurement differ from each other by their signs. According to the sign, the errors of measurement are divided into: positive errors, if the approximate value of the measured quantity is greater than its true value, i.e. if $x_i > x_o$, and negative errors, if the approximate value of the measured quantity is less than its true value, i.e. if $x_i < x_o$.

For example, if the true value of the measured distance is 3000m, and the individual results of the measurement are

x_3 = 3100m and x_7 = 2850m,

then the errors of these results:

Δ_3 = 3100-3000 = +100m (positive error) and
Δ_7 = 2850-3000 = -150m (negative error).

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* Reference is here made to the accuracy of individual results and not to the accuracy of the method of measurement.

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105. The magnitude and the sign of the error are its basic characteristics.

Inevitability of errors in measurements is governed by the fact that any measurement, independent of its thoroughness, accuracy of the instruments employed, and corrections for the effect of various factors, is always carried out under conditions of infinitely large number of sources of errors. We will name some of these sources.

Imperfection of construction of the measuring instruments - inaccuracy of scales (division values), eccentricity of dials, backlash, instability of tripods etc.

Imperfection of our senses, for example, of our sight when taking readings from scales, aligning of cursors with divisions on scales, aligning of crosses with sight heads, levelling by means of spirit levels etc.; imperfections of hearing during audible notching, etc.

Meteorological conditions - clouds, wind, air temperature, convection currents etc.

Each source of errors provides, in measurement, its, so termed, elementary error. The sum, however, of all such elementary errors will
106. finally provide the error of a given result of a measurement.

There are some groups of sources of errors which give, during measurement, elementary errors which are constant in magnitude and sign. It is obvious, that the total error of this group of error sources will likewise be constant in magnitude and in sign.

It is possible, using simple methods, to determine in advance, the magnitude and the sign of such a constant, or, as it is sometimes termed, systematic error. Knowing the magnitude and sign of the systematic error, the latter can be accounted for and eliminated from a given result of a measurement by the introduction of a corresponding correction.

EXAMPLE 1. In checking a balance weight of 1kg. it is found that its actual weight is 992g. In order to remove the known error it is necessary, when using this balance weight, to add 8g. for every kilogram.

EXAMPLE 2. When checking a 20 metre measuring tape, it is found that its true length is 20.1 m. Therefore the systematic error of this tape, distributed over its whole length, will be

$$\frac{20-20.1}{20.1} \approx -0.005 \quad \text{or} \quad 0.5\%.$$

When using this tape, a correction should be made to effect an increase of 0.5% on the obtained result of the measurement.

EXAMPLE 3. In checking a telestereoscope, a graph of its constant errors with respect to its readings, is plotted (fig.24). Using this graph, we can eliminate the error of a given telestereoscope, from an obtained reading. Thus, if the reading happens to be 25,09, then, taking into account the correction of -0,02 from the graph, we use the reading

$$25,09-0,02 = 25,07;$$

and having obtained a reading of 54,87 we correct it by +0,01:

$$54,87+0,01 = 54,88 \text{ etc.}$$

Fig.24.

A systematical error can also take place as a result of working to nearest (rounded) numbers and of making allowances in calculations associated with measurements.

EXAMPLE 4. An arc equal to 1/955 of distance corresponds to an angle of 0,01. In translating linear values into divisions of a goniometer, the value 1/955 is rounded to 1/1000 for the purpose of facilitating calculations and thereby a constant error on the large side of approximately 5% of the calculated result is committed.

107. In the above examples are given typical cases of systematic errors which occur and are accounted for.

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107. Such sources which give elementary errors of constant magnitude and sign, are comparitively few.

By far the greatest number of sources of errors gives, under conditions of a given measurement, elementary errors varying in magnitude and sign, these errors having a chance character. The chance character of these elementary errors, obviously determines also the chance character of errors of individual results of a measurement, as well as total errors.

It is quite evident that a combination (summation) of chance elementary errors will give a chance total error which will ultimately be the error of our measurement. Having repeated the measurement we will obtain another combination of chance elementary errors and another total error.

By chance errors are terme. errors which being the results of combined action of a great number of sources of errors, attain new chance values with every new measurement.

The magnitude and sign of a chance error can not be determined beforehand and therefore it is not possible, from a given result of a measurement, to exclude this error in advance by a corresponding correction.

To chance errors belong errors which arise in measurement: of distances by different methods, of angles by different instruments, of barometric pressure, of temperature, of weights etc.

Also to chance errors belong such magnitudes of chance character as deviations, inaccuracies and departures. Thus, if the true distances D = 5200m, and if in measurement a distance D_1 = 5050m is obtained, we say that there is a deviation of

$$D_1 - D = 5050 - 5200 = -150\text{m}.$$

If a shell must have a length l_0 = 1010mm when its actual length is l_1 = 1012mm, we say that departure or inaccuracy in the dimensions is

$$l_1 - l_0 = 1012 - 1010 = +2\text{mm}.$$

From determination of a chance error it can be seen that the latter can be regarded as a chance result of some occurrence. This means, that to chance errors we can apply all the deductions which we made in the part "Principles of the law of probability", commencing with the deduction on the rule of finite, chance results of occurrences.

The part of the law of probability which treats of chance errors and the rule which governs the errors, are known as the theory of chance errors or simply the theory of errors.

108. The theory of errors outlines the rules which govern the chance errors, and from these rules develops the methods, by the systematic application of which the optimum results of measurements can be obtained.

Errors which are characterised only by magnitude and signs are termed linear errors or errors which are independent of direction, in contrast to errors which are characterised not only by magnitude and signs, but also by direction (errors in planes and in space).

26. Laws of chance errors.

The conditions under which chance error originates do not allow a previous determination of its magnitude and sign and hence any correction of the obtained result of the measurement. This does not essentially signify that chance errors can not generally be accounted for. On the contrary, these errors have to be calculated or else the obtained measurements can not otherwise be utilized. It is only necessary to determine what this calculation involves and what it comes to.

Let us assume that we have produced a series of measurements of the distance to a target situated in direct proximity from our front line. We have measured the distance to this target. As an error in the measurement is

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108. unavoidable and can not be eliminated, then, in opening the fire on the sight corresponding to the measured distance, we may be firing on our own forces. But if we know the magnitude of this limited chance error, which can be, in practice, allowed in the given method of measurement of distance, then, by increasing the sight by this value we guarantee the safety of our forces.

Another example. The target is not being observed. Firing for adjustment is not possible. The target coordinates are taken from an aerial photograph. Making the necessary measurements and calculations, we have determined the sight settings necessary for opening the fire. As the error can not be avoided and removed we are compelled to fire on the target from several sights - firing on an area. A question arises with regard to the dimensions of this area. To cover the area by fire in correspondence to limited errors is inefficient - the expenditure of shells is too high, and the time required to execute this firing task is too great. Consequently, we have to confine the firing only to those zones in the region of possible positions of the target (RUPT), where the probabilities of location of the target (finding the target) are considerable, and for this purpose we must know the probabilities of obtaining errors differing in magnitude and sign, when we are determining the sight settings for opening the firing.

The above examples show that the calculation of chance errors is not only possible, but is also essential, as without it, it is impossible to solve correctly the most important practical problems. For this calculation it is necessary to know what is the relationship, with a given method of measurement, between the magnitude and sign of the
109. chance error and the probability of its arising. This relationship is expressed by the corresponding laws of chance errors.

The relationship between absolute magnitude (value) and sign of a chance error and the probability of its arising, is termed the law of chance errors.

The laws of chance errors are numerous. Each one of them express a definite conformity to rule of chance errors which originates from the conditions in which these errors arise.

For example, the law of equal probability expresses the rule for chance errors arising when readings from instrument scales are rounded. The law of arc-sine expresses the rule for chance errors which arise as a result of eccentricities of goniometric instruments, etc.

The rules for chance errors can be found in two ways:

1. By experiment. For this purpose, a large number of measurements is made of a quantity, the true value of which is not known beforehand. For each individual result, its error is determined. After distribution of the obtained errors according to their magnitudes and signs, a relationship between the limits of errors and the frequencies of obtaining the errors within these limits, is determined. And, finally, from the law of high numbers these frequencies are taken as equal to their corresponding probabilities.

2. Analytical method. By analysing the conditions in which chance errors of the given law, originate.

The rules established as a result of such investigations can be expressed analytically or graphically.

Analytically, the law of chance errors is expressed in a general form, by the equation

$$p = F(\Delta),$$

where Δ - magnitude (value) of error;
p - probability of its occurrence;
F (function) - denoting the relationship which exists between Δ and p, for example $p = 1/2\Delta^2$.

If the form of the function $F(\Delta)$ is known, the law of errors may also be expressed graphically. The law of errors is also expressed graphically when it has been derived from the results of a large number of measurements.*

In practical artillery measurements, we have to deal almost exclusively with the following two laws of chance errors: Gauss law

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is given in section 27.

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the law of equal probability.

110. Out of these two laws we are mostly concerned with the Gauss law.* It is sufficient to say that the Gauss law applies to the dispersion of shells and to the errors of such measurements as the measurements of distances by visual method, measurements from maps and by range finder, measurements of azimuths, measurements of temperatures, weights etc

27. Derivation of Gauss law of chance errors.

As has been mentioned previously, the derivation of the law of chance errors may be made in two ways - experimentally and analytically.

Let us first find the main premises for the Gauss law of chance errors by the experimental method, by calculating the results of a sufficiently high number of measurements, the chance errors of which follow this law.

For this purpose we will use the following example. 170 equally trained reconnaissance men, when carrying out the same reading of a visual route, had pace-measured the distance between the same points on ground, and had converted the results of their measurements into metres. In this way 170 approximations of the value of this distance were obtained.

This same distance was measured with instruments with such an accuracy, that in comparison with the method of pace-measurement, the obtained result, neglecting special discrepancies, can be taken as the true value of the given distance. This result was found to be 3000m.

Using this accurate result, we will determine the errors of individual results of pace-measurement and we will enter all our calculations in the following table (table 4).

Table 4.

111.

In order to establish the relationship between the value and the law of chance error and the frequency of its occurrence, we will break up the errors of the series of measurements into groups of 50m. according to their value and sign. Thus, into the 1st group we take all the negative errors of values from 201m to 250m, in the 2nd group - all the negative errors of the value of 151m to 200m. etc. We will determine the number of errors in each such group and we will find the corresponding frequencies of their occurrence (table 5).

112.

Table 5.

Groups.	
Magnitude of error From ... to	
Number of errors.	
Frequency of occurrence of errors in %.	

From the data contained in this table, it can be seen how frequently the chance errors, within definite limits of magnitude and sign, were obtained. It must be noted that in dividing the errors into groups of every 50m, we do so only from considerations of convenience of such a division - the limit of errors in magnitude is approximately 250m, and this figure is divisable by 5 without remainder.

For greater clarity we reproduce the relationships given in table 5 in the form of a graph. For this purpose we mark off along the axis OX, in both directions from the point O, the limits of errors every 50m, to a selected scale, and along the axis OY - to another chosen scale, the frequencies of occurrence of the errors within these limits (fig.25).

* Gauss - [illegible]

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Fig.25.

In this graph the area of every rectangle expresses separately, the frequency of occurrence of a chance error within definite limits of magnitude and sign, and the area of the whole graph - the sum of frequencies of occurrence of all chance errors.

Before turning to the general forms, let us make use of experimental data of yet another series of repeated measurements - 580 results of measurements of one and the same distance by means of a range finder (table 6) and let us plot a graph of frequencies (fig.26) for this series of measurements.

Table 6.

Limits of errors of distance in %	Negative errors		Positive errors	
	Number of errors	Frequencies in %	Number of errors	Frequencies in %

Fig.26.

We have plotted two graphs from the results of a large and at the same time different number of measurements carried out by different methods. It is easy to note, in comparing these graphs, that they are very similar in form. This allows us to conclude that the relationship between the magnitude and sign of an error, on the one side, and the frequency of occurrence of the error, on the other side, in all these cases, when the chance errors of measurements follow the Gauss law, is established according to the same general rule:

1. The greater the error, the less the frequency of its occurrence - the heights of the rectangles in both graphs decrease as the magnitude of the error increases.

2. The frequencies of negative errors and the frequencies of positive errors, contained within equal limits of magnitude, are approximately equal to each other - the rectangles, equally spaced from the axis Of, are approximately of the same height, i.e. the rectangles with respect to the axis Of are distributed (in height) approximately symmetrically. Comparing the second graph with the first, we note, that with a large number of measurements the graph obtained is more symmetrical. From this we may assume, that the graph, plotted from the results of a very (infinite) large number of measurements, must be entirely symmetrical, i.e. the frequencies of errors, equal in absolute magnitude, and different in sign, must be equal to each other (fig.27).

Fig.27.

3. Every method of measurement has its own limit of errors - both graphs have no breaks, and the rectangles, corresponding to frequencies of limit errors in magnitude, tend to approach the axis Ox (this is especially noticeable in the second graph).

These three assumptions determine the Gauss law of chance errors derived from calculation of experimental data.

Using the law of high numbers we can apply our deductions to the relationship between the magnitude and sign of an error and the probability of its occurrence, i.e. we can assume that:

- as the error increases, its probability decreases, the less the error, the greater its probability;
- errors which are equal in absolute magnitude, but different in sign, are equally probable - the probability of occurrence of a positive error is equal to the probability of occurrence of a negative error equal to the first in absolute magnitude.

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sufficiently small to be normally considered very close to zero and such errors are ignored.

Briefly these three assumptions can be stated thus: errors are distributed non-uniformly, symmetrically and within limits.

This is a general expression of the Gauss law.

If the centre points of the upper sides of the rectangles are joined by a curve (see fig.27), then this curve, which is termed the Euler curve, will express graphically these same three bases for the assumptions for the Gauss law.

The probability of occurrence of an error within given limits, is expressed by the relationship of the area bound by the part of the curve, contained between the corresponding ordinates, to the area bound by the whole curve.

Thus, the probability of occurrence of a positive error between the limits from a to b (see fig.27) is expressed by the ratio of the area abed to the area ABC; the probability of occurrence of a negative error between the limits from f to e - by the ratio of the area efkl to the area ABC etc.

Denoting these areas, corresponding to the limits of errors, by s_1, s_2, s_3,..., s_8 and adding these areas, we obtain:

$$s_1+s_2+s_3+\ldots s_8 = S,$$

where S - the area bound by the whole curve.

Dividing the left hand and the right hand sides of this equation by S, we have

$$\frac{s_1}{S}+\frac{s_2}{S}+\frac{s_3}{S}+\ldots+\frac{s_8}{S} = \frac{S}{S} = 1.$$

This means, that the sum of the probabilities of all the possible chance errors within the conditions of a given series of measurements is equal to unity, which must be according to the statement of the theorem of addition of probabilities.

As the area, bound by the whole curve, represents the sum of probabilities which is equal to unity, then this area can likewise be taken as equal to unity. It then follows that the elementary, small areas will numerically be equal to the probabilities which they represent.

From the expression of the probability of occurrence of an error, by means of an area which is contained within ordinates, corresponding to given limits, it follows that the probability of occurrence of an error of definite magnitude and sign, is equal to zero, as in this case the ordinates, which contain the area, will merge and such a probability will be represented by one ordinate - a line. Therefore it is possible to speak of probability of occurrence of an error only within some limits, no matter how small these limits may be, for example, we can speak of a probability of occurrence of an error within the limits from 4.99990 to 5.00010m etc.

Fig.28.

The curve of the Gauss law very clearly confirms all the basic assumptions of this law with which we are now acquainted. Let us assume that the differences of the limits of errors, the probabilities of occurrence of which are represented by areas s_1, s_2, s_3 and s_4, are equal to each other (fig.28):

$$l_1 = l_2 = l_3 = l_4 = l.$$

Then:

- the smaller the error, the greater the probability of its occurrence: $s_1 > s_2$ and $s_2 > s_3$;
- positive errors and negative errors, contained within limits

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116. $s_2 = s_4$;

- the probability of occurrence of an error, exceeding in magnitude a certain limit (-OA or +OC), is equal to zero, because the curve ABC will, at some distance from the axis OY (at points A and C), merge with the axis OX.

For different methods (series) of measurements the Gauss law curves will likewise be different.

But as the established general relationship between the limits of errors and the probabilities of their occurrence within these limits is expressed by the same assumptions, the curves will always retain their character.

117. In order to allow a comparison of the probabilities of occurrence of errors within given limits for different series of measurements, it is necessary to plot the curves corresponding to these series of measurements so, that the areas, bound by each of these curves, are of equal magnitude. This requirement originates naturally from the fact that these areas in all cases are taken to be equal to unity. To comply with this requirement the curves must be plotted to the same scale along the axis OX, and along the axis OY to the scale changed by the amount by which the base of one curve is greater than (less than) the base of another curve, i.e. it is necessary to maintain approximately the principle of construction which was earlier applied to the construction of the graphs for the probabilities of combinations.

Fig.29.

In fig.29 are shown curves of the Gauss law for two different methods (series) of measurements.

This diagram permits us to draw the following conclusions:

1. Each of these curves fully confirms the assumptions of the Gauss law.

2. Probabilities of occurrence of an error within definite limits in magnitude and sign, for example from a to b, for each method of measurement are peculiar to that measurement, as the areas

abcd ≠ abef.

3. The method of measurement, the law of errors of which is represented by the curve $A_1B_1C_1$ is more accurate than the method, the law of errors of which is represented by the curve ABC, as the first method has smaller limits of errors than the second:

$OA_1 < OA$ and $OC_1 < OC$.

4. From the conditions of construction of both curves, the areas bound by them are of equal magnitude. Hence the smaller the base of the curve (the smaller the error), the greater is its highest ordinate, i.e. the greater is the accuracy of method of measurement. Consequently,
118. it is possible to judge from the highest ordinate of a given Gauss law, the accuracy of the method of measurement, to compare it with the accuracy of another method, if, of course, the correct scales in construction of the curves have been maintained.

A very clear representation regarding the Gauss law of chance errors is given by, the, so called, Galton apparatus (fig.30).

Fig.30.

A large quantity of small shot (pellets) is dropped into the funnel A. Those pellets, falling along the inclined surface of the bottom of the box B encounter thin rods (pins or needles) stuck into the bottom of the box. As the pellets impinge on these rods, which in this apparatus play the role of sources of errors (deviations), they change their direction with each such impact and fall into different compartments. As the impacts of pellets against the rods are chaotic, the number of such pellets as a result of these impacts, are

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obviously of chance character, the result of the distribution of the pellets in the compartments is very similar to the distribution of chance errors in measurements in the case when these errors follow the Gauss law. In fig.30, such a distribution is shown by the dotted curve.

We have derived the main statements of the Gauss law of chance errors by means of calculation and analysis of experimental data. We will arrive at the same conclusions by analysing the conditions governing the origin of chance errors treated by this law.

Let us suppose that in a given series of measurements there are eight sources of errors. Each one of these sources gives, with each individual measurement, elementary errors: $\alpha_1 = +1$ or $\alpha_2 = -1$, i.e. the elementary errors, equal in absolute magnitude and differing only in sign.

Let us also suppose that the probabilities of occurrence of positive and negative errors are equal:

$$p_{\alpha_1} = p_{\alpha_2} = \frac{1}{2}.$$

Two conditions have been allowed here: a small number of sources of errors has been taken and an assumption made that all these sources give the elementary error of the same absolute magnitude. Therefore,

119. the magnitude of the elementary error is deliberately not expressed in the conventional units of measurement. It will further be seen that these conditions do not reflect on the correctness of the deductions.

Any one of the eight sources of errors can, with each individual measurement, give either a positive error or a negative error. As a result of combined effect of all the eight sources of errors, one of the following total errors may be obtained:

$$\begin{array}{lll} \Delta_1 = +8; & \Delta_4 = +2; & \Delta_7 = -4; \\ \Delta_2 = +6; & \Delta_5 = 0; & \Delta_8 = -6; \\ \Delta_3 = +4; & \Delta_6 = -2; & \Delta_9 = -8. \end{array}$$

The total error $\Delta_1 = +8$ will be obtained if all the eight sources of errors will give only positive elementary errors:

$$\Delta_1 = +1+1+1+1+1+1+1+1 = +8.$$

The total error $\Delta_2 = +6$ will be obtained in the case when seven sources of errors give positive elementary errors and one source - negative:

$$\Delta_2 = +1+1+1+1+1+1+1-1 = +6.$$

The total error $\Delta_3 = +4$ will be obtained if six sources of errors give positive, and two sources of errors - negative elementary errors:

$$\Delta_3 = +1+1+1+1+1+1-1-1 = +4 \text{ etc.}$$

As the elementary errors of all our sources of errors differ only in sign it follows that any of the above enumerated total errors can be regarded as a <u>combination</u> made up of two elementary errors <u>opposite</u> in sign when one and the same source of errors acts eight consecutive times.

Hence, when determining the probabilities of occurrence of these total errors, it is possible to use the formula for the probability of occurrence of a combination made up of two opposite events with eight-fold repetition of the trial with the condition that $p = q = 1/2$:

$$P_i = \frac{8!}{m!n!} p^m q^n = \frac{8!}{m!n!} \left(\frac{1}{2}\right)^m \left(\frac{1}{2}\right)^n = \frac{8!}{m!n!} \left(\frac{1}{2}\right)^8,$$

where m - number of sources, which gave positive elementary errors;
n - number of sources, which gave negative elementary errors.

120. For example, the probability of obtaining (occurrence) of the total error $\Delta_3 = +4$, will be

$$P_3 = \frac{8!}{6!2!} \left(\frac{1}{2}\right)^8 = \frac{28}{256} = 0.1094;$$

the probability of obtaining the total error $\Delta_6 = -2(-1+1+1-1-1+1-1-1)$

$$P_6 = \frac{8!}{3!5!} \left(\frac{1}{2}\right)^8 = \frac{56}{256} = 0.2188 \text{ etc.}$$

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120. Let us calculate the probabilities of obtaining all the possible total errors, within the conditions of our example, and let us tabulate these calculations (table 7).

Table 7.

Adding these probabilities, we obtain:

$$\sum_{1}^{9} P_i = 1,$$

which shows, in the first place, that all the possible total errors in magnitude and sign have been considered and, in the second place, that the calculations are correct.

The data in this table show, that:

- with the increase in error, the probability of its occurrence decreases;
- errors which are equal in absolute magnitude and different in sign, are equally probable;
- the errors do not exceed, in magnitude, the limits ±8.

This, therefore, means, that we have arrived at the same statements of the Gauss law of chance errors which we had previously established experimentally - that the errors are non-uniform, symmetrical and within limits.

Fig.31.

These three statements are more clearly expressed by a graph plotted from the data in table 7 (fig.31).

121. In deriving the Gauss law of chance errors we made two assumptions:

1. The number of sources of errors is limited (8).
2. All the sources of errors give elementary errors of the same absolute magnitude: $\alpha_1 = +1$ and $\alpha_2 = -1$.

We will investigate whether these two assumptions influence the correctness of the deductions we have just made.

For this purpose we will establish, in the first place, the relationship between the magnitude and sign of an error and the probability of its occurrence in the case when the number of sources of errors begins to increase.

Let us suppose, that in the conditions of a given measurement, sixteen, and not eight, sources of errors exist, and that each one of them gives elementary errors equal in absolute magnitude: $\alpha_1 = +1$ and $\alpha_2 = -1$.

Let us calculate all the possible total errors and the probabilities of their occurrence, in the given conditions. We will represent these calculations in the form of a graph (fig.32).

Fig.32.

This graph expresses the Gauss law more fully. With eight sources of errors the probability of occurrence of the error at the limit magnitude (±8) can not yet be considered small enough (0.0039) to be ignored, but with sixteen sources, the probabilities of occurrence of errors ±16 (0.00002), ±14 (0.00024) and even ±12 (0.00183) are so small that such errors can be ignored.

Obviously, with further increase in the number of sources of errors, the statements of the Gauss law derived by us will only be further confirmed and there is no need of a great number of sources of errors for the construction of graphs.

Comparing graphs of figs.31 and 32, we see, that with the increase in the number of sources of errors the "steps" of the graph begin to lessen. It is

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...that the "steps" form of the graph will be less, the greater the number of the sources of errors, and with an infinitely large number of sources of errors the "step" form must disappear - the top sides of rectangles of the graphs must form a smooth curve which was obtained in the calculation of the experimental data (see fig.27). The same form would have been assumed by the curve of the distribution of probabilities of combinations, with an infinitely large number of recurrent (repeated) trials. Therefore, at that time, this curve was termed by us as the Gauss curve.*

Thus we have established the basic statements of the Gauss law, assuming a limited number of sources of errors, and this assumption has not reflected on the correctness of our deductions. Let us now see in what way these deductions will be affected by the presence, in a given measurement, of several groups of sources of errors differing in magnitudes of their elementary errors.

Let us suppose, that in conditions of a given measurement there are three groups of sources of errors:

- first group consisting of eight sources of errors with elementary errors $\varepsilon'_1 = +1$ and $\varepsilon'_2 = -1$;
- second group consisting of six sources of errors, the elementary errors of which are: $\varepsilon''_1 = +2$ and $\varepsilon''_2 = -2$;
- third group consisting of five sources of errors with elementary errors $\varepsilon'''_1 = +3$ and $\varepsilon'''_2 = -3$.

The total errors of the first group of sources of errors and the probabilities corresponding to these errors were found by us above (see table 7).

The effect of the sources of errors of the second group only, can produce the following total errors: $\Delta''_1 = +12$, $\Delta''_2 = +8$, $\Delta''_3 = +4$, $\Delta''_4 = 0$, $\Delta''_5 = -4$, $\Delta''_6 = -8$ and $\Delta''_7 = -12$. The probabilities of these errors, as probabilities of combinations consisting of two opposite events with a six-fold repetition of the trial, are:

$$P''_1 = \frac{1}{64}, \quad P''_2 = \frac{6}{64}, \quad P''_3 = \frac{15}{64}, \quad P''_4 = \frac{20}{64}, \quad P''_5 = \frac{15}{64}, \quad P''_6 = \frac{6}{64} \text{ and } P''_7 = \frac{1}{64}.$$

The effect of the sources of errors of the third group only, can have total errors of: $\Delta'''_1 = +15$, $\Delta'''_2 = +9$, $\Delta'''_3 = +3$, $\Delta'''_4 = -3$, $\Delta'''_5 = -9$ and $\Delta'''_6 = -15$, and their probabilities:

$$P'''_1 = \frac{1}{32}, \quad P'''_2 = \frac{5}{32}, \quad P'''_3 = \frac{10}{32}, \quad P'''_4 = \frac{10}{32}, \quad P'''_5 = \frac{5}{32} \text{ and } P'''_6 = \frac{1}{32}.$$

In combined effect of all the three groups of sources of errors, any coincidences of total errors of these groups can take place. Each such chance coincidence will give also a chance resultant error equal to the sum of total errors.

For example, if the first group of sources of errors gives an error $\Delta'_3 = +4$, the second group, $\Delta''_6 = -8$ and the third group, $\Delta'''_1 = +15$, then the resultant error for the three groups, is

$$\Delta = \Delta'_3 + \Delta''_6 + \Delta'''_1 = +4-8+15 = +11;$$

other coincidences will give other resultant errors in magnitude and sign.

Resultant error of limit magnitude will occur when the total errors of limit magnitude coincide:

$$\Delta'_1 + \Delta''_1 + \Delta'''_1 = +8+12+15 = +35 \text{ and}$$

$$\Delta'_5 + \Delta''_7 + \Delta'''_6 = -8-12-15 = -35.$$

The remaining resultant errors which can take place in combined effects of three groups of sources of errors, will, naturally, be within these limits: +35, +33, +31,...+5, +3, +1, -1, -3, -5,...-31, -33, -35.

It is easy to confirm that the resultant errors will actually be as stated above, if the addition of individual total errors be made to a definite system according to the following table (table 8).

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* See section 12 fig.19.

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123. The resultant error, occurring in a combined effect of several groups of sources of errors, is a compound event. Therefore, also the probability of obtaining such a resultant error, as the probability of a compound event, is equal to the product of probabilities of total errors of which it consists.

Thus, the probability of the resultant error +35 is equal to the product of probabilities of the constituent total errors:

$$P_{+35} = P'_1 \cdot P''_1 \cdot P'''_1 = \frac{1}{256} \cdot \frac{1}{64} \cdot \frac{1}{32} = \frac{1}{524288} = 0.000002.$$

Examining table 8, we can note, that by far the largest majority of resultant errors has several variants.

Thus, for example, the resultant error +29 occurred in two variants:

$$+2+12+15 = +29 \quad \text{and} \quad +8+12+9 = +29,$$

and the resultant error +23 occurred in three variants:

$$-4+12+15 = +23; \quad +2+12+9 = +23 \quad \text{and} \quad +8+12+3 = +23 \quad \text{etc.}$$

Table 8.

124. _____

125. From here it follows, that in determining the probability of occurrence of the resultant error, it is necessary to take into account all its variants, by applying the theorem of addition of probabilities.

The table 8 is not complete, as in it is shown only the method of determining the magnitudes of resultant errors. In a full table, the resultant error +29 has one other variant: +6+8+15 = +29. Thus, the probability of its occurrence:

$$P_{29} = \frac{56}{256} \cdot \frac{1}{64} \cdot \frac{1}{32} + \frac{1}{256} \cdot \frac{1}{64} \cdot \frac{5}{32} + \frac{8}{256} \cdot \frac{6}{64} \cdot \frac{1}{32} = \frac{109}{524288} \approx 0.000208.$$

The resultant error +27 has four variants:

1. $+8+4+15 = +27 \quad \ldots \quad P_1 = \frac{1}{256} \cdot \frac{15}{64} \cdot \frac{1}{32} = \frac{15}{524288};$

2. $+6+12+9 = +27 \quad \ldots \quad P_2 = \frac{8}{256} \cdot \frac{1}{64} \cdot \frac{5}{32} = \frac{40}{524288};$

3. $+4+8+15 = +27 \quad \ldots \quad P_3 = \frac{28}{256} \cdot \frac{6}{64} \cdot \frac{1}{32} = \frac{168}{524288};$

4. $0+12+15 = +27 \quad \ldots \quad P_4 = \frac{70}{256} \cdot \frac{1}{64} \cdot \frac{1}{32} = \frac{70}{524288};$

whence the probability of the resultant error

$$P_{27} = \frac{15+40+168+70}{524288} = \frac{293}{524288} = 0.000559 \quad \text{etc.}$$

126. To systematise such calculations, it can be recommended that in table 8, vertical graphs for probabilities of the variants of resultant errors be added, after which, all that remains is to add the probabilities of the variants similar in magnitude and sign. All such calculations, for the purpose of our example, are given in fig.33.

_____

_____

Fig.33.

In examining this graph, it can be seen, that when several groups of error sources take place, the established three basic assumptions [illegible] error is [illegible]

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§28. Numerical expression of the Gauss law (scale of errors).

For practical utilisation of the three derived assumptions of the Gauss law of chance errors, it is necessary to establish a numerical expression for the assumptions, i.e. it is necessary to establish, what is the numerical relationship between the set limits of errors and the probability of occurrence of errors within these limits.

The assumptions of the Gauss law are some for all methods (series) of measurements, the chance errors of which obey this law. It is obvious, that the numerical expression of these assumptions must also be same. This means, that the limits, with respect to which the probabilities of occurrence of errors, are determined, must be expressed in some unit measures which are common for all series of measurements, independently of the metrical value of the errors (the method of measurement and the character of the measured quantity). Only in this way is it possible to assess both the accuracy of individual results independently of the method of measurement, and the accuracy of the methods themselves.

The accuracy (precision) of a given method of measurement is characterised by the values of errors arising with that method, - the more accurate the method, the less the errors, the greater the probability of occurrence of such errors. Therefore, for a general unit measure, expressing the limits of the error for all methods of measurement, and hence also for the measure of accuracy of any method of measurement, it is necessary to take a value of some error.

Let us suppose, that two methods of measurement of the same quantity have given the following two series of errors:

1st method: +5, -7, -2, +4, +8, -3 and +9;
2nd method: -12, -6, +16, +1, -9, +12 and -10.

It is obvious, that the first method in comparison with the second, is more accurate as the former gave errors of smaller value.

If, as the measure of accuracy, we take individual chance errors from these two series of measurements, then our conclusions will be most contradictory. Thus, from the errors of the first results of measurement (+5 and -12) we can conclude, that the 1st method is more accurate than the 2nd; yet the error of the second result (-7 and -6) leads to the opposite conclusion. Neither can the smallest errors (-2 and +1) be taken, as in this case the 2nd method will be more accurate than the 1st, which obviously does not correspond to the actuality. From this it follows, that we can not take an individual chance error as a measure of accuracy of a given method of measurement.

When it is essential to give a characteristic to a series (grouping) of various quantities (objects), some mean value (quantity) is generally taken.

For example, if out of a group of 17 apples of different size, we wish to give some approximate representation of the size of the apples in the whole group, using one apple without showing the others, we, naturally, would select an apple which is of the average size in the group.

In the same way, when assessing the accuracy of a given number of measurements we take the mean error as a measure of accuracy characterising this series of measurements.

As will be seen later, there are several mean errors; in artillery practice it is usual to take the so called middle error out of all the mean errors, as a measure of accuracy. Such a term for this error is explained by the fact that it is in the middle of a series of errors which lie in an increasing or decreasing order of their absolute magnitudes.

Let us return to the two series of errors and let us distribute these errors according to their absolute magnitudes, in an increasing order:

1st method: 2, 3, 4, 5, 7, 8, 9;
2nd method: 1, 6, 9, 10, 12, 14, 16.

In the first row, the error, occupying the middle position, is 5 [illegible] is seen, that the 1st method is definitely more accurate than the 2nd.

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127. To assess the group of apples, the apple of the mean size was taken. In positioning all the apples in an increasing order of magnitude (fig.34), such a mean (middle) apple was found to be the ninth.

Fig.34.

Occupying the middle position, this apple in size is greater than any of the apples of one (right) half and less than any of the apples of the other (left) half of the whole group of apples. From this analogy, the middle error is characterised by the same property.

By the middle error is termed such an error which by its absolute value is greater than each of the errors of one half and less than each of the errors of the other half of all the errors positioned (distributed) in a row in an increasing or decreasing order.

Table 9.

128.

129. From this determination, it is possible to find experimentally the value of the middle error which characterises the accuracy of a given series (method) of measurements.

From the determination of the middle error it follows, that the probability of obtaining the error which in absolute value is greater (or less) than the middle error, is a half.

Thus, if the middle error of a method of measurements is 25m, then the probability of obtaining an error of more than 25m (or less than 25m) is equal to a half.

These properties of the middle error make it a measure of accuracy (precision), and hence a measure by means of which the numerical relationship is expressed between the limits of the error and the probabilities of its occurrence within these limits.

To ascertain such a numerical relationship, the same methods that were used in arriving at the assumptions of the Gauss law, are applicable.

In the first place, we will find this relationship by calculation of the experimental data, making use of the numerical examples given in section 27.

In the first of these examples we had 170 results of pace-measurement of distance (see table 4).

Let us arrange, in an increasing order of absolute magnitude of errors, these 170 measurements (table 9) and let us find the middle error for this series of measurements commencing with the determination of the middle error.

With 170 measurements, the middle error in the series of errors must occupy a place between 85th and 86th errors. The absolute value of both the 85th and 86th errors is 60m; consequently, the middle error of this series of measurements is likewise 60m.

In order to determine the frequencies of occurrence of an error within the limits expressed in middle errors, we break up all the errors into groups of every 6cm (table 10), and we denote here the value of the middle error by the letter E.

Table 10.

In metres from ... to												
Errors. In terms of the value of middle error from ... to												
Number of errors.												
Frequencies of errors in %.												

130. Let us now make use of the data in the second example, in which were given 500 results of measurements of a distance by means of a

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130. distance, as it is this error that occupies the place between 290th and 291st errors. This is easily confirmed by adding the quantities of all the positive and negative errors obtained within the limits of 0 to +0.5% of distance:

76+72+70+66 = 290.

Let us divide the errors of this series into groups of every 0.5% of distance and let us determine, for this case, the frequencies of occurrence of the error within the limits, expressed in middle errors (table 11).

Table 11.

Errors.	In percentages of distance from ... to								
	In terms of the value of the middle error from ... to								
Number of errors.									
Frequencies of errors in %.									

Comparing the bottom lines in tables 10 and 11, we can note that the frequencies of occurrence of errors within the same limits, expressed in middle errors, are approximately equal to each other.

Fig.35.

131. Taking the mean values of the frequencies, let us suppose, that with a very large number of measurements, the frequencies of occurrence of errors, which are equal in absolute magnitude and which are different in sign, are equal to each other. Then, applying the law of large numbers, we can represent the numerical relationship between the limits of errors, expressed in mean errors, and the probabilities of occurrence of errors within these limits, by the graph shown in fig.35.

The sources of errors, which are obtained in any type of measurement, is infinitely great. The possibility, that the sources of errors may give in individual measurements either only positive, or only negative elementary errors, is not excluded. In such cases, despite the fact that the elementary errors in magnitude are infinitely small, the total error can be of any magnitude - theoretically from $-\infty$ to $+\infty$.

The probabilities of occurrence of such errors are negligibly small, and they can be ignored. Therefore the base of the Gauss curve is limited between 2σ (see fig.35).

But the probabilities of occurrence of errors within the limit from -6σ to -4σ, and from +4σ to +6σ are likewise very small:

0.0004+0.0031+0.0031+0.0004 = 0.007 = 0.7%.

Such errors may also be usefully ignored, and then the Gauss law can numerically, with certain rounding, be expressed by the scale shown in fig.36.

Fig.36.

This scale is termed an error scale. This error scale is used, as a rule, in calculations, when dealing with artillery fire on the theoretical basis.

From the error scale can be seen a confirmation of the basic assumptions of the Gauss law, mainly:

1. With the increase in the error, the probability of its occurrence decreases (0.25 - 0.16 - 0.07 - 0.02).

2. The probabilities of occurrence of negative errors and positive [illegible] (0.25 and 0.25; 0.16 and 0.16 etc).

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131. 3. A practical limit of error for any method (series) of measurements, is the limit ±4E.

It is possible to arrive at the same scale of errors by a different method - by the method of analising the origination of chance errors which obey the Gauss law.

132. In fig.33, section 27, are given the probabilities of occurrence of chance errors when three groups of sources of errors act jointly.

Let us find for this series of errors such a limit for which the probabilities of occurrence of a large magnitude and a small magnitude error are equal to each other, i.e. are equal to one half.

Adding the probabilities of occurrence of errors: -5, -3, -1, +1, +3 and +5, we obtain

$$2(0.088421+0.084316+0.076605) = 0.498684.$$

The result of the addition is very close to 0.5, and hence, the limit we seek, lies between 5 and 7, i.e. is equal to 6.

Taking the value of the middle error as 6, we break up all the errors into groups of every 6 (units) and we will determine the probabilities of occurrence of the error for each such group (table 12).

Table 12.

Limits of errors, Probabilities of occurrence of errors in %.							

Neglecting the probabilities of occurrence of errors within the limits of ±4E and ±6E, and taking to the nearest whole numbers of percentage, we obtain the same results: 2%, 7%, 16%, 25%, 25%, 16%, 7% and 2%.

The limits of the scale of errors given above, are expressed in whole numbers of E. In calculations which require greater precision use must be made of a scale, the limits of errors of which are expressed in parts of E: 0.5E or 0.25E (table 13).

This table shows that the Gauss law can definitely be expressed as a curve, because, as the limits of errors decrease, the probabilities of their occurrence differ less and less from each other in magnitude, and the graph made up from small parts of E will doubtlessly have a shape of a curve.

The scale of errors can also be made up somewhat differently (fig.37).

It is easy to see that the numbers 0.50, 0.82, 0.96 and 1.00 show the probabilities of occurrence of the error within the limits of 0 to ±E, 0 to ±2E etc.

It is quite obvious, that such a scale can be made up to any desired accuracy - in any fractions of E. A scale of errors, made up
133. on this principle, in hundredths of E (with an accuracy up to 0.01E), has the appearance of a table shown in the Appendix 1, (see Appendix 1, table of values $\Phi(\beta)$). This table is used in calculations which require special accuracy. In addition, this table in comparison with the usual scale of errors, in some cases simplifies the procedure of calculations.

Table 13.

Fig.37.

The scale of errors, whatever form it has, permits us to solve the following two basic problems:

1. To determine the probability of occurrence of the error within given limits.

2. To determine the limits of errors, knowing its probability of occurrence, knowing its middle error.

Let us show this by means of examples.

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133. We will commence with determination of probability of occurrence of the error within given limits. Let us suppose that the distance to the target estimated by the artilleryman, is 4000m, and the middle error of the given method of measurement is 80m.

We will find, in the first place, what errors could have been admitted by the artilleryman and what are the probabilities of these errors.

We will make up, for this purpose, a scale of errors expressed in values of middle error (fig.38).

Fig.38.

134. In examining this scale, the following deductions can be made.

1. The greatest error which the artilleryman could reasonably admit, can not exceed 320m both on the high and on the low side (±4E) i.e. the true value of the measured distance is within the limits 4000m ±320m, viz. 3680 to 4320m.

2. The probability that the artilleryman errs on the low side, is not greater than 80m, i.e. the probability that the true value of the measured distance is contained within the limits of 4000m to 4080m, is 0.25 = 25%. But the probability that the error was admitted on the high side from 160m to 240m, i.e. the probability that the true value of the measured distance is contained within the limits of 3760m to 3840m, is 0.07 = 7% etc.

3. The probability of occurrence of the error not exceeding 80m, independently from its sign, i.e. the probability that the true value of the measured distance is contained within the limits of 4000m ±80m (from 3920m to 4080m), is 0.25+0.25 = 0.50 = 50%. But the probability of the error which does not exceed 160m in both directions (±2E), i.e. the probability that the true value of the measured distance is contained within the limits of 3840m to 4160m, is 0.16+0.25+0.25+0.16 = 0.82 = 82% etc.

On the same scale we can determine the probability of occurrence of an error in any stipulated limits, i.e. in the limits not coinciding with whole number values of E.

Let us find, for example, the probability of occurrence of an error within the limits of -50m to -100m.

As can be seen, such an error partially takes in two limits (fig.39).

Fig.39.

Having allowed that within the limits of each E, the errors are distributed uniformly, we can form the following proportions:

80m-0.25	and	80m-0.16
30m-p1		20m-p2

Whence

$$p_1 = \frac{30 \times 0.25}{80} = 0.09375 \quad \text{and} \quad p_2 = \frac{20 \times 0.16}{80} = 0.04.$$

But the probability we seek

$$P = p_1 + p_2 = 0.09375 + 0.04 = 0.13375 \approx 13.4\%.$$

135. A slightly more accurate result will be obtained if the error scale, expressed in fractions of E, is taken, for example in 0.5E (fig.40):

$$p_1 = \frac{30 \times 0.12}{40} = 0.09; \quad p_2 = \frac{20 \times 0.09}{40} = 0.045;$$

$$P = 0.09 + 0.045 = 0.135 = 13.5\%.$$

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Fig.40.

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135. And, finally, a more accurate result is obtained from the table of values of $\Phi(\beta)$. In this table are given probabilities within the limits of 0 to $\pm\beta$, these limits being expressed in hundredths of E (every 0.01E). The procedure of solving, using the table of values of $\Phi(\beta)$ is as follows (fig.41):

Fig.41.

1. Let us express the limits of our error in fractions of E, by dividing each one of them by 80m:

$$50:80 = 0.625E \quad \text{and} \quad 100:80 = 1.25E.$$

2. We find from the table of values of $\Phi(\beta)$ the probabilities of occurrence of the error within the limits of 0 to $\pm 0.625E$ and 0 to $\pm 1.25E$:

$$p_1 = 0.32419+0.00246 = 0.32665 \quad \text{and} \quad p_2 = 0.60083.$$

3. We subtract the smaller probability from the greater, i.e. we find the probability of occurrence of the error within the limits of ±50m to ±100m (independent of the sign):

$$0.60083-0.32665 = 0.27418.$$

4. We determine the required probability by dividing the obtained result by 2, because positive and negative errors, contained within similar limits, are equally probable:

$$p = \frac{0.27418}{2} = 0.13709 \approx 13.7\%.$$

136. The greater accuracy of solution when using the table of values of $\Phi(\beta)$ is explained by the fact that the admission of uniformity of distribution of errors within the limits, expressed in hundredths of E, is very insignificant, as the limits themselves are small.

Knowing the magnitude of the middle error of a given method of measurement, as well as the magnitude of the error of individual result, it is possible, using the error scale, to assess the accuracy of this result.

Let us assume, that the actual distance to the target is 8000m. The gunner, having applied the method, the middle error of which is 10% of the distance being measured, has determined the distance to the target as 8500m, i.e. he has admitted an error of 500m.

In referring to the error scale, made up from the given middle error (fig.42) it is possible to draw the following conclusion.

Fig.42.

The error made by the gunner does not come outside the limits of ±600m, i.e. beyond the limits ±4. The probability of occurrence of the error within these limits is 0.50, i.e. the error, which does not come outside these limits, can occur on the average in 50 cases out of 100, and therefore we can consider that the determination of the distance by the gunner was performed with a sufficiently high accuracy.

If, however, the distance to the target was determined by the gunner, as 4500m, i.e. with an error of 1500m, then such a result for a given method of measurement must be considered as one of low accuracy, as the probability of occurrence of the error within the limits of ±1200m (0 ±8E) is only 0.14, i.e. such an error can occur only in 14 cases out of 100 on the average. The determination of the distance [illegible] exception) that the gunner is not performing properly the given method of measurement.

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assessment of the accuracy in the preparation of the original data when carrying out artillery fire.

28. Arithmetical mean and quadratic mean of an error and the relationship between them and the middle (median) error.

Determination of the magnitude of the middle (median) error by its position in the row of absolute values of errors (is denoted by E_{pp}), with a small number of measurements, does not ensure the required
137. accuracy. In such conditions, the possibility is not excluded that even one additional measurement may considerably change our judgement of the magnitude of the middle error, and hence also our judgement of the accuracy of a given method of measurement.

Let us suppose, that as a result of 11 measurements of the same quantity by some method, we obtained a series of errors and we distributed the absolute values of these errors in an increasing order:

$$1, 2, 3, 3, 4, 6, 8, 10, 14, 17, 20;$$

the middle error of this series

$$E_{pp} = 6.$$

Let us suppose, that one more measurement is made - twelfth. This measurement can, apparently, give an error which will have an absolute value less or greater than the middle error.

If the 12th measurement gives an error less than the middle error, for example 2, then the middle error of the new series (1,2,2,3,3,4,6, 8,10,14,17,20)

$$E_{pp} = \frac{4+6}{2} = 5.$$

If, however, the 12th measurement gives an error, greater than the middle error, for example 13, then the middle error of such a series (1,2,3,3,4,6,8,10,13,14,17,20)

$$E_{pp} = \frac{6+8}{2} = 7.$$

The method of measurement was the same, but the middle error depending on the result of only one additional measurement, assumed a value of either 5 or 7 instead of 6. The difference, as can be seen, is substantial.

Therefore when determining the magnitude of the middle error from a small number of measurements - and in practice we have to deal, as a rule, with such a number - it is necessary to take into consideration the numerical values of all the errors of a given series of measurements, i.e, instead of distributing the errors as a progressing series, to apply a method of calculation. Then the results of individual measurements will become less significant so far as the magnitude of the middle error is concerned, as was the case in the above example.

When determining the magnitude of the middle error from the results of measurements by the method of calculation, use is made, as a rule, of the values of middle errors: arithmetical mean and quadratic mean.

The manner in which these calculations are made, will be discussed in detail later - in the chapter on "Calculation of results of measurements". We will only establish here, what numerical relationships exists in the Gauss law (which follow this law) between the arithmetical
138. mean and the quadratic mean errors and the middle error as generally accepted, in artillery fire, as a measure of accuracy (precision).

By the arithmetical mean error (θ) is termed the mathematical expectation of the absolute value of the error.

Or in general form:

$$\theta = M(|\Delta|) = \int_{-\infty}^{+\infty} |\Delta| \varphi(\Delta)\, d\Delta = 2\int_{0}^{+\infty} \Delta \varphi(\Delta)\, d\Delta,$$

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138. ... arithmetical mean error;

$|\Delta_i|$ - absolute values of errors;

p_i - probabilities corresponding to these errors.

From this determination we find the relationship between the arithmetical mean error Δ_1 and the middle error E.

For the simplification of calculation we will assume that within the limits, expressed in whole numbers of E, the magnitudes of errors are distributed uniformly, i.e. for these limits we can take the mean values of the error (fig.43), expressed in fractions of E.

Fig.43.

Thus, for limits of 0 to $\pm E$ such a mean value of the error can be taken as equal to $\pm 1/2$ E; for limits of $\pm E$ to $\pm 2E$ - can be taken as equal to ± 1 1/2 E etc.

The probabilities of such absolute mean values of the error according to the error scale:

$$p_{\pm\frac{1}{2}E} = 0.25+0.25 = 0.50;$$

$$p_{\pm 1\frac{1}{2}E} = 0.16+0.16 = 0.32 \text{ etc.}$$

Then the mean arithmetical error, expressed as mathematical expectation of absolute value of the error:

$$\Delta_1 = \tfrac{1}{2}E\times 0.50+1\tfrac{1}{2}E\times 0.32+2\tfrac{1}{2}E\times 0.14+3\tfrac{1}{2}E\times 0.04 =$$

$$= \frac{50+96+70+28}{200}E = \frac{244}{200}E = \frac{122}{100}E \approx \frac{6}{5}E.$$

Whence

$$E = \frac{5}{6}\Delta_1.$$

139. If we allowed uniform distribution of magnitudes of errors within the limits, expressed not in whole numbers of E, but in fractions of E, the result would have been more accurate. Making use of the table of values of $\Phi(t)$, we obtain a more accurate result:

$$E = 0.84535\Delta_1 \approx \frac{5}{6}\Delta_1.$$

Let us show the truth of the established relationship between E and Δ_1 by a numerical example. In fig.33 section 27, are given the values of errors and the probabilities of their occurrence when three groups of error sources are present:

$$\Delta_1 = 1\times 0.068421\times 2+3\times 0.084316\times 2+5\times 0.076603\times 2+7\times 0.060295\times 2+$$
$$+9\times 0.054577\times 2+11\times 0.042646\times 2+13\times 0.031562\times 2+15\times 0.022091\times 2+$$
$$+17\times 0.014530\times 2+19\times 0.008953\times 2+21\times 0.005129\times 2+23\times 0.002704\times 2+$$
$$+25\times 0.001301\times 2+27\times 0.000559\times 2+29\times 0.000208\times 2+31\times 0.000063\times 2+$$
$$+33\times 0.000015\times 2+35\times 0.000002\times 2 = 7.067888.$$

Whence

$$E = 0.84535\times 7.067888 \approx 5.975.$$

Earlier (see page 132) it was established that the middle error of this series can be taken as equal to 6.

EXAMPLE 1. Determine the value of the middle error in a method of measurement, if the mean arithmetical error is 30m.

Solution. $E = 0.84535\times 30 = 25.3605$m, or $E = 5/6\times 30 = 25$m.

EXAMPLE 2. Determine the value of the mean arithmetical error, if the middle error in a method of measurement, is 20 divisions on the goniometer.

Solution. $\Delta_1 = 20:0.84535 = 23.661$ goniometer divisions or $\Delta_1 = 20:5/6 = 24$ goniometer divisions.

In ... quadratic error which it is necessary to denote by Δ_2.

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139. The mean quadratic error, is taken the value which is equal to the square root of the mathematical expectation of the square of the error.

$$\Delta_2 = \sqrt{\Delta_1^2 p_1 + \Delta_2^2 p_2 + \Delta_3^2 p_3 + \ldots + \Delta_s^2 p_s} = \sqrt{\sum_1^s \Delta_i^2 p_i},$$

from which

$$\Delta_2^2 = \Delta_1^2 p_1 + \Delta_2^2 p_2 + \Delta_3^2 p_3 + \ldots + \Delta_s^2 p_s = \sum_1^s \Delta_i^2 p_i.$$

140. From this last equation, we will establish the relationship between the mean quadratic error and the middle error. For this purpose, we take the mean values of errors, expressing them as fractions of Δ, in the same way as was done above when deriving the relationship between Δ_1 and Δ (see fig.43).

Having carried out the corresponding substitutions in the general formula for Δ_2^2, we obtain:

$$\Delta_2^2 = (\tfrac{1}{2}\Delta)^2 \times 0.50 + (1\tfrac{1}{2}\Delta)^2 \times 0.32 + (2\tfrac{1}{2}\Delta)^2 \times 0.14 + (3\tfrac{1}{2}\Delta)^2 \times 0.04 = \tfrac{1}{4} \times 0.50 \times \Delta^2 +$$
$$+ \tfrac{9}{4} \times 0.32 \times \Delta^2 + \tfrac{25}{4} \times 0.14 \times \Delta^2 + \tfrac{49}{4} \times 0.04 \times \Delta^2 = \frac{50 + 288 + 350 + 196}{400}\Delta^2 = \frac{221}{100}\Delta^2.$$

Let us extract the square root from both sides of the equation:

$$\Delta_2 = 1.4866\Delta \approx 1.5\Delta = \tfrac{3}{2}\Delta,$$

whence

$$\Delta = \tfrac{2}{3}\Delta_2.$$

More accurate calculations from the table of values of $\Phi(t)$ will give:

$$\Delta = 0.67449 \ \Delta_2 \approx \tfrac{2}{3}\Delta_2.$$

We will show the truth of this relationship, using the same numerical example as for Δ_1:

$$\Delta_2^2 = 1^2 \times 0.176842 + 3^2 \times 0.166632 + 5^2 \times 0.153210 + 7^2 \times 0.132592 + 9^2 \times 0.109154 +$$
$$+ 11^2 \times 0.085292 + 13^2 \times 0.063164 + 15^2 \times 0.044182 + 17^2 \times 0.02906 + 19^2 \times 0.017906 +$$
$$+ 21^2 \times 0.010238 + 23^2 \times 0.005408 + 25^2 \times 0.002602 + 27^2 \times 0.001118 + 29^2 \times 0.000416 +$$
$$+ 31^2 \times 0.000130 + 33^2 \times 0.000030 + 35^2 \times 0.000004 = 76.999904.$$

Hence

$$\Delta_2 = \sqrt{76.999904} = 8.775.$$

Then

$$\Delta = 0.67449\Delta_2 = 0.67449 \times 8.775 = 5.91864985 \approx 5.92.$$

As can be seen, the result is very close to the one obtained when determining Δ from Δ_1 and resolving into series.

EXAMPLE 3. Determine the value of the middle error in a method of measurement, if the mean quadratic error is equal to 300m.

Solution. $\Delta = 0.67449 \times 300 = 202.347$m, or $\Delta = 2/3 \times 300 = 200$m.

141. EXAMPLE 4. The middle error is equal to 0.04. Find the value of the mean quadratic error.

Solution. $\Delta_2 = 4:0.67449 \approx 5.9304$ of a goniometer division, or $\Delta_2 = 4:2/3 = 6$ goniometer divisions.

The relationship between the mean arithmetical and mean quadratic errors can be easily established through the middle error:

$$\Delta = \tfrac{2}{3}\Delta_2 = \tfrac{5}{6}\Delta_1,$$

from which

$$\Delta_1 = \tfrac{4}{5}\Delta_2, \quad \text{or} \quad \Delta_2 = \tfrac{5}{4}\Delta_1,$$

or more accurately

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$$\Delta_1 = 0.79787\Delta_2 \quad \text{or} \quad \Delta_2 = 1.25331\Delta_1.$$

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141. Example 3. The mean arithmetical error in a method of measurement is equal to 0.24. Find the value of the mean quadratic error.

Solution. $\varepsilon_2 = 3/4\varepsilon_1 = 0.30$.

It must be noted, that the mean arithmetical and the mean quadratic errors can be, and are, utilized (especially the latter) as parameters characterising the accuracy of a method of measurement.

X. The law of equal probability.

In practice, when conducting artillery measurements, we frequently meet with the necessity to round the results of our measurements, which, naturally, leads to errors - errors of rounding.

Such errors arise, in the first place, in cases when, in taking readings from scales of measuring instruments, we have, for various reasons, to take these readings in whole divisions on the scales, i.e. we can not or do not wish to take the apparent fractions of the divisions.

For example, in measuring distance by means of a range finder, the divisions on the scale of which, are for every 1-0m, and rounding the results to the nearest hundred metres, we will be admitting errors differing in value and sign depending on the position, at the moment of taking the reading, which the pointer occupies with respect to the space between two adjacent divisions of the scale.

Thus, if the pointer happens to be in position "b" (fig.44) and the distance will be taken as equal to 2100m, an error will be admitted on the lower side, of approximately 25m. If, however, the pointer, when taking the reading, will occupy the position "a", the distance will be determined as 2200m; the error admitted will then be on the larger side,

142. by 25m. When the position of the pointer is "c", the distance may be rounded to 2100m as well as to 2200m - the errors of rounding in this case will be equal to either -50m or +50m, and, finally, if the pointer is in the positions of either "a" or "c", the readings taken will not be rounded, i.e. the errors will be zero.

This example shows, that the errors of rounding can be negative or positive and that in magnitude, these errors can not exceed some quite definite limit - the limit equal to a half of the value of the division of the instrument scale (±100/2).

Fig.44.

Fig.45.

It is quite obvious, that when taking readings, the pointer can, with the same probability, happen to come opposite any point on the scale within the limits of its division. Consequently, the errors of rounding are equally probable. It is therefore accepted, that the errors of rounding obey the law of equal probability.

The errors of rounding occur also when it is necessary to translate the results of measurements to other instruments.

Thus, for example, when setting the calculated distance to a target on to the sighting equipment, the distance is expressed in whole divisions of the sight and the level, frequently omitting to consider the remainder or excess distance. In this case the errors of rounding would not exceed a half of the value of the division of a level (from $-1/2\Delta_{level}$ to $+1/2\Delta_{level}$) and will be equally probable.

Thus the law of equal probability consists essentially in that the probability of occurrence of errors which obey this law, does not depend on the magnitude of the errors themselves, and that for some limit, there exists a constant magnitude.

For clearer demonstration we will represent this law in the form

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We will denote the value of one division of the instrument by 2l.

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142. [illegible] in which circulating from the point O, divisions equal to L to any suitable scale. This means, that the errors must not come outside the limits of 0 to 2L.

143. The errors are equally possible, independently of their values. Consequently, the probabilities of their occurrence must be expressed by ordinates of the same height. If we draw such ordinates for all the possible errors in value and sign, within the limits of 0 to 2L, and if we join their tops by a common line, we will obtain a straight line CD which is parallel to the axis OA. The fact that these lines are parallel, expresses the equality in probability of the errors.

An graphical representation of the probabilities of occurrence of errors within various limits in magnitude and sign, these probabilities are taken as areas bounded by the ordinates which correspond to the set limits.

For example, the probability of occurrence of errors within the limits of Δ_1 and Δ_2, are taken as equal to the area abcd:

$$p = \text{abcd} = ab \cdot h = (\Delta_2 - \Delta_1) \cdot h.$$

In this expression, h is not known. We will find the relationship between h and L, because it is L, being a half of the value of the division of the instrument scale, that characterizes the accuracy of the scale.

The sum of the probabilities of occurrence of all the possible errors in magnitude and sign, in the case of the law of equal probability, can be expressed as the area ABCD, and this area must be equal to unity:

$$p = ABCD = AB \cdot h = 2L \cdot h = 1,$$

whence

$$h = \frac{1}{2L}.$$

Then the probability of occurrence of the error within specific limits

$$p = (\Delta_2 - \Delta_1) \cdot h = (\Delta_2 - \Delta_1) \cdot \frac{1}{2L} = \frac{\Delta_2 - \Delta_1}{2L}.$$

EXAMPLE 1. The divisions on the range finder scale are for every 100m. What is the probability that in the measurement of the distance, an error will be admitted, on the high side, of not less than 20m?

Solution. According to the condition: $2L = 100m$, $\Delta_1 = 0$ and $\Delta_2 = +20m$. Substituting the numerical values of 2L, Δ_1 and Δ_2 in the general formula, we obtain:

$$p = \frac{\Delta_2 - \Delta_1}{2L} = \frac{+20-0}{100} = 0.2.$$

EXAMPLE 2. In the conditions in the example 1 above, determine the probability of admitting an error on the low side, of not less than 10m and not more than 40m.

Solution. Here $\Delta_1 = -40m$ and $\Delta_2 = -10m$. Then

$$p = \frac{-10-(-40)}{100} = \frac{+30}{100} = 0.3.$$

EXAMPLE 3. What is the probability that in a sight setting an error will be admitted, not exceeding 10m, if $2L = 50m$?

Solution. $2L = 50m$, $\Delta_1 = -10m$ and $\Delta_2 = +10m$. Then

$$p = \frac{+10-(-10)}{50} = \frac{20}{50} = 0.4.$$

144. Let us now turn to the measure of accuracy for the law of equal probability. We will find the expression for the middle error of this law.

The middle error E is characterized by the fact that the probability of occurrence of an error within the limits of zero and $-E$ is 0.5.

Taking Δ_1 [illegible] and Δ_2 [illegible] law of equal probability, and substituting these values of Δ_1 and Δ_2

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144. in the general formula, we obtain:

$$\frac{1}{2} = \frac{+E^p-(-E^p)}{2L} = \frac{2E^p}{2L} = \frac{E^p}{L},$$

from which

$$E^p = \frac{1}{2}.$$

Generally, when discussing the accuracy of the law of equal probability, use is made of its other characteristic - the mean quadratic error E_2^p.

The mean quadratic error is equal to the square root of the mathematical expectation of the square of the error:

$$E_2^p = \sqrt{\Delta_1^2 p_1 + \Delta_2^2 p_2 + \ldots + \Delta_S^2 p_S}.$$

As for the law of equal probability

$$p_1 = p_2 = p_3 = \ldots = p_S = p = \frac{1}{2L},$$

then

$$E_2^p = \sqrt{(\Delta_1^2 + \Delta_2^2 + \ldots + \Delta_S^2)\frac{1}{2L}} = \sqrt{\frac{\Delta_1^2+\Delta_2^2+\ldots+\Delta_S^2}{2L}}.$$

Let us express the errors $\Delta_1, \Delta_2, \ldots \Delta_S$ in fractions of L. For this purpose we break up one division of the scale (2L) into 8 equal parts - into limits (fig.46), and we take for each such limit, the mean value of the error. Thus, for the limit of 0 to $\pm 1/4$ L such a mean value, is

$$\Delta_1 = (0 \pm \tfrac{1}{4}L) : 2 = \pm\tfrac{1}{8}L;$$

Fig.46.

145. for the limit of from $\pm\frac{1}{4}L$ to $\pm\frac{2}{4}L$

$$\Delta_2 = (\pm\tfrac{1}{4}L + \pm\tfrac{2}{4}L) : 2 = \pm\tfrac{3}{8}L \quad \text{etc.}$$

The probabilities of occurrence of the error in each of these limits, are equal to each other, and as according to the condition there are 8 limits, then the probability of occurrence of the error will be

$$p = \frac{1}{2L} = \frac{1}{8}.$$

Substituting Δ_1, Δ_2... etc, in the above formula, by the corresponding mean values and substituting for p = 1/8, we obtain

$$E_2^p = \sqrt{\left[2(\tfrac{1}{8}L)^2 + 2(\tfrac{3}{8}L)^2 + 2(\tfrac{5}{8}L)^2 + 2(\tfrac{7}{8}L)^2\right]\frac{1}{8}} = \sqrt{L^2 \cdot \frac{21}{64}} \approx \sqrt{L^2 \cdot \tfrac{1}{3}} = \frac{L}{\sqrt{3}}.$$

In this expression the coefficient 2 shows that any of the errors can be positive or negative.

In rounding 21/64 to 1/3, an error was admitted:

$$\frac{1}{3} - \frac{21}{64} = \frac{64-63}{192} = \frac{1}{192}$$

If the division 2L were divided into 16 equal parts instead of 8, then

$$E_2^p = \sqrt{L^2 \frac{2(1+9+25+49+81+121+169+225)2}{256} \cdot \frac{1}{16}} = \sqrt{L^2 \cdot \frac{85}{256}} \approx \sqrt{L^2 \cdot \frac{1}{3}} = \frac{L}{\sqrt{3}}$$

In this case, however, rounding of 85/256 to 1/3 will give an error:

$$\frac{1}{3} - \frac{85}{256} = \frac{256-255}{768} = \frac{1}{768}.$$

When 2L is divided into an infinite number of equal parts, an

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145. exact relationship is obtained:

$$E_2^p = \frac{1}{\sqrt{3}}$$

146. EXAMPLE. Determine the middle and the mean quadratic errors when taking readings from the protractor ring of a tele-stereoscope.

Solution. 2L = 0-01; L = 0-00.5 = 0.5 of the protractor division. Then

$$E^p = \frac{L}{2} = \frac{0.5}{2} = 0.25 \text{ of protractor division;}$$

$$E_2^p = \frac{1}{\sqrt{3}} = \frac{0.5}{\sqrt{3}} \approx 0.29 \approx 0.3 \text{ of protractor division.}$$

31. Problems on linear errors.

PROBLEM 68. The following values of errors in individual measurements of an angle are found to be: +0-10, -0-05, +0-07, -0-03, -0-01, +0-04, +0-06, -0-02, +0-01, 0-00 and +0-02. Determine the value of the middle error from its position in the series of errors.

Answer. 0-03.

PROBLEM 69. In measuring the same distance, the following errors of individual results were obtained: -20m, +40m, +10m, -15m, +5m, +20m, -5m and +25m. Determine the value of the middle error from its position in the series of errors.

Answer. 17.5m.

PROBLEM 70. The middle error of a method of measurement of distance, is equal to 100m. What is the probability that in measurement of the distance by this method an error will be admitted: a) of not more than 100m; b) of not more than 200m; c) of not more than 50m; d) of not less than 100m and not more than 200m; e) of not less than 25m and not more than 75m; f) of not more than 50m on the low side and not more than 100m on the high side?

Answer. a) 0.5 (50%)*; b) 0.82226 (82%); c) 0.26407 (26%); d) 0.32266 (32%); e) 0.25314 (25.4%); f) 0.38204 (38%).

PROBLEM 71. The middle error in a method of determining of a goniometer setting, is 0-05. What is the probability that, in determining the goniometer setting by this method, an error is admitted: a) of not more than 0-02; b) of not more than 0-08; c) within the limits from ±0-03 to ±0-10; d) within the limits from -0-04 to -0-12; e) within the limits from -0-06 to +0-03.

Answer. a) 0.21268 (21%); b) 0.71949 (71%); c) 0.50836 (51%); d) 0.24199 (24%); e) 0.44800 (44%).

PROBLEM 72. Find the value of the middle error if the mean arithmetical error of the method of measurement is 180m.

Answer. 152.163m.

PROBLEM 73. What is the probability of occurrence of an error not exceeding 0-15, if the mean arithmetical error of the method of measurement is 0-12?

Answer. Δ = 10.1442 goniometer divisions; P = 0.68141 (67.2%).

PROBLEM 74. Determine the mean arithmetical error of a given method of measurement, if its middle error is 30m.

Answer. 35.5m.

PROBLEM 75. Find the value of the middle error, if the mean quadratic error of the method of measurement is equal to 40 divisions on the goniometer.

Answer. About 27 goniometer divisions.

147. PROBLEM 76. What is the probability of occurrence of an error within the limits from -20m to -50m, if the mean quadratic error of the method of measurement is 49m?

Answer. Δ = 35.05m approximately equals 33m; P = 0.18794 (18.2%).

PROBLEM 77. The middle error is 150m. What is the value of the mean quadratic error of this method of measurement?

Answer. $E_2 \approx 222.4$m.

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* In this and the following problems the answers are given from the table of $\Phi(t)$ [brackets in original] to four decimal places.

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147. PROBLEM 78. The mean arithmetical error of a method of measurement is 0-07. What is the mean quadratic error of this method?

Answer. $E_2 \approx 8.77$ goniometer divisions.

PROBLEM 79. The mean quadratic error of a method of measurement is 53m. Determine the value of the mean arithmetical error of this method.

Answer. $E_1 \approx 42.3$m.

PROBLEM 80. What is the probability that in setting a level, an error of not more than 5m is admitted, if one division of the level indicator is equal to 20m?

Answer. 0.5.

PROBLEM 81. A compass has divisions for every 1-00. What is the probability that in taking azimuth reading, an error of not more than 0-30 is admitted?

Answer. 0.6.

PROBLEM 82. With the same condition as in problem 81, determine the probability of occurrence of an error within the limits from -0-20 to +0-50.

Answer. 0.7.

PROBLEM 83. Determine the middle and the mean quadratic errors in sight setting, if $\Delta x = 60$m.

Answer. $E^p = 15$m; $E_2^p = 17.3$m.

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148.

Chapter 6.

ADDITION OF LAWS OF CHANCE ERRORS.

32. Addition of Gauss laws.

If, when measuring, there occur errors from several sources of errors which are themselves independent from each other, then the errors of measurement, being the result of a combined effect of these sources, will be total errors.

Such total errors, will obviously follow some new, combined (added) law of errors. It is also obvious, that this new law of errors must differ from each of the laws governing the errors of individual sources (groups of sources) of errors.

The question arises, how is this new law to be found, knowing the laws which act jointly, i.e. the question arises about the addition of these interacting laws.

Any law of chance errors is characterised, in the first place, by a definite relationship between the value of the error and the probability of its occurrence, which relationship is expressed by a corresponding graph, and, in the second place, by the value of the middle error, being a measure of the accuracy of this law. Consequently, it is necessary to find this relationship and its middle error, also for the new combined law.

Let us establish the appearance of this law which is obtained when several Gauss laws are added.

Let us recall the numerical example, in which it was established that when three groups of error sources are present, the resultant errors follow the Gauss law, i.e. the same law which governed the errors of these individual groups (section 27, fig.33). Further (section 29), this conclusion was confirmed by that in the case of a series of such resultant errors, the numerical relationships between E, E_1 and E_2 proved to be such that conform to the Gauss law. It is possible to consider the laws which govern the errors of different groups of sources as independent Gauss laws, and therefore it is possible to draw a
149. conclusion that, when adding several Gauss laws, the new law of errors will be also a Gauss law.

There remains to find, for this new law, its middle error.

Let us first find for the new Gauss law, its mean quadratic error E_2, and from it - its middle error E, proceeding from the fact that in the Gauss law there exists a relationship between the middle error and the mean quadratic error: $E = 2/3\ E_2$ or $E_2 = 3/2\ E$.

Let us suppose that in conditions of a given measurement, two Gauss laws exist, the mean quadratic errors of which, expressed by the mathematical expectation of the squares of the errors Δ' and Δ'' corresponding to these laws, are as follows:

$$\mu'^2 = \ldots(\Delta'^2) \quad \text{and} \quad \mu''^2 = \ldots(\Delta''^2).$$

Each resultant error Δ of the new law is equal to the sum of the errors of mutually acting laws:

$$\Delta = \Delta' + \Delta'',$$

therefore, also the mean quadratic error E can be expressed by the mathematical expectation of the square of this sum:

$$\ldots(\Delta^2) = \ldots[(\Delta' + \Delta'')^2] = \ldots[\Delta'^2 + 2\Delta'\Delta'' + \Delta''^2].$$

Applying the theorem of addition of mathematical expectations, we obtain:

$$[\ldots(\Delta^2) = \ldots(\Delta'^2) + \ldots(\Delta''^2)]$$

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149. In this expression $M.E.(\Delta'^2)$ and $M.E.(\Delta''^2)$ are known from the condition:

$$M.E.(\Delta'^2) = E_2'^2 \quad \text{and} \quad M.E.(\Delta''^2) = E_2''^2 .$$

From the theorem of multiplication of mathematical expectations

$$M.E.(2\Delta'\Delta'') = 2 \cdot M.E.(\Delta') \cdot M.E.(\Delta'').$$

But the Gauss law is symmetrical - positive and negative errors which are equal in value, are equally probable. Therefore the mathematical expectation of the first degree of the error (taking into consideration its sign) of the law is equal to zero, i.e.

$$2 \cdot M.E.(\Delta') \cdot M.E.(\Delta'') = 2 \cdot 0 \cdot 0 = 0.$$

150. Then

$$E_2^2 = M.E.(\Delta'^2)+M.E.(2\Delta'\Delta'')+M.E.(\Delta''^2) = E_2'^2+0+E_2''^2 = E_2'^2+E_2''^2,$$

whence

$$E_2 = \sqrt{E_2'^2+E_2''^2}.$$

Substituting in this expression $E_2 = 3/2\ E$, $E_2' = 3/2\ E'$ and $E_2'' = 3/2\ E''$, we obtain:

$$\frac{3}{2}E = \sqrt{(\frac{3}{2}E')^2+(\frac{3}{2}E'')^2};$$

after contraction

$$E = \sqrt{E'^2+E''^2}.$$

Applying the same method of calculation to three Gauss laws, we obtain:

$$E = \sqrt{E'^2+E''^2+E'''^2},$$

adding, in the first place, two Gauss laws and then adding the obtained result to the third Gauss law.

It is obvious, that we will arrive at the same conclusion for any number of mutually acting Gauss laws:

$$E = \sqrt{\sum_1^n E^{12}}.$$

<u>When adding several independent Gauss laws, the middle error of the new, combined Gauss law is equal to the square root of the sum of the squares of the middle errors of the added laws</u>.

Let us verify this deduction, making use of the same numerical example for obtaining chance errors when three groups of error sources act jointly (see section 27, fig.33).

In that example, there were three Gauss laws which differed in their errors and the probabilities of their occurrence. Let us determine, for each individual law, its mean quadratic error.

<u>First group</u>. Possible errors: -8, -6, -4, -2, 0, +2, +4, +6 and +8; the corresponding probabilities of the occurrence of these errors:

$$\frac{1}{256}, \frac{8}{256}, \frac{28}{256}, \frac{56}{256}, \frac{70}{256}, \frac{56}{256}, \frac{28}{256}, \frac{8}{256} \quad \text{and} \quad \frac{1}{256}.$$

Then the mean quadratic error of this law

$$E_2' = \sqrt{64 \cdot \frac{1}{256} \cdot 2+36 \cdot \frac{8}{256} \cdot 2+16 \cdot \frac{28}{256} \cdot 2+4 \cdot \frac{56}{256} \cdot 2+0 \cdot \frac{70}{256}} =$$

$$= \sqrt{\frac{2048}{256}} = \sqrt{8}.$$

151. <u>Second group</u>. Possible errors: -12, -8, -4, 0, +4, +8, +12; the probabilities of their occurrence are respectively: 1/64, 6/64, 15/64, 20/64, 15/64, 6/64, and 1/64. The mean quadratic error of this law

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151. $E_2'' = \sqrt{144 \cdot \frac{1}{64} \cdot 2+64 \cdot \frac{6}{64} \cdot 2+16 \cdot \frac{15}{64} \cdot 2+0 \cdot \frac{20}{64}} = \sqrt{\frac{1536}{64}} = \sqrt{24}.$

Third group. Possible errors: -15, -9, -3, +3, +9, and +15; corresponding probabilities: 1/32, 5/32, 10/32, 10/32, 5/32 and 1/32. Then

$$E_2''' = \sqrt{225 \cdot \frac{1}{32} \cdot 2+81 \cdot \frac{5}{32} \cdot 2+9 \cdot \frac{10}{32} \cdot 2} = \sqrt{\frac{1440}{32}} = \sqrt{45}.$$

Substituting the squares of the obtained mean quadratic errors, in the general formula, we obtain:

$$E_2 = \sqrt{E_2'^2+E_2''^2+E_2'''^2} = \sqrt{8+24+45} = \sqrt{77} = 8.775.$$

In determining the value of the mean quadratic error from the resultant errors (section 29, page 140) the same answer was obtained: $E_2 = 8.775$, which confirms the correctness of the formula derived by us, because if the equation

$$E_2 = \sqrt{E_2'^2+E_2''^2+E_2'''^2}$$

is true, then on the basis of the relationship $E_2 = 3/2\ E$, the equation

$$E = \sqrt{E'^2+E''^2+E'''^2}$$

will be true.

EXAMPLE 1. We are dealing with two Gauss laws, the middle errors of which are: $E' = 200m$ and $E'' = 60m$. What is the middle error of the combined Gauss law?

Solution. Here we have mutual action of two Gauss laws, the middle errors of which, according to the condition of the example, are: $E' = 200m$ and $E'' = 60m$. Substituting the numerical values in the general formula, we obtain the middle error of the new combined Gauss law:

$$E = \sqrt{E'^2+E''^2} = \sqrt{200^2+60^2} = \sqrt{43600} \approx 209m.$$

EXAMPLE 2. In determining the calculated distance, the middle errors of the Gauss law are as follows: when meteorological conditions of firing are considered - 0.5% of distance; when ballistic conditions of firing are considered - 0.3%; and when determining the topographical distance - 0.8%. Determine the value of the total middle error.

Solution. Substituting the numerical values of the middle errors of the mutually acting Gauss laws, in the general formula, we obtain:

$$E = \sqrt{E'^2+E''^2+E'''^2} = \sqrt{0.5^2+0.3^2+0.8^2} = \sqrt{0.25+0.09+0.64} =$$

$$= \sqrt{0.98} \approx 1\% \text{ of distance.}$$

152. 33. Addition of the Gauss law and the law of equal probability.

Most frequently encountered in artillery measurements is the simultaneous effect of the Gauss laws and the laws of equal probability.

For example, in measurement of angles by means of stereoscope, i.e. in setting the stereoscope, we admit errors which follow the Gauss law, and in taking readings from the scale of the same stereoscope, the errors obtained follow the law of equal probability, and so on.

Let us see, what form will be the combined law of errors which is obtained when these laws act mutually.

We will commence by the addition of the Gauss law and the law of equal probability, when $\Delta < E$.

Let us state that between the parameters E and Δ of the Gauss law and the law of equal probability, there exists the following relationship: $\Delta = E/2$.

Let us find the resultant errors which obtain when the above stated laws act jointly. For simplification of calculations, we will

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152. add the mean values of errors for the limits which are equal in magnitude. We will here express the mean errors, which follow the law of equal probability, as parts of the middle error of the Gauss law, having in that L = E/2 (table 14).

Table 14.

In the top horizontal column of the graph A are given the mean errors, which follow the Gauss law, and in the left vertical column of the graph B - the mean errors conforming to the law of equal probability; and at the intersection of these graphs - the resultant (total) errors.

Let us now calculate the probabilities of occurrence of each of these resultant errors.

The resultant error -63/16 E obtains when the mean error of the Gauss law -3 1/2 E is added to the mean error of the law of equal probability -7/16 E. The probability of occurrence of the error -3 1/2 E, is 0.02. The probability of occurrence of the error -7/16 E, is 0.125. The probability of occurrence of the total error -63/16 E, according to the theorem of multiplication, is 0.02x0.125 = 0.00250.

The probability of occurrence of the resultant error -47/16 E, being the sum (-2 1/2 E)+(-7/16 E), is 0.07x0.125 = 0.00875 etc.

We will write all the calculations in the general table (table 15).

Table 15.

Having added the probabilities of occurrence of the errors, in the vertical columns of the graphs, we note that the probability of occurrence of an error within the limits from -63/16 E to -49/16 E, is 0.02; the probability of occurrence of an error within the limits from
154. -47/16 E to -33/16 E is 0.07; within the limits from -31/16 E to -17/16 E, the probability is 0.16 etc. We obtain a distribution of probabilities closely conforming to the Gauss law. Consequently, in the case, when the parameter of the law of equal probability L = E/2, <u>the new combined law, when the Gauss law is added to the law of equal probability, is also a Gauss law</u>.

This condition will also be true for the case when the parameter L of the law of equal probability is equal to the middle error E of the Gauss law, i.e. when L = E. Carrying out calculations similar to those when L = E/2, we obtain a graph of such a total (combined) law (fig.47).

Fig.47.

As can be seen, the distribution of the probabilities of occurrence of the error in this combined law differs very slightly from the distribution which follows the Gauss law.

Consequently, if when adding the Gauss law to the law of equal probability the value of the parameter of the law of equal probability L does not exceed the value of the parameter of the Gauss law, i.e. if $L \leq E$, then the combined law can be taken as the Gauss law.

When $L > E$ the combined law will not be a Gauss law.

Fig.48.

Thus, for example, when L = 2E, although the combined law is expressed graphically by a curve somewhat similar to the Gauss curve,
155. actually this combined law cannot be taken as the Gauss law. The distribution of probabilities in the combined law differs from the distribution of the probabilities in the Gauss law.

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155. When the difference between the parameter of the law of equal probability and the middle error of the Gauss law is great, for example, when $L = 4E$, the combined law does not resemble, even graphically, the Gauss law (fig.49).

Fig.49.

When adding the Gauss law and the law of equal probability, the mean quadratic error can be expressed as

$$E_2 = \sqrt{E_2'^2 + E_2''^2},$$

where E_2' - the mean quadratic error of the Gauss law,
E_2'' - the mean quadratic error of the law of equal probability.

Substituting for E_2'' the accepted expression for the mean quadratic error of the law of equal probability $L/\sqrt{3}$, we obtain:

$$E_2 = \sqrt{E_2'^2 + \frac{L^2}{3}}.$$

We had established, that when adding the Gauss law and the law of equal probability for the case when $L < E$, the combined law can be taken as the Gauss law. Therefore, it is possible to change from the mean quadratic error E_2, of such a combined law, to its middle error E, on the basis of the well known relationship between E_2 and E:

$$E = 0.67449 E_2 = 0.67449 \sqrt{E_2'^2 + \frac{L^2}{3}},$$

or, introducing the coefficient 0.67449 inside the radical sign:

$$E = \sqrt{(0.67449 E_2')^2 + \frac{0.67449^2 L^2}{3}} = \sqrt{E'^2 + 0.1516 L^2}$$

where E' - the middle error of the combined Gauss law.

EXAMPLE 1. Show, what combined law will be obtained when a Gauss law with the middle error $E' = 10m$, is added to the law of equal probability, the parameter of which $L = 5m$, and find its middle error.

Solution. We will determine in the first place, the form of the combined law. The parameter L of the law of equal probability is less than the middle error E' of the Gauss law. Consequently, the combined
156. law can be taken, with sufficient accuracy for practical purposes, as the Gauss law.

From the derived formula, the middle error of the combined law, is

$$E = \sqrt{E'^2 + 0.1516 L^2} = \sqrt{100 + 0.1516 \times 25} = \sqrt{103.79} \approx 10.2m.$$

EXAMPLE 2. The middle error of the Gauss law $E' = 0.02$. The parameter of the law of equal probability $L = 0.05$. Determine the form of the combined law, obtained when adding these two laws, and find its characteristic.

Solution. Here is given a case, when $L > E$ ($L = 2.5E$). This means that the combined law is not a Gauss law, and that the formula

$$E = \sqrt{E'^2 + 0.1516 L^2}$$

does not apply to this law. For this law it is only possible to determine its mean quadratic error:

$$\sqrt{E'^2 + \frac{L^2}{3}} = \sqrt{\left(\frac{1}{0.67449}\right)^2 \frac{L^2}{3}} = \sqrt{\frac{4}{0.4549} \frac{25}{3}} \approx 4.14 \text{ goniometer divisions.}$$

Comparing the above case of addition of the Gauss law, and the addition of the law with the law of equal probability, we can draw the following conclusions:

1. The sum of several laws is that when adding several Gauss laws, the sum is also a Gauss law.

2. It is possible the necessity of taking in other laws of

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156. when adding the Gauss law with the law of equal probability the combined law can be taken as a Gauss law.

These properties of the Gauss law ensure its very wide application; it was believed, at one time, that no laws of chance errors existed other than the Gauss law. It is these properties of the Gauss law that permit us to treat the errors, arising in the process of artillery measurements, as errors which obey the Gauss law, although in these measurements there are other laws of errors apart from those of the Gauss law.

34. Problems on the addition of laws of errors.

PROBLEM 84. Determine the characteristics of the law of errors which is obtained as a result of addition of three Gauss laws, the middle errors of which are 4, 6 and 12m.

Answer. $E = 14m$; $E_1 = 16.6m$; $E_2 = 20.8m$.

157. PROBLEM 85. Determine the characteristics of the law of errors obtained as a result of addition of the Gauss laws, the middle errors of which are 5, 11, 15 and 23m.

Answer. $E = 30m$; $E_1 = 35.5m$; $E_2 = 44.5m$.

PROBLEM 86. In measurement of an angle, two Gauss laws act mutually, the middle errors of these laws being 0-06 and 0-08. What is the probability that the angle measured will have an error not exceeding 0-15?

Answer. $E = 0\text{-}10$; $P = 0.68833$ (68%).

PROBLEM 87. In determination of the calculated distance, the middle errors are as follows: when meteorological conditions of firing are considered - 30m; when ballistic conditions of firing are considered - 26m; and when topographical distance - 45m. What is the probability that the calculated distance determined will have an error not exceeding 50m.

Answer. $E = 60m$; $P = 0.42593$ (42%).

PROBLEM 88. In a given measurement, two Gauss laws act mutually, the middle error of one of the laws being 32m. Determine the value of the middle error of the other Gauss law, if the combined middle error is equal to 40m.

Answer. 24m.

PROBLEM 89. Indicate what combined law will be obtained when a Gauss law, the middle error of which is 12m is added to the law of equal probability, the parameter of which L is 4m.

Answer. The law follows closely that of Gauss law: $E = 12.1m$.

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158. Chapter 7.

CALCULATION OF THE RESULTS OF MEASUREMENTS.

35. Adequate value of the measured quantity.

Artillery firing, commencing with preparation of the data for the opening of fire and finishing with the final command, is built up on the basis of utilisation of the results of various types of measurements of: distances, angles, deviations of explosions etc.

Errors in measurements are unavoidable; a measurement never provides a true (absolutely accurate) value of the measured quantity. Therefore, when working out the results of measurements, the first question that arises is: what value, in the result of the measurements made, should be taken as the true value of the measured quantity; what value will be most adequate to this true value, i.e. the value that will have the least error if taken as the true value. We therefore commence to discover such an adequate value.

Let us suppose, that when measuring some unknown quantity by means of a method, the errors of which follow the Gauss law, the following results have been obtained:

$$x_1 x_2 x_3 \ldots x_{S-1} x_S.$$

We will denote the true value of the measured quantity by x_0. Then the errors of these results

$$\begin{aligned} \Delta_1 &= x_1 - x_0 \\ \Delta_2 &= x_2 - x_0 \\ \Delta_3 &= x_3 - x_0 \\ &\ldots\ldots \\ &\ldots\ldots \\ \Delta_S &= x_S - x_0 \end{aligned}$$

159. Adding the terms on both sides of these equations, we determine the true value of the measured quantity.

$$\Delta_1 + \Delta_2 + \Delta_3 + \ldots + \Delta_S = x_1 + x_2 + x_3 + \ldots + x_S - S x_0,$$

whence

$$x_0 = \frac{x_1 + x_2 + x_3 + \ldots + x_S}{S} - \frac{\Delta_1 + \Delta_2 + \Delta_3 + \ldots + \Delta_S}{S}$$

The Gauss law is symmetrical - with a sufficiently large number of measurements S, to each positive error will correspond a negative error approximately equal to it in absolute magnitude, and in adding all these errors, the algebraic sum will be close to zero. If, however, a small remainder is obtained, this remainder when divided by S will become S times smaller. Therefore, the last term of the right hand side of the above equation can be neglected, and then

$$x = \frac{x_1 + x_2 + x_3 + \ldots x_S}{S} = \xi$$

For the unknown true value of the measured quantity is taken the mean arithmetical value of all the separate results of measurements, i.e. the mean result.

EXAMPLE. Distance to a target was measured by the same method 10 times. The results of measurements, given as divisions of the sight, are as follows: 125, 128, 116, 127, 121, 118, 123, 130, 120 and 122. Determine the adequate value of the measured distance.

Solution. Let us substitute in the general formula the numerical values of the obtained results of measurement

$$x_{mean} = \xi = \frac{1230}{10} = 123.$$

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159. Consequently, the true value of the measured distance can be taken as the distance corresponding to 123 divisions on the sight.

In taking the mean result ξ as the true value of the measured distance x_0, we will, also naturally, be determining the errors, of separate results, with respect to the mean result. Denoting such errors by λ, we have

$$\begin{aligned} \lambda_1 &= x_1 - \xi \\ \lambda_2 &= x_2 - \xi \\ \lambda_3 &= x_3 - \xi \\ &\cdots \\ &\cdots \\ \lambda_s &= x_s - \xi \end{aligned}$$

The errors, worked out with respect to ξ, in contrast to the true errors, worked out with respect to x_0, are normally termed apparent errors.

160. The algebraic sum of apparent errors is always zero.

Let us add the terms in the above equations:

$$\sum_1^s \lambda_1 = \sum_1^s x_1 - s\xi;$$

according to the condition

$$\frac{\sum_1^s x_1}{s} = \xi, \quad \text{or} \quad \sum_1^s x_1 = s\xi$$

and then actually

$$\sum_1^s \lambda_i = \sum_1^s x_i - s\xi = 0.$$

Let us check this statement. In the above example there were 10 separate results of measurements, and the mean result was found to be equal to 123 divisions of the sight. Calculating the apparent errors of separate results with respect to the mean, we obtain:

$$\sum_1^{10} \lambda_1 = +2+5-7+4-2-5+0+7-3-1 = 0.$$

If the true errors $\Delta_1, \Delta_2, \Delta_3 \ldots$ in the conditions of a given measurement follow Gauss law, then also the apparent errors $\lambda_1, \lambda_2, \lambda_3 \ldots$ in the same conditions must follow the same law, being the errors of the same order.

In the numerical example for 170 results of measurements of distance (section 27, table 4) the errors of individual results were determined with respect to the true value of the measured distance which was 3000m.

Let us suppose, that the true value was unknown and that the errors of individual results were determined with respect to the mean result:

$$\xi = \frac{\sum_1^{170} x_i}{170} = \frac{510008}{170} = 3000.047.$$

Rounding the obtained result to 3000 (neglecting the remainder of 47m), we obtain the apparent errors in magnitude and sign, which do not differ from their corresponding true errors, and the investigation of combining of the apparent errors will lead to the same deductions.

161. In deriving the formula for the mean result as being an accurate value of the measured quantity, we assumed that

$$\Delta_1 \cdot \Delta_2 \cdot \Delta_3 \cdot \ldots \cdot \Delta = \ldots$$

This assumption is completely true only for an infinitely large number of measurements, and only for this case will each positive error have a corresponding negative error, and the sum of all the errors

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161. [illegible] in practice, in taking the mean result ξ as the true value of the measured quantity x_0, we make an error - an error of the mean result

$$\delta = \xi - x_0 = \frac{\Delta_1 + \Delta_2 + \Delta_3 + \cdots + \Delta_S}{S}$$

It is obvious, that this error will be smaller, the larger the value of S. We have calculated above, the mean value of the distance from the 170 individual results of measurement; the error of the mean result in this case

$$\delta = \frac{510008}{170} - 3000 = +\frac{8}{170} \approx +0.047\text{m}.$$

The value of the error of the mean result being so negligibly small, once again confirms the correctness of the above assumption.

With the decrease in the value S, the value of the error of the mean result becomes more and more substantial. Thus, the error of the mean result taken from the first 85 results of the measurement in the same example

$$\delta = \frac{255447}{85} - 3000 = +\frac{447}{85} \approx +5.259\text{m},$$

but, from the first 35 results

$$\delta = \frac{105687}{35} - 3000 = +\frac{687}{35} \approx 19.629\text{m}.$$

Apart from the number of measurements, the value of the error of the mean result depends also on the accuracy of the method of measurement itself - the more accurate the method, the smaller the individual errors, the smaller their algebraic sum and the smaller $\delta = \Sigma\xi/S$.

In taking the mean result as the true value of the measured quantity, it is necessary to take into account the error of the mean result.

162. 36. Adequate value of the middle (median) error.

We had established above (see section 30) the relationship between the true errors of individual results of measurement and the mean arithmetical and mean quadratic errors, and from these, also the middle (median) error, being the basic characteristic of the Gauss law.

In taking the mean result, when calculating the results of measurements, as the true value of the measured quantity, we obtain the apparent and not the true errors. Hence, if the value of the median error of a given method of measurement is not known, then it is necessary to establish rules for determining the adequate value of the mean error from these apparent errors.

Let us commence with establishing the expression for the adequate value of the mean quadratic error, and from this we will proceed to the median error.

Let us establish, in the first place, the relationship between the true and the apparent errors.

The true and the apparent errors of any individual result of measurement are determined by the formulae:

$$\Delta_i = x_i - x_0 \quad \text{and} \quad \lambda_i = x_i - \xi,$$

or

$$\Delta_i - x_0 = x_i \quad \text{and} \quad \lambda_i - \xi = x_i,$$

whence

$$\Delta_i - x_0 = \lambda_i - \xi,$$

$$\Delta_i = \lambda_i - (\xi - x_0) = \lambda_i - \delta,$$

i.e. the true error is equal to the apparent error plus the error of the mean result.

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162. Let us assume that we have a series of measurement of the same quantity:

$$x_1 x_2 x_3 \ldots x_S,$$

the errors of which obey the Gauss law.

Then, from the relationship just established,

$$\Delta_1 = \lambda_1 + \delta$$
$$\Delta_2 = \lambda_2 + \delta$$
$$\Delta_3 = \lambda_3 + \delta$$
$$\ldots\ldots$$
$$\Delta_S = \lambda_S + \delta$$

163. The errors Δ_i, λ_i and δ can be both positive and negative. To remove the effect of the signs, we will raise these equations to the second degree:

$$\Delta_1^2 = \lambda_1^2 + 2\lambda_1\delta + \delta^2$$
$$\Delta_2^2 = \lambda_2^2 + 2\lambda_2\delta + \delta^2$$
$$\Delta_3^2 = \lambda_3^2 + 2\lambda_3\delta + \delta^2$$
$$\ldots\ldots$$
$$\Delta_S^2 = \lambda_S^2 + 2\lambda_S\delta + \delta^2$$

Adding the terms on each side of these equations, we obtain:

$$\sum_1^S \Delta_i^2 = \sum_1^S \lambda_i^2 + 2\delta \sum_1^S \lambda_i + S\delta^2$$

We know, that the algebraic sum of the apparent errors of the first degree is always zero. Hence, also the mean term of the right hand side of the equation

$$2\delta \sum_1^S \lambda_i = 0,$$

and then

$$\sum_1^S \Delta_i^2 = \sum_1^S \lambda_i^2 + S\delta^2.$$

In this equation, according to the condition, $\Sigma \Delta_i^2$ and δ^2 are not known. In order to eliminate δ^2, we make use of the expression

$$\delta = \frac{\Delta_1 + \Delta_2 + \Delta_3 + \ldots + \Delta_S}{S}$$

and we square this expression:

$$\delta^2 = \left(\frac{\Delta_1 + \Delta_2 + \Delta_3 + \ldots + \Delta_S}{S}\right)^2$$

According to the general rule of raising a polynomial to the second degree, we have:

$$(\Delta_1 + \Delta_2 + \Delta_3 + \cdots + \Delta_S)^2 = \sum_1^S \Delta_i^2 + \sum_1^S 2\Delta_i\Delta_i' ,$$

where

$$\sum_1^S 2\Delta_i\Delta_i' = 2(\Delta_1\Delta_2 + \Delta_1\Delta_3 + \cdots + \Delta_1\Delta_S) + 2(\Delta_2\Delta_3 + \Delta_2\Delta_4 + \cdots + \Delta_2\Delta_S) + 2(\Delta_3\Delta_4 + \Delta_3\Delta_5 + \cdots + \Delta_3\Delta_S) +$$

$$+ 2(\Delta_{S-2}\Delta_{S-1} + \Delta_{S-2}\Delta_S) + 2\Delta_{S-1}\Delta_S$$

164. All the products being the products of chance errors, which follow the Gauss law, can have "+" or "−" signs, and with a sufficiently large number of measurements their algebraic sum will be very close to zero. Therefore, ignoring the last term in the expression

$$\delta^2 = \left(\frac{\Delta_1 + \Delta_2 + \Delta_3 + \cdots + \Delta_S}{S}\right)^2 = \frac{\sum_1^S \Delta_i^2}{S^2} + \frac{\sum_1^S 2\Delta_i\Delta_i'}{S^2}$$

we have:

$$\delta^2 = \frac{\sum_1^S \Delta_i^2}{S^2} = \frac{\sum_1^S \Delta_i^2}{S} \cdot \frac{1}{S}$$

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164. Approximately, it is equal to the square of the mean quadratic error. Hence

$$\delta^2 = E_2^2 \cdot \frac{1}{S}.$$

Let us return to the equation

$$\sum_1^S \Delta_1^2 = \sum_1^S \lambda_1^2 + S\delta^2;$$

the left hand side of this equation can be represented as

$$\sum_1^S \Delta_1^2 = \frac{\sum_1^S \Delta_1^2}{S} S = E_2^2 S,$$

and after substitution in the right hand side $\delta^2 = E_2^2 \cdot 1/S$, we obtain:

$$SE_2^2 = \sum_1^S \lambda_1^2 + E_2^2,$$

whence

$$SE_2^2 - E_2^2 = \sum_1^S \lambda_1^2, \quad \text{or} \quad E_2^2(S-1) = \sum_1^S \lambda_1^2,$$

or, solving with respect to E_2,

$$E_2^2 = \frac{\sum_1^S \lambda_1^2}{S-1};$$

then

$$E_2 = \sqrt{\frac{\lambda_1^2+\lambda_2^2+\lambda_3^2 \ldots +\lambda_S^2}{S-1}}.$$

165. The adequate value of the mean quadratic error, when working out the results of measurements, is taken as equal to the square root of the sum of the squares of the errors of individual results, divided by the number of measurements less one.

To change from this mean quadratic error to the median error, we will make use of the well known relationship

$$r = 0.67449 E_2 \approx \frac{2}{3} E_2.$$

In deriving the expression for the mean quadratic error from the results of the calculations of measurements, it was taken that the sum $\sum z\Delta_i\Delta_i'$ is close to zero and that the mathematical expectation is replaced by the mean value of the square of the error, which is permissible only with a very large (more correctly - infinitely large) number of measurements. With a limited number of measurements these assumptions give rise to an error in determination of the mean quadratic error, and hence also in the median error. Therefore the working out of the results of measurements can give only the adequate value of the median error, which will be the nearer to the true value, the greater the number of measurements made.

Example 1. In pace measuring of a base the following individual results of the measurement in metres were obtained: 400, 360, 390, 370, 350, 360, 380, 420, 390, 400, 380, 410, 370, 340, 370 and 390. Determine the adequate value of the median error from the mean quadratic error.

Solution. The procedure for solving such problems is as follows: we first determine the mean result which we take to be the true value of the measured quantity; we then calculate the apparent errors of individual results and raise these errors to the second degree; we then find the mean quadratic error from the formula derived by us; finally, we determine the value of the median error. The most convenient method of solving is to arrange all the data in the form of a table.

Table

$$\xi = \frac{6080}{16} = 380\text{m}$$

$$E_2 = \sqrt{\frac{1200}{15}} \approx 21.91\text{m}.$$

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$E = 0.67449 \times 21.91 \approx 14.78$m.

EXAMPLE 2. In preparation of the original data, the following calculated distances to the same target, in metres were obtained: 4625, 4590, 4560, 4620, 4600, 4655, 4635, 4610, 4640, 4585, 4615, 4595, 4620, 4575 and 4660. Determine the number of these prepared results which were carried out as being "excellent", "good", "indifferent" and "bad", if the median error for the determination of the distance in the given conditions is not known, and in order to obtain "excellent" mark, the error must not exceed one median error, to obtain "good" - two median errors, to obtain "indifferent" - three median errors and to obtain "bad" - more than three median errors.

Solution.

i	x_i	ξ	λ_i	λ_i^2	Mark.
					excellemt.
					good.
					indifferent.
					excellent.
					excellent.
					good.
					excellent.
					excellent.
					good.
					good.
					excellent.
					good.
					excellent.
					indifferent.
					good.

$$\xi = \frac{69285}{15} = 4619\text{m};$$

$$E_2 = \sqrt{\frac{13360}{14}} = 30.89\text{ m};$$

$$E = 0.67449 \times 30.89 = 20.835\text{m};$$

$$E = 20.835\text{m};\quad 2E = 41.67\text{m};\quad 3E = 62.505\text{m}.$$

Result: "excellent" -7, "good" - 6, "indifferent" - 2.

Let us now turn to the obtaining of the adequate value of the mean arithmetical error.

For true values of errors, the mean arithmetical error is

$$E_1 = \frac{\sum_1^S |\Delta_i|}{S}$$

167. and the mean quadratic error

$$E_2 = \sqrt{\frac{\sum_1^S \Delta_i^2}{S}}$$

With a sufficiently large number of measurements the apparent errors in value and sign are equal to their corresponding true errors. On this basis, we substitute λ_i for Δ_i in the indicated expression, and then

$$E_1' = \frac{\sum_1^S |\lambda_i|}{S}, \qquad \text{and} \qquad E_2' = \sqrt{\frac{\sum_1^S \lambda_i^2}{S}}$$

The adequate value of the mean quadratic error is given by the formula

$$E_2 = \sqrt{\frac{\sum_1^S \lambda_i^2}{S-1}}$$

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167. Assuming that between the adequate values of the mean arithmetical and the mean quadratic errors the same relationship exists as in the case of the true values, i.e.

$$E_1' = 0.79787\varepsilon_2' \quad \text{and} \quad E_1 = 0.79787\varepsilon_2 \quad \text{(see page 141)},$$

we write down the proportion:

$$\varepsilon_1 : \varepsilon_2 = \varepsilon_1' : E_2',$$

whence the adequate value of the mean arithmetical error

$$E_1 = E_1' \frac{E_2}{E_2'},$$

or, carrying out corresponding substitutions,

$$E_1 = \frac{\sum_1^S |\lambda_i|}{S} \cdot \frac{\sqrt{\frac{\sum_1^S \lambda_i^2}{S-1}}}{\sqrt{\frac{\sum_1^S \lambda_i^2}{S}}} = \frac{\sum_1^S |\lambda_i|}{S} \cdot \sqrt{\frac{S}{S-1}} = \sum_1^S |\lambda_i| \cdot \sqrt{\frac{S}{S^2(S-1)}},$$

and in the final form

$$E_1 = \frac{\sum_1^S |\lambda_i|}{\sqrt{S(S-1)}}.$$

EXAMPLE 3. Determine in the same conditions as in the example 2, the adequate value of the median error from the mean arithmetical error.

168. Solution. We add the absolute values of the apparent errors of individual results:

$$\sum_1^{15} |\lambda_i| = 6+29+59+1+19+36+16+9+21+34+4+24+1+56+41 = 356.$$

Then the adequate value of the mean arithmetical error

$$\varepsilon_1 = \frac{356}{\sqrt{15(15-1)}} = \frac{356}{\sqrt{210}} = \frac{356}{14.4914} \approx 24.5663\text{m},$$

and the adequate value of the median error

$$\varepsilon = 0.84535 \times 24.5663 \approx 20.867\text{m}.$$

Frequently instead of the expression derived by us, use is made of a simpler formula

$$\varepsilon_1' = \frac{\sum_1^S |\lambda_i|}{S}$$

from which we obtain somewhat smaller values of the mean arithmetical error and hence the median error.

17. Median error of the mean result.

In taking the mean result as the true value of the measured quantity, we admit an error in the mean result.

It is very easy to confirm that this error, with the same method of measurement, the same number of measurements, will be different in magnitude and sign for each individual mean result, if the mean result is to be obtained from different individual results of the measurement.

Let us take once more of the 17c results in the example dealing with direct measurement (see section 27 table 4).

Find the mean result from the first 15 results:

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$$\xi_1 = \frac{51431}{17} = 3025.35\text{m},$$

from which the error of this mean result

$$\delta_1 = \xi_1 - x_0 = 3025.35 - 3000 = +25.35\text{m};$$

the second group of the following 17 results (from No.18 to No.34) gives us:

$$\delta_2 = \xi_2 - x_0 = \frac{51231}{17} - 3000 = 3013.59 - 3000 = +13.59\text{m},$$

the error of the mean result of the third group of the 17 separate results (from No.35 to No.51) will be

$$\delta_3 = \xi_3 - x_0 = \frac{50854}{17} - 3000 = 2991.41 - 3000 = -8.59\text{m}.$$

etc.

169. Each of the errors δ_1, δ_2, δ_3, etc, calculated by this method, undoubtedly bears a chance character, because the sequence of occurrence of individual results in repeated measurement of the same quantity is random. Therefore, none of these errors can be taken as a measure characterising the accuracy of the determination of the mean result with a given method of measurement and a given number of measurements.

It is obvious, that the accuracy of the mean result must be characterised by some kind of its own median error - the median error of the mean result.

The median error E of a given series of measurements is determined from the combination of the errors of individual results of the measurements, comprising this series. Consequently, analagously, it is possible to state that also the median error of the mean result must likewise be determined from the combination of the errors of individual mean results for the condition, of course, that the method of measurement is the same and that each individual mean result is calculated for the same number of measurements.

Let us find the expression for such a median error.

Let us assume, that the same quantity was measured by the same method N times (for example, 170) and that all the individual results of the measurement in the order of sequence of their occurrence have been divided into K groups (for example, 10) with S results (17) in each group.

Let us find for each of these groups its mean result, and for each such mean result, its error:

$$\delta_1, \delta_2, \delta_3, \ldots \delta_K$$

In the previous section (see page 164) it was established that the error of the mean result is connected with the mean quadratic error, taken from the squares of the errors of individual results, by the following relationship:

$$\delta^2 = E_2^2 \frac{1}{S}.$$

The values of individual mean results, calculated for the same [illegible] different in magnitude. Therefore, the mean [illegible] them in magnitude will also be different, i.e.

$$\delta_1^2 = E_2^2 \cdot \frac{1}{S}$$

$$\delta_2^2 = E_2^2 \cdot \frac{1}{S}$$

$$\delta_3^2 = E_2^2 \cdot \frac{1}{S}$$

.....

$$\delta_K^2 = E_2^2 \cdot \frac{1}{S}$$

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169. Let us add the terms on each side of these equations:

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$$\delta_1^2+\delta_2^2+\delta_3^2+\ldots+\delta_K^2=\frac{1}{S}(E_2'^2+E_2''^2+E_2'''^2+\ldots+E_2^{K2}).$$

170. In the right hand side of this equation we have the sum of individual chance values of the square of the mean quadratic error.

When we have a series of individual chance values of the measured quantity, we take the mean result as its adequate value. It is obvious, that, in having a series of individual chance values of the square of the mean quadratic error, we can also take, for its adequate value, the mean result, i.e. we can take the mean value of the square of the mean quadratic error:

$$E_2^2=\frac{E_2'^2+E_2''^2+E_2'''^2+\ldots+E_2^{K2}}{K},$$

whence

$$E_2'^2+E_2''^2+E_2'''^2+\ldots+E_2^{K2}=KE_2^2.$$

Then the sum of the squares of the errors of individual mean results

$$\delta_1^2+\delta_2^2+\delta_3^2+\ldots+\delta_K^2=\frac{K}{S}E_2^2.$$

Dividing both sides of this equation by K, we obtain:

$$\frac{\delta_1^2+\delta_2^2+\delta_3^2+\ldots+\delta_K^2}{K}=\frac{E_2^2}{S}.$$

In the same way, as was the case in the errors of individual results of a measurement, the sum of the squares of the errors of individual mean results, divided by the number of such results, is no other but the square of the mean quadratic error of the mean result. Denoting this error by R_2, we obtain:

$$R_2^2=\frac{E_2^2}{S}.$$

The median error of the mean result, denoted by r, is related to the mean quadratic error of the mean result by the same relationship as in the case of the median and the mean quadratic errors E and E_2:

$$E=0.67449E_2 \quad \text{and} \quad r=0.67449R_2.$$

Therefore, substituting the mean quadratic errors by their corresponding median errors, we obtain:

$$r^2=\frac{E^2}{S},$$

or

$$r=\frac{E}{\sqrt{S}}.$$

<u>The median error of the mean result is equal to the median error of the method of measurement, divided by the square root of the number of measurements.</u>

Let us demonstrate the truth of this formula by a numerical example. For this, we will find the errors of all the 10 mean results, which were mentioned above, and from these errors we will determine the
171. value of r by all the methods known to us. We will enter our calculations in a general table (table 16).

Table 16.

$\Sigma|\delta|=164.5$ and $\Sigma\delta^2=4[illegible].6.$

The median error of the mean result, from the data in this table,

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171. 1. From the mean arithmetical error:

$$R_{E_1} = \frac{0.84535 \times 164.5}{\sqrt{10 \times 9}} = 14.64\text{m}.$$

2. From the mean quadratic error:

$$R_{E_2} = 0.67449\sqrt{\frac{4164.66}{9}} = 14.52\text{m}.$$

3. From the position in the series of absolute values of errors:

$$R_{pp} = \frac{13.59 + 15.49}{2} = 14.54\text{m}.$$

Above (see section 28, table 9) it was established, that the median error of the method of measurement in the conditions of the given example is equal to 60m. Then, from the derived formula

$$R = \frac{60}{\sqrt{17}} = 14.55\text{m},$$

As can be seen, all these numerical values of the error of the mean result are very close to each other in magnitude.

EXAMPLE 1. Determine the median error of the mean result, if the median error of the method of measurement is 40m and the mean result was found from four measurements.

Solution. According to the condition $E = 40$ and $S = 4$; substituting these numerical values in the general formula, we obtain:

$$R = \frac{40}{\sqrt{4}} = 20\text{m}.$$

172. EXAMPLE 2. The same angle was measured by the same method 16 times. Let us find the median error of the mean result, if the median error of the method of measurement is 0-02.

Solution. $E = 0\text{-}02$ and $S = 16$, then

$$R = \frac{2}{\sqrt{16}} = 0.5 \text{ goniometer divisions.}$$

EXAMPLE 3. In measurement of the same angle by the same method, the following results were obtained: 8-69, 8-71, 8-70, 8-72, 8-66, 8-70, 8-71, 8-69, 8-68, 8-74, 8-69, 8-70, 8-72 and 8-67. Let us determine the accuracy of this measurement.

Solution. We will find the median error of the mean value of the measured angle (of the mean result). The median error of the method of measurement is not given - we will determine it from the results (errors) of individual measurements, and thereafter from the value of E we will find R.

Table

$$\Sigma x_i = 121\text{-}80;$$

$$\xi = \frac{121\text{-}80}{14} = 8\text{-}70;$$

$$\Sigma|\lambda_i| = 0\text{-}24;$$

$$E_{E_1} = 0.84535 \times \frac{24}{\sqrt{14 \times 13}} = 1.51 \text{ goniometer divisions.}$$

$$\Sigma\lambda_i^2 = 0\text{-}66.$$

$$E_{E_2} = 0.67449\sqrt{\frac{66}{13}} = 1.52 \text{ goniometer divisions.}$$

$$E_{pp} = \frac{0\text{-}01 + 0\text{-}02}{2} = 1.50 \text{ goniometer divisions.}$$

Three numerical values of the median error of the method (more accurately - of the series) of measurement were obtained. We take the rounded goniometer divisions as $E = 1.5$, and then

$$R = \frac{1.5}{\sqrt{14}} = 0.4 \text{ goniometer divisions.}$$

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172. If the median error of the method of measurement E serves as the results of measurement conform, then the median error of the mean result R will also characterise, in the same way, the Gauss law, but as applied to the errors of individual mean results.

The fact that the errors of individual mean results obey the Gauss law, can be seen from the actual formula of the median error of the mean result. From this formula the median error of the mean result R depends only on the value of the median error of the method of measurement E and on the number of measurements S. If, however, the median error of the

173. method of measurement E is a parameter of the Gauss law, then also the median error of the mean result R will likewise be a parameter of the Gauss law, but a parameter which is $\sqrt{S}$ times smaller.

From here it follows, that knowing the value of the median error of the mean result and making use of the error scale or the table of values $\Phi(\beta)$, it is possible to calculate the probabilities of occurrence of the error of the mean result, within some set limits, or, in other words, the probabilities that within these limits is contained the true value of the measured quantity.

EXAMPLE 4. The median error of the mean result is 10m. Let us find the probabilities of occurrence of the error of the mean result for various limits of magnitude and sign, in whole numbers of R, or, in other words, let us find the probabilities that the true value of the measured quantity is contained within such limits.

Solution. For better representation we will draw a scale of errors in R = 10m (fig.50).

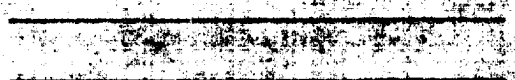

Fig.50.

In examining this scale, the following deductions can be made:

1. The maximum error of the mean result can be practically taken as the error which is equal to $\pm 4R = \pm 40$m. This means, that taking, in the conditions of the given example, the mean result as the true value of the measured quantity, we make an error which is not greater than ±40m. Within the limits from $\xi - 4R$ to $\xi + 4R$ will be contained this true value.

2. The probability of occurrence of the error of the mean result within the limits from 0 to ±R, i.e. from 0 to ±10m, will, according to the theorem of addition, be equal to 0.25+0.25 = 0.5. In other words, 0.5 is the probability that the true value of the measured quantity is contained within the limits from $\xi - 10$m to $\xi + 10$m. The probability that the true value of the measured quantity is contained within the limits from $\xi - 20$m to $\xi + 20$m (the probability of the error from 0 to ±2R) is 0.16+0.25+0.25+0.16 = 0.82 (from the table of values $\Phi(\beta)$ = 0.82266) etc.

3. The probability of occurrence of the error of the mean result within the limits from 0 to +R, i.e. from 0 to 10m, is 0.25; within the limits from +R to +2R (from +10 to +20m) the probability is 0.16 etc; the same values of the probability will apply for the limits from 0 to -R, from -R to -2R etc.

EXAMPLE 5. In 16 measurements of distance, the mean result is 3200m. Find the probability that the true value of the measured distance is contained within the limits:

a) from 3180m to 3220m,
b) from 3170m to 3230m,
c) from 3155m to 3245m,
d) from 3140m to 3260m,
e) from 3220m to 3250m,
f) from 3150m to 3225m,

if the mean error of the method of measurement of the distance, is 80m.

Solution. We will first find the median error of our mean result:

$$R = \frac{80}{\sqrt{16}} = 20\text{m}.$$

174. We will make up a scale of errors for this value of R (fig.51):

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the probability that the true value of the measured distance is contained within the limits from 3180 to 3220m, is equal to the probability of occurrence of the error from 0 to ±20m (from 0 to ±1.0R)

$$P_a = 0.25+0.25 = 0.50;$$

Fig.51.

b) the probability that the true value of the distance is within the limits from 3170 to 3230m, or the probability of occurrence of the error from 0 to ±30m (from 0 to ±1.5R)

$$P_b = 0.09+0.25+0.25+0.09 = 0.68;$$

more accurately, from the table of values $\Phi(\beta)$:

$$P_b = \Phi(1.5) = 0.68833;$$

c) the probability of the error being within the limits from 0 to ±45m, or - in parts of R - from 0 to ±2.25R, is

$$P_c = 0.024+0.16+0.25+0.25+0.16+0.024 = 0.868 \text{ (from the scale)};$$

$$P_c = \Phi(2.25) = 0.87088 \text{ (from the table)};$$

d) the probability of the error being within the limits from ±40m to +60m, i.e. from +2R to +3R, is

$$P_d = 0.07,$$

or

$$P_d = \frac{1}{2}\left[\Phi(3.0)-\Phi(2.0)\right] = \frac{0.95698-0.82266}{2} = 0.06716;$$

e) the probability of the error being within the limits from 0 to -50m, or from 0 to -2.5R, is

$$P_e = 0.25+0.16+0.04 = 0.45,$$

or

$$P_e = \frac{1}{2}\Phi(2.5) = 0.45413;$$

f) the probability of the error being within the limits from -2.5m to +50m, i.e. from -1.25R to +2.5R, is

$$P_f = 0.05+0.25+0.25+0.16+0.04 = 0.75,$$

or

$$P_f = \frac{1}{2}\left[\Phi(1.25)+\Phi(2.5)\right] = \frac{0.60083+0.90825}{2} = 0.75454.$$

175. Up to now the value of the median error of the mean result r was determined from a set value of the median error of the method of measurement E, and a set number of measurements n. Of course, it is possible to take definite values of r and n, and from them to find the number n which would satisfy the set relationship between r and n, i.e. it is possible to find the required number of measurements at which the median error of the mean result will have the desired numerical value.

Let us solve the equation $R = r/\sqrt{n}$ with respect to n:

$$\sqrt{n} = \frac{r}{R}, \quad \text{or} \quad n = \left(\frac{r}{R}\right)^2.$$

EXAMPLE 6. How many measurements must be made, for the median error of the mean result to be 0.05, if the median error of the method of measurement is 0.20?

Solution. $n = (r/R)^2 = (20/5)^2 = 4^2 = 16$ measurements.

EXAMPLE 7. The median error of a method of measurement of a distance, is 50m. How many measurements must be made, for the median error, in determination of the distance, to be

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175. Solution. $S = (50/20)^2 = 2.5^2 \approx 6$ measurements.

In practice, it is usually sufficient to take the limit error of the mean result; taking such an error, as a rule, equal to $\pm 4R$.

EXAMPLE 8. The median error of a method of measurement, is 0-05. How many measurements must be made, for the mean result to have an error not exceeding 0-10?

Solution. We will take the limit error of the mean result

$$\delta_{max} = \pm 4R = \pm 0\text{-}10,$$

whence

$$R = \frac{10}{4} = 2.5 \text{ goniometer divisions,}$$

and the number of measurements we require

$$S = \left(\frac{5}{2.5}\right)^2 = 4.$$

If the requirement were for the limit error to be only 0-05 instead of 0-10, then

$$R = \frac{5}{4} = 1.25 \text{ goniometer divisions} \quad \text{and} \quad S = \left(\frac{5}{1.25}\right)^2 = 16.$$

The accuracy of the result of a measurement can be increased by increasing the accuracy of the method of measurement itself, i.e. by decreasing its median error or by increasing the number of measurements.

176. In concrete cases, the value of the median error of a method of measurement can not be changed, as this value is determined by the properties of the measuring instruments, the qualities of the operator, the circumstances in which the measurement is being taken, etc. There remains therefore the second approach - to raise the accuracy of the result of a measurement by increasing the number of measurements.

The increase of the number of measurements is related, in the first place, to the expenditure of time, and in measurements dealing with deviations of explosions, it is also related to the expenditure of shells. It is therefore necessary to confine, the measurements, as a rule, to a comparitively small number.

Let us see, what increase in the number of measurements will be most effective from the point of view of increasing the accuracy of the result of a measurement. For this purpose, we will work out the median errors of the mean result R for various numerical values of S and we will find the variations of the median error of the mean result ΔR in parts of E and in percentages for each consecutive measurement (table 17).

Table 17.

This table shows that the most substantial increase in accuracy can be obtained only with initial measurements. Thus, with one measurement $R = E$; with two measurements $R = 0.707E$, which compared with the first result shows a decrease in the value of R by 0.293E, or by 29.3%; with three measurements $R = 0.577E$, which in comparison with the previous result gives a decrease in R by 0.130E, or by 18.4%, etc. An exceptionally sharp decrease in R is shown by the second and third measurements. Therefore, whenever possible, a measurement of a quantity should be made not less than 2 - 3 times.

After four measurements the accuracy increment falls off sharply. Therefore, in firing, when determining the position of the mean trajectory from the deviations of explosions, the measurements are limited to four, and only for cases requiring special accuracy (precision) are the measurements raised to nine. Further increase in the number of measurements produces such an insignificant increase in accuracy of the result of the measurement, that as a rule it is only made when the measurements carried out are of the nature of scientific research.

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177. 38. Adequate value of the median error from the differences between individual results.

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The adequate value of the median error ε can also be found directly from the results of separate measurements, without preliminary determination of numerical values of apparent errors. This method consists in that the median error is determined from the differences between adjacent individual results of the measurement. Cases when such a method is not only more advantageous, but also the only method possible, will be shown later.

Let us suppose, that there is a series of individual results of a measurement of the same quantity by the same method:

$$x_1 x_2 x_3 \ldots x_s.$$

We will take the condition that the errors of these results obey the Gauss law.

Let us calculate the apparent errors of these results:

$$\begin{aligned} \lambda_1 &= x_1 - \xi \\ \lambda_2 &= x_2 - \xi \\ \lambda_3 &= x_3 - \xi \\ &\cdots \\ \lambda_s &= x_s - \xi \end{aligned}$$

whence

$$\begin{aligned} x_1 &= \lambda_1 + \xi \\ x_2 &= \lambda_2 + \xi \\ x_3 &= \lambda_3 + \xi \\ &\cdots \\ x_s &= \lambda_s + \xi \end{aligned}$$

Let us take the differences between two adjacent results, in each case subtracting the preceding result from the following, and let us denote these differences by u

$$\begin{aligned} x_2 - x_1 &= (\lambda_2 + \xi) - (\lambda_1 + \xi) = \lambda_2 - \lambda_1 = u_1 \\ x_3 - x_2 &= (\lambda_3 + \xi) - (\lambda_2 + \xi) = \lambda_3 - \lambda_2 = u_2 \\ x_4 - x_3 &= (\lambda_4 + \xi) - (\lambda_3 + \xi) = \lambda_4 - \lambda_3 = u_3 \\ &\cdots \\ x_s - x_{s-1} &= (\lambda_s + \xi) - (\lambda_{s-1} + \xi) = \lambda_s - \lambda_{s-1} = u_{s-1} \end{aligned}$$

In this way, we have substituted the diffrences between individual results by the differences between their corresponding apparent errors and we have obtained s-1 values of such differences.

Let us raise these differences to the second degree:

$$\begin{aligned} \lambda_2^2 - 2\lambda_2\lambda_1 + \lambda_1^2 &= u_1^2 \\ \lambda_3^2 - 2\lambda_3\lambda_2 + \lambda_2^2 &= u_2^2 \\ \lambda_4^2 - 2\lambda_4\lambda_3 + \lambda_3^2 &= u_3^2 \\ &\cdots \\ \lambda_s^2 - 2\lambda_s\lambda_{s-1} + \lambda_{s-1}^2 &= u_{s-1}^2 \end{aligned}$$

178. and adding their terms:

$$(\lambda_2^2 + \lambda_3^2 + \lambda_4^2 + \ldots + \lambda_s^2) - 2(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_4 + \cdots + \lambda_{s-1}\lambda_s) + (\lambda_1^2 + \lambda_2^2 + \lambda_3^2 + \cdots + \lambda_{s-1}^2) =$$
$$= u_1^2 + u_2^2 + u_3^2 + \cdots + u_{s-1}^2$$

From the symmetry of the Gauss law, we can neglect the sum of the paired products of apparent errors

$$\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_4 + \cdots + \lambda_{s-1}\lambda_s ,$$

as this sum will, with sufficiently large number of measurements, be close to zero, and then

$$\sum_2^s \lambda_i^2 + \sum_1^{s-1} \lambda_i^2 = \sum_1^{s-1} u_i^2 .$$

Let us see what does the left hand side of this equation represent. Let us commence with the second term

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$$\sum_1^{s-1} \lambda_i^2 .$$

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178. If we had a result of the measurement, and hence also apparent errors, then from the well known formula the adequate value of the mean quadratic error, will be

$$E_2 = \sqrt{\frac{\sum_1^S \lambda_i^2}{S-1}}$$

But, in the expression

$$\sum_1^{S-1} \lambda_i^2$$

we have the sum of the squares of S less one, instead of S apparent errors; with such a number of apparent errors the adequate value of the mean quadratic error must be expressed:

$$E_2 = \sqrt{\frac{\sum_1^{S-1} \lambda_i^2}{(S-1)-1}} = \sqrt{\frac{\sum_1^{S-1} \lambda_i^2}{S-2}},$$

whence

$$E_2^2(S-2) = \sum_1^{S-1} \lambda_i^2 .$$

The component

$$\sum_2^S \lambda_i^2$$

also represents the sum $(S-1)$ of the squares of apparent errors. Therefore, we can also substitute the component in the expression

$$E_2^2(S-2),$$

and then, the equation

$$\sum_2^S \lambda_i^2 + \sum_1^{S-1} \lambda_i^2 = \sum_1^{S-1} u_i^2$$

179. will take the form

$$\sum_1^{S-1} u_i^2 = E_2^2(S-2) + E_2^2(S-2) = 2E_2^2(S-2).$$

Solving this equation with respect to E_2, we obtain:

$$E_2^2 = \frac{1}{2} \frac{\sum_1^{S-1} u_i^2}{S-2},$$

or

$$E_2 = \frac{1}{\sqrt{2}} \sqrt{\frac{\sum_1^{S-1} u_i^2}{S-2}}.$$

In the Gauss law the median error E is related to the mean quadratic error E_2 by the following relationship

$$E = 0.67449 E_2.$$

Assuming, that the differences u, being the differences between chance errors following the Gauss law (see page 177), likewise follow this law, we substitute E_2 by E and we obtain:

$$E = \frac{0.67449}{\sqrt{2}} \sqrt{\frac{\sum_1^{S-1} u_i^2}{S-2}},$$

or, after dividing 0.67449 by $\sqrt{2}$,

$$E = 0.4769 \sqrt{\frac{u_1^2 + u_2^2 + u_3^2 + \ldots + u_{S-1}^2}{S-2}}$$

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179. $$u_1 = \lambda_2 - \lambda_1,$$
$$u_2 = \lambda_3 - \lambda_2$$
$$u_3 = \lambda_4 - \lambda_3$$
$$\cdots\cdots\cdots$$
$$u_{s-1} = \lambda_s - \lambda_{s-1}$$

and let us add the terms in these equations:

$$u_1 + u_2 + u_3 + \cdots + u_{s-1} = \lambda_2 - \lambda_1 + \lambda_3 - \lambda_2 + \lambda_4 - \lambda_3 + \cdots + \lambda_s - \lambda_{s-1}.$$

Then, after reduction of similar terms, we obtain:

$$\sum_1^{s-1} u_i = \lambda_s - \lambda_1 = x_s - x_1.$$

The algebraic sum of the differences is equal to the difference between the last and the first results of the measurement. We will use this equation in checking the correctness of our calculations of individual differences.

180. EXAMPLE 1. In taking readings of intersection on the enemy battery by means of BZA*, the following individual results of 10 measurements in metres were obtained: 5750, 5870, 5940, 5780, 5860, 5840, 5790, 5900, 5800 and 5820. Determine the adequate value of the median error of this series of measurements.

Solution.

A check:

Table. 5820-5750 = +70m.

$$E_u = 0.47694\sqrt{\frac{76700}{8}} = 46.31\text{m}.$$

EXAMPLE 2. In determining the initial velocity by firing, the following results of 12 separate measurements in m/sec were obtained: 577.9, 582.1, 580.3, 581.6, 578.7, 579.6, 580.8, 578.2, 582.0, 581.1, 578.8 and 580.5. Find the adequate value of the median error.

Solution.

A check:

Table. 580.5-577.9 = +2.6m/sec.

$$E_u = 0.47694\sqrt{\frac{63.43}{10}} = 1.2\text{m/sec}.$$

The method of determining the adequate value of the median error from the differences between individual results is recommended for use in the cases when the conditions, under which the measurement is being made, are unstable. Such conditions, in particular, could have existed in the above two examples - instability of meteorological conditions in firing by means of BZA*, and the difference in temperatures of combat charges when determining the initial velocity.

This method should be used when during process of variation the measured quantity itself varies, for example when determining the distance to a moving target. In these conditions, the determination of the adequate value of the median error from the apparent errors leads to incorrect deductions.

181. We will show this by an example.

Let us assume, that the angle between the target and the orientation point is being measured by the same method, and we will discuss two cases: a stationary target and a moving target, and in both cases, errors, similar in magnitude and sign, will correspond to the same results of the measurement (in the consecutive order in which they are obtained).

The measurement of the angle between the orientation point and the stationary target gave the following results: 1-09, 1-03, 1-01, 1-04, 1-00, 1-05, 1-04, 1-01, 0-94, 1-03, 1-00, 1-06, 0-98, 1-04, 1-02 (mils).

In the case when the target is moving, we will assume, for simplification of the calculations, that the first measurement has

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181. coincided with the instant when the target commences to move, and that
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[illegible] in such a way, that with each consecutive measurement the true value of the measured angle was increasing by 0-02. In this way, the first result was such as for the stationary target (1-09), the second result obtained an increment of 0-02 (1-03+0-02 = 1-05), the third result obtained an increment of 0-04 (1-01+0-04 = 1-05), the fourth - 0-06 (1-04+0-06 = 1-10) etc.

Let us determine the adequate value of the median error for both cases, by all the methods known to us, and let us compare the obtained results. We will enter all the calculations in a general table (table 18).

Table 18.

182. We will first find the adequate value of the median error for the stationary target when $\xi = 1616:16 = 1\text{-}01$:

1. From the mean arithmetical error

$$E_{E_1} = 0.84535\frac{54}{\sqrt{16 \times 15}} = 2.95 \text{ goniometer divisions.}$$

2. From the mean quadratic error

$$E_{E_2} = 0.67449\sqrt{\frac{288}{15}} = 2.96 \text{ goniometer divisions.}$$

3. From the position in the series of absolute values of errors

$$E_{pp} = 3 \text{ goniometer divisions.}$$

4. From the differences between adjacent results

$$E_u = 0.47694\sqrt{\frac{540}{14}} = 2.96 \text{ goniometer divisions.}$$

All the calculations have given us the same numerical value of the median error, which we will take as the adequate value of the median error of the method of measurement we are discussing.

We will now find the adequate value of the median error for the moving target when $\xi = 1856:16 = 1\text{-}16$:

1. From the mean arithmetical error

$$E_{E_1} = 0.84535\frac{124}{\sqrt{16 \times 15}} = 6.76 \text{ goniometer divisions.}$$

2. From the mean quadratic error

$$E_{E_2} = 0.67449\sqrt{\frac{1228}{15}} = 6.1 \text{ goniometer divisions.}$$

3. From the position in the series of absolute values of errors

$$E_{pp} = 7 \text{ goniometer divisions.}$$

4. From the differences between the adjacent results

$$E_u = 0.47694\sqrt{\frac{536}{14}} = 2.95 \text{ goniometer divisions.}$$

Comparing these results, we see, that only the method using differences has provided us with the adequate value which is close to the accepted value of the median error of the method of measurement. Determination of the adequate value of the median error from the apparent errors has given us incorrect and varying values E.

Thus, in the cases when the measured quantity changes during measurement, only the method of differences should be employed for the determination of the adequate value of the median error.

It is necessary to lay stress on that when determining the adequate value of the median error, using the differences between the adjacent results of the measurement, it is necessary to set out these results in the consecutive order in which they were obtained during measurement.

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82. [illegible] the changes of the measured quantity is upset, which leads to false deductions.

83. **39. Applicability of the Gauss law and elimination of abnormal results of measurement.**

Up to now, in all the cases of calculation of the results of the measurement, we have taken the errors of the series of measurements, we were discussing, as following the Gauss law. All the deductions made were on the basis of this assumption.

But apart from the Gauss law, other laws of errors can also exist in the given measurements, and the combined law of errors will not always be the Gauss law. In the working out of the results of a measurement, a question naturally arises: are the errors of these results such errors which obey the Gauss law, i.e. is the Gauss law applicable to the series of measurements? If they do not, the application of the formulae used for the Gauss law will lead to gross errors.

It was established earlier, that the relationships between the mean arithmetical, the mean quadratic and the median errors, expressed as follows, are specific to the Gauss law:

$$E = 0.84535E_1, \quad E = 0.67449E_2 \quad \text{and} \quad E_1 = 0.79787E_2,$$

or when rounded to the second decimal place:

$$\frac{E}{E_1} = 0.85, \quad \frac{E}{E_2} = 0.67 \quad \text{and} \quad \frac{E_1}{E_2} = 0.8.$$

These relationships are used when discussing the applicability of the Gauss law. It is considered, for practical purposes, that the Gauss law is applicable to errors of a given series of measurements, if the relationships between E, E_1 and E_2, obtained by the calculation of the results of measurements, do not differ from the relationships, specific to the Gauss law, by more than 10%, i.e. if

$$\frac{E}{E_1} - 0.85 \leqslant 0.085,$$

$$\frac{E}{E_2} - 0.67 \leqslant 0.067$$

and

$$\frac{E_1}{E_2} - 0.8 \leqslant 0.08.$$

Here the adequate value E is determined from the position in the series of absolute values of errors or from the differences between the adjacent results (E_{pp} and E_u).

EXAMPLE 1. The same angle was measured by the same method 15 times, and the following individual results were obtained: 1-30, 1-36, 1-33, 1-28, 1-36, 1-41, 1-45, 1-38, 1-27, 1-34, 1-29, 1-37, 1-31, 1-47 and 1-33. Check the applicability of the Gauss law to the given series of measurements and find the adequate values of the angle and of the median error.

84. Solution. Let us work out the results of the measurement

[illegible]

$$\xi \quad \frac{[illegible]}{[illegible]} \quad 1-$$

$$[illegible] = \frac{7\xi}{\sqrt{[illegible]}}$$

$$E_2 = \sqrt{\frac{\xi}{[illegible]}}$$

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184. E_{pp} = 4 goniometer divisions;

$$E_u = 0.47694\sqrt{\frac{271}{13}} = 4.12 \text{ goniometer divisions.}$$

$E_{pp}:E_1 = 4.00:4.83 = 0.82$; $0.82-0.85 = 0.03$; $0.03<0.085$,
$E_u:E_1 = 4.12:4.83 = 0.85$; $0.85-0.85 = 0$.
$E_{pp}:E_2 = 4.00:5.94 = 0.67$; $0.67-0.67 = 0$.
$E_u:E_2 = 4.12:5.94 = 0.71$; $0.71-0.67 = 0.4$; $0.04<0.067$.
$E_1:E_2 = 4.83:5.94 = 0.81$; $0.81-0.80 = 0.01$; $0.01<0.08$.

Conclusion. The Gauss law is completely applicable to the given series of measurements.

EXAMPLE 2. Working out of the results of the measurement of distance gave the following numerical values of parameters: $E_{pp} = 20$m; $E_u = 19.3$m; $E_1 = 23.5$m and $E_2 = 21.8$m. Establish the applicability of the Gauss law to these results.

185. Solution. We will find the numerical values of the relationships between the parameters:

$$\frac{E}{E_1} = \frac{19.3}{23.5} = 0.82,$$

$$\frac{E_u}{E_2} = \frac{19.3}{28.1} = 0.69$$

and

$$\frac{E_1}{E_2} = \frac{23.5}{28.1} = 0.84.$$

Let us take the differences between these relationships and the relationships which theoretically correspond to the Gauss law:

$0.82-0.85 = -0.03$; $0.03<0.085$;
$0.69-0.67 = +0.02$; $0.02<0.067$;
$0.84-0.8 = +0.04$; $0.04<0.08$.

All the differences are less than the permissible; consequently, the Gauss law is applicable to these results.

To check, we will compare the relationships calculated with respect to E_{pp}.

$$\frac{E_{pp}}{E_1} = \frac{20}{23.5} = 0.85 \quad \text{and} \quad \frac{E_{pp}}{E_2} = \frac{20}{28.1} = 0.71.$$

Then, the differences between the relationships:

$0.85-0.85 = 0$;

$0.71-0.67 = 0.04$; $0.04<0.067$,

i.e. this check confirms the applicability of the Gauss law.

[illegible] in working out the results of the measurement of the speed of an aircraft, the following numerical values for parameters in [illegible] were obtained. $E_{pp} = 4.5$; $E_u = 3.9$; $E_1 = 4.7$ and $E_2 = 7.5$. [illegible] establish if the Gauss law is applicable.

[illegible]lution.

[illegible] $0.96-0.85 = 0.11$; $0.11>0.085$;

[illegible] $0.52-0.67 = -0.15$; $0.15 \approx 0.067$;

[illegible] $0.71-0.85 = -0.14$; $0.14<0.85$;

[illegible] $0.52-0.67 = -0.15$; $0.15>0.067$;

[illegible] $0.71-0.67 = 0.04$; $0.04<0.067$.

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184. $E_{pp} = 4$ goniometer divisions;

$$E_u = 0.47694\sqrt{\frac{971}{13}} = 4.12 \text{ goniometer divisions.}$$

$E_{pp}:E_1 = 4.00:4.83 = 0.82$; $0.82-0.85 = 0.03$; $0.03<0.085$.
$E_u:E_1 = 4.12:4.83 = 0.85$; $0.85-0.85 = 0$.
$E_{pp}:E_2 = 4.00:5.94 = 0.67$; $0.67-0.67 = 0$.
$E_u:E_2 = 4.12:5.94 = 0.71$; $0.71-0.67 = 0.4$; $0.04<0.067$.
$E_1:E_2 = 4.83:5.94 = 0.81$; $0.81-0.80 = 0.01$; $0.01<0.08$.

Conclusion. The Gauss law is completely applicable to the given series of measurements.

EXAMPLE 2. Working out of the results of the measurement of distance gave the following numerical values of parameters: $E_{pp} = 20$m; $E_u = 19.3$m; $E_1 = 23.5$m and $E_2 = 21.8$m. Establish the applicability of the Gauss law to these results.

185. Solution. We will find the numerical values of the relationships between the parameters:

$$\frac{E_{pp}}{E_1} = \frac{19.3}{23.5} = 0.82,$$

$$\frac{E_u}{E_2} = \frac{19.3}{28.1} = 0.69$$

and

$$\frac{E_1}{E_2} = \frac{23.5}{28.1} = 0.84.$$

Let us take the differences between these relationships and the relationships which theoretically correspond to the Gauss law:

$$0.82-0.85 = -0.03; \quad 0.03<0.085;$$
$$0.69-0.67 = +0.02; \quad 0.02<0.067;$$
$$0.84-0.8 = +0.04; \quad 0.04<0.08.$$

All the differences are less than the permissible; consequently, the Gauss law is applicable to these results.

To check, we will compare the relationships calculated with respect to E_{pp}.

$$\frac{E_{pp}}{E_1} = \frac{20}{23.5} = 0.85 \quad \text{and} \quad \frac{E_{pp}}{E_2} = \frac{20}{28.1} = 0.71.$$

Then, the differences between the relationships:

$$0.85-0.85 = 0;$$

$$0.71-0.67 = 0.04; \quad 0.04<0.067,$$

i.e. this check confirms the applicability of the Gauss law.

EXAMPLE 3. In working out the results of the measurement of the speed of an aircraft, the following numerical values for parameters in m/sec were obtained. $E_{pp} = 4.5$; $E_u = 3.9$; $E_1 = 4.7$ and $E_2 = 7.5$. Establish if the Gauss law is applicable.

Solution.

$\frac{E_{pp}}{E_1} = \frac{4.5}{4.7} = 0.96$; $0.96-0.85 = 0.11$; $0.11>0.085$;

$\frac{E_u}{E_2} = \frac{3.9}{7.5} = 0.52$; $0.52-0.67 = -0.15$; $0.15 \approx 0.067$;

$\frac{E_{pp}}{E_2} = \frac{4.5}{7.5} = 0.6$; $0.6-0.85 = -0.25$; $0.25<0.85$;

$\frac{E_u}{E_1} = \frac{3.9}{4.7} = 0.83$; $0.83-0.67 = 0.15$; $0.15>0.067$;

$\frac{E_1}{E_2} = \frac{4.7}{7.5} = 0.63$; $0.63-0.8 = -0.17$; $0.17>0.08$.

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185. The majority of the relationships, calculated from the results of measurements, exceeds the relationships, specific to Gauss law, by more than 10%, and in the case of the relationships E_0/E_2 and E_1/E_2 this excess is almost 20%. Consequently, in the given case the Gauss law is not applicable.

186. When working out the results of measurements, the question of applicability of the Gauss law to a given series of measurements, is connected with the question on the elimination, from this series, of the so called <u>abnormal</u> results.

In carrying out a measurement of the same quantity by the same method, we obtain, not infrequently, individual results which differ in magnitude from other results. It is apparent that such results must influence the results of the calculation, and frequently this influence is sufficiently high completely to change the appreciation of the law of errors of a given series of measurements.

Let us suppose, that in the example mentioned above, another result, equal to 1-67, is added to the 15 results of the measurement of the angle.

In this case the mean result will no longer be 1-35, but 1-37. As a result of this the values of the apparent errors will change, and then

$$E_{pp} = 5 \text{ goniometer divisions},$$

$$E_1 = \frac{106}{\sqrt{16 \times 15}} = 6.84 \text{ goniometer divisions}$$

and

$$E_2 = \sqrt{\frac{1454}{15}} = 9.85 \text{ goniometer divisions},$$

and the relationships:

$$\frac{E_{pp}}{E_1} = \frac{5}{6.84} = 0.73; \quad 0.73-0.85 = -0.12; \quad 0.12>0.085;$$

$$\frac{E_{pp}}{E_2} = \frac{5}{9.85} = 0.51; \quad 0.51-0.67 = -0.16; \quad 0.16>0.067;$$

$$\frac{E_1}{E_2} = \frac{6.84}{9.85} = 0.69; \quad 0.69-0.80 = -0.11; \quad 0.11>0.08.$$

In working out the 15 results of the measurement of the angle, we arrived at the conclusion that the Gauss law is applicable to them. In introducing into our calculations the 16th result, we had to retract from this conclusion, because all the differences of the relationships between E, E_1 and E_2 had exceeded the admissible limits for the applicability of the Gauss law.

If sufficiently substantial grounds exist to suppose that the results of such a kind are due to causes inherent in the given method of measurement (Coarse errors in readings and in construction of graphs, mathematical errors in calculating, sighting errors in firing etc), then elimination of such abnormal results does not raise any doubts. For example, if in determining the distance to the target by a brief method, it is found that in one case the target point was entered in the wrong square on the chart, then obviously, when finding the adequate values of the distance and the median error of the method such a result should not be taken into consideration during the working out.

But not always is it possible to establish the cause of large deviations of individual results, and then great care should be exercised when considering their elimination from the series of
187. measurements, as it is well known that in quite normal circumstances of measurement, the errors of individual results, and hence their deviations, can be very great.

In such cases we must proceed from the numerical values of probabilities of occurrence of the errors which correspond to the individual results, eliminating those, the probabilities of the errors of which are negligibly small.

So far in all calculations we have been taking, as the practically allowable limit, [illegible]

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187. method of measurement, and we considered the probabilities of occurrence of the error above ±4E as being negligibly small.

The possibility of such an admission when working out the results of a measurement, when our problem is to determine the form and the basic characteristics of the law of errors, can not satisfy us.

Actually, from the table of values $\Phi(\beta)$ the probability of occurrence of the error within the limits ±4E.

$$\Phi(4.0) = 0.99302,$$

whence the probability of occurrence of the error in excess of ±4E, is

$$1-0.99302 = 0.00698.$$

The probability does not appear to be great, but we are interested not in the probability of occurrence of one error in excess of ±4E, but in the probability of occurrence of at least one such error.

Let us assume, that 20 measurements are made of the same quantity. Then the probability of occurrence of at least one error in excess of ±4E

$$p_I = 1-(1-0.00698)^{20} = 1-0.99302^{20} = 0.13072 \approx 13.1\%;$$

this denotes, that on the average, for every 100 series of 20 analagous measurements, we will obtain, in 13 series, results with errors exceeding ±4E. It is obvious, that there are no grounds to refer such results to the category of abnormal results.

We will take the practical limits of the admissible errors as ±6E. The probability of occurrence of one error within the limits of ±6E.

$$\Phi(6.0) = 0.9999482,$$

from which the probability of occurrence of at least one error in excess of ±6E, with 20 measurements

$$p_I = 1-0.9999482^{20} = 0.000961 \approx 0.1\%.$$

This means, that for every 1000 series of 20 measurements we will obtain, on the average, only one result having an error exceeding ±6E. Such a result can be ignored.

We know, that with the decrease in the number of repetitions of the trial, the probability of occurrence of the event at least once, likewise decreases. Therefore with a small number of measurements we
188. can take, for the abnormal results, those results the errors of which exceed ±5E.

The probability of occurrence of one error within the limits of ±5E

$$\Phi(5.0) = 0.99926.$$

Let us assume, that 10 measurements have been made; then the probability of occurrence of at least one error in excess of ±5E

$$p_I = 1-0.99926^{10} = 0.00734 \approx 0.7\%,$$

and for 5 measurements such a probability will be even smaller:

$$p_I = 1-0.99926^5 = 0.00367 \approx 0.4\%.$$

The occurrence of separate results with an error exceeding ±5E with such a number of measurements is sufficiently improbable, and they can be completely excluded from the series of measurements.

From this we have the following rule: <u>abnormal results are taken to be those results, the errors of which exceed ±6E, and with a small number of measurements (less than 10) - also the results the errors of which exceed ±5E.</u>

In cases when in the series of measurements, individual abnormal

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188. combination (group) of results of the measurements is as follows.

In the first place we exclude from the calculation those results which differ sharply in magnitude from other results. We then determine, from the remaining results, the adequate values of the measured quantity and of the median error.

The errors of eliminated results are calculated with respect to the mean result obtained and, comparing these errors with the median error we establish which of these results should be completely eliminated, and which should be again introduced into the series of measurements. After this, further calculation of the results of the measurement is made.

EXAMPLE 4. In determining the distance to the same target by the same method, the following results in sight divisions were obtained: 131, 132, 133, 135, 130, 129, 148, 133, 137, 131, 122, 130, 133, 135, 131, 130, 132 and 130. Determine, which result of the measurement should be counted as abnormal.

Solution. Two results stand out sharply: 148 and 122; these we will exclude from the first working out of the results of the measurement.

Table.

189. The mean result without the excluded results

$$\xi = \frac{2112}{16} = 132.$$

The median error taken from its position in the series of errors

$$E_{pp} = \frac{1+2}{2} = 1.5\Delta.$$

The errors of excluded results:

$$\lambda_7 = 148-132 = +16\Delta \quad \text{and} \quad \lambda_{11} = 122-132 = -10\Delta.$$

These errors, expressed in terms of E:

$$\lambda_7 = \frac{16}{1.5} = 10.7E \quad \text{and} \quad \lambda_{11} = \frac{10}{1.5} = 6.7E.$$

Consequently, both results are abnormal and should not be introduced in the calculation of the measurements.

We used the formulae which applied only to the Gauss law. Let us examine if this law applies to the given series of measurements (without those excluded).

$$E_1 = \frac{28}{\sqrt{16\times15}} = 1.81\Delta \quad \text{and} \quad E_2 = \sqrt{\frac{74}{15}} = 2.22\Delta;$$

then the relationships between E, E_1 and E_2:

$$\frac{E}{E_1} = \frac{1.5}{1.81} = 0.83; \quad 0.83-0.85 = -0.02; \quad 0.02<0.085;$$

$$\frac{E}{E_2} = \frac{1.5}{2.22} = 0.68; \quad 0.68-0.67 = 0.01; \quad 0.01<0.067;$$

$$\frac{E_1}{E_2} = \frac{1.81}{2.22} = 0.82; \quad 0.82-0.80 = 0.02; \quad 0.02<0.08.$$

The differences of the relationships are less than the permissible - the rule is applicable.

Example 5. The point of intersection to the target taken by means of theodolite gave the following individual results of the measurement in [illegible], 4150, 4160, 4180, 4220, 4465, 4120, 4175, 4250, 4325, 4150, 4140, [illegible], 4170, 4145, [illegible], 4155 and 4090. Determine the adequate value of the measured distance and of the median error of this measurement.

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189. [illegible] Under the first working out, the results: 4465, 4325 and 3920m, and we will determine the adequate value of the measured distance from the remaining 15 results:

$$\xi = \frac{62475}{15} = 4165\text{m}.$$

[illegible] find the apparent errors with respect to this mean result [illegible] will arrange them in an increasing order of absolute values:

[illegible], 5, 10, 10, 15, 20, 20, 30, 40, 45, 55, 60, 65, 75, 85.

From this the median error from its position in the series

$$E_{pp} = 30\text{m}.$$

190. Let us check the eliminated results by comparing their errors with E_{pp}.

4465-4165 = +300m; 300:30 = 10E - abnormal result;
4325-4165 = +160m; 160:30 = 5.3E - permissible result;
3920-4165 = -245m; 245:30 = 8.2E - abnormal result.

We will introduce the result 4325m back into the series of measurements and we will work out the results in their final form.

Table.

The adequate value of the measured distance

$$D = \frac{66800}{16} = 4175\text{m}.$$

The adequate value of the median error from its position in the series of absolute values of errors

$$E_{pp} = \frac{30+45}{2} = 37.5\text{m}.$$

The adequate value of the median error from the differences between adjacent results

$$E_u = 0.4769\sqrt{\frac{86525}{14}} = 37.494 = 37.5\text{m}.$$

Let us check the applicability of the Gauss law to our series of measurements, although this need not be done, as $E_{pp} = E_u$.

The mean arithmetical and the mean quadratic errors

$$E_1 = \frac{710}{\sqrt{16\times15}} = 45.9\text{m} \quad \text{and} \quad E_2 = \sqrt{\frac{53500}{15}} = 59.8\text{m}.$$

191. then the relationships:

$$\frac{E_u}{E_1} = \frac{37.5}{45.9} = 0.82;$$

$$\frac{E_u}{E_2} = \frac{37.5}{59.8} = 0.63;$$

$$\frac{E_1}{E_2} = \frac{45.9}{59.8} = 0.77.$$

These relationships differ very little from the relationships which obey the Gauss law, and this confirms the applicability of the law to the examined series of measurements.

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191. 10. Problems on the Working Out of the Results of Measurements

PROBLEM 90. In measuring an angle by means of a stereoscope, the following results were obtained: 2-41, 2-47, 2-45, 2-40, 2-43, 2-38, 2-49, 2-44, 2-42 and 2-46. Determine the mean value of the measured angle.

Answer. 2-42.5.

PROBLEM 91. In measuring the distance to a datum mark, the following results of the measurement in metres were obtained: 5445, 5384, 5430, 5515, 5418, 5406, 5392, 5525, 5468, 5424, 5452, 5439, 5398 and 5464. Determine the adequate value of the measured distance.

Answer. 5440.

PROBLEM 92. In measuring the same angle by binoculars, the following results of individual measurements were obtained: 1-59, 1-64, 1-61, 1-55, 1-62, 1-58, 1-52, 1-60, 1-72, 1-60, 1-65, 1-56, 1-60, 1-56, 1-67, 1-50, 1-63, 1-58, 1-49, 1-64 and 1-69. What is the value of the measured angle?

Answer. 1-60.

PROBLEM 93. The following values of apparent errors of individual results of measurement in metres were obtained: -13, -5, +16, +2, +24, -18, -20, +8, -12, +10 and +8. What is the median error of the method of measurement?

Answer. E_{pp} = +12m; E_{E_1} = 11.02m; E_{E_2} = 9.83m.

PROBLEM 94. In measurement of a base by means of a measuring tape, the following results of individual measurements in metres were obtained: 178.24, 178.31, 179.02, 178.64, 178.45, 178.56, 178.81, 178.54, 178.62 and 178.41. Determine the value of the median error of the method of measurement.

Answer. E_{pp} = 0.155m; E_{E_1} = 0.166m; E_{E_2} = 0.162m.

PROBLEM 95. Measurement of distance to a target gave the following individual results in divisions of the sight: 108, 112, 105, 106, 104, 105, 109, 105, 104, 106, 102, 107, 101, 109 and 107. What is the probability that the next measurement of this distance by the same method will have an error of:

a) not more than $2\Delta_A$;

b) not more than $5\Delta_A$?

Answer. $E_{E_2} = 1.908\Delta_A$; P_a = 0.52065; P_b = 0.92280.

PROBLEM 96. In taking readings on the goniometer, the following results of individual measurements were obtained: 55-90, 55-45, 56-00, 55-80, 55-65, 56-00, 56-10, 56-15, 55-50, 56-05, 55-85, 55-95, 56-25, 56-20, 55-70, 55-95, 56-70, 56-20, 56-00 and 55-90. What is the probability that the next reading on the goniometer of the same target and by the same method will have an error of:

a) not more than 0-10;

b) not more than 0-35;

c) not less than 0-50?

Answer. E_2 = 21.55 goniometer divisions; P_a = 0.34163; P_b = 0.72666; P_c = 0.11763.

192. PROBLEM 97. What is the median error of the mean result, if the value of this mean result was obtained from the data of 16 separate results of a measurement, and if the median error of the method of measurement is 80m?

Answer. 20m.

PROBLEM 98. The median error of a method of measurement is ±15. What is the median error of the mean result, the value of which is determined from the data of 25 separate measurements?

Answer. 6-03.

PROBLEM 99. Determine the median error of the mean result, if in the measurement of the angle the following values of the errors of separate measurements, in goniometer divisions, were obtained: -2, -4, +6, -5, +1, -4, -2, +8 and -6.

Answer. E_{E_2} = 3.39 goniometer division; E = 1.13 goniometer divisions.

PROBLEM 100. In the measurement of a base, the following results of separate measurements in metres, were obtained: 69, 75, 77, 86, [illegible] mean result.

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192. Answer. E_{E_2} = 26.28m; R = 8.27m.

PROBLEM 101. In measuring the distance to the target by a range-finder, the following results of individual measurements in metres, were obtained: 4890, 4860, 4820, 4860, 4810 and 4870. Determine the smallest limits which contain the true value of the measured distance, with probability being 50%, if the median error of the method of measurement is 20m.

Answer. R ≈ 8.0m; from 4841.8m to 4858.2m.

PROBLEM 102. In measurement of an angle, the following results of separate measurements in goniometer divisions were obtained: 1-61, 1-69, 1-66, 1-60, 1-58, 1-65, 1-72, 1-64, 1-69, 1-65, 1-63, 1-54, 1-67, 1-68 and 1-64. Determine the smallest limits which contain the true value of the measured angle, with a probability of 0.82266.

Answer. E_{E_2} ≈ 3.1 goniometer divisions; R ≈ 0.8 goniometer divisions from 1-62.4 to 1-65.6.

PROBLEM 103. In determining by fire of the initial velocity, the following results of individual measurements in m/sec were obtained: 426.3, 428.1, 424.2, 426.7, 421.5, 420.8, 425.4, 422.1, 426.5, 421.8, 424.4, 427.1, 423.7, 425.9, 421.7 and 427.0. Find the adequate value of the initial velocity and the median error of the method by which it is determined.

Answer. v_0 = 424.2m/sec; E_u = 2.053m/sec.

PROBLEM 104. In determination of distance to the target by means of D&R*, the following results of individual measurements in metres were obtained: 7135, 6915, 7095, 7350, 7015, 7210, 6970, 7320, 7400, 7040, 7165, 6960, 7285, 7150, 6995, 7250, 7290, 7035, 7165 and 6955. Find the adequate value of the distance and the median error.

Answer. D = 7136m; E_u = 113.8m.

PROBLEM 105. In measurement of a distance to a sounding target, the following results of individual measurements in metres were obtained: 2034, 2071, 2142, 2104, 2123, 2082, 2067, 2095, 2066, 2075, 2085, 2044, 2099, 2072, 2100 and 2043. What is the probability that, in taking the mean result as the true value of the distance, we admit an error not exceeding 10m.

Answer. E_u = 18.28m; R = 4.57m; r = 0.86.

PROBLEM 106. In determination of distance the following results of individual measurements in metres were obtained: 4255, 4205, 4135, 4175, 4210, 4315, 4150, 4220, 4290, 4195, 4170, 4215, 4305, 4225, 4275, 4165, 4220, 4165, 4185, 4285, 4260 and 4130.

Check the applicability of the Gauss law to the working out of the results of the measurements and find the adequate value of the distance and of the values of the characteristic of the given law of errors.

Answer. The Gauss law is applicable; D = 4215m; E_1 = 48.4m; E_{E_2} = 45m; E_u = 45.24m.

PROBLEM 107. In determining the distance, the following results of individual measurements in metres were obtained: 6188, 6195, 6130, 6168, 6167, 6111, 5973, 6156, 6208, 6257, 6121, 6035, 6169, 5274, 6090, 6115, and 6171.

Check the applicability of the Gauss law to the working out of the results of measurements, and find the adequate value of the distance and the characteristic of the given law of errors.

Answer. The Gauss law is applicable; R = 6.42m; E_1 = 65.9m; E_{E_2} = 79.0m; E_{E_3} = 53m; E_u = 54.3m.

PROBLEM 108. In determination of a base, the following results of individual measurements in metres were obtained: 84.9, 83.6, 84.8, 87.6, 83.4, 85.4, 88.5, 94.0, 85.7, 84.0, 86.1, 92.8, 86.4, 84.9, 87.3 and 91.7. Find the adequate value of the dimension of the base and indicate which results of these measurements are abnormal.

Answer. 94.0 and 92.8.

PROBLEM 109. Find the adequate value of the median error and establish the applicability of the Gauss law for the working out of 18 following results of the measurement of aircraft speed: 31.5, 32.0, 33.2, 34.1, 36.5, 30.5, 33.5, 36.0, 35.5, 31.5, 38.0, 34.0, 39.0, 35.5, 34.5, 37.0, 35.5 and 40.0m/sec.

Answer. The Gauss law is applicable; speed 34.8m/sec; R = 1.6m/sec.

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* Three letters transliterated from Russian.

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Chapter VI

ERROR IN A PLANE AND IN SPACE.

41. Error-vectors.

It has been shown above that all chance errors can be divided into the following two categories:

1. Errors which are independent of the direction, or linear errors (otherwise termed scalar errors).

2. Errors which are dependent on direction, or errors in a plane and in space.

The errors of the first category are determined by magnitude (numerical value) and by sign. For example, having measured the barometric pressure and having established the error of the measurement $\Delta = -2$mm, we obtain a complete picture as to the accuracy of the measurement made. Apart from the number 2 and the sign "minus", no other characteristics of the error are required.

To errors, which are independent of direction, i.e. which are not connected with any direction, belong errors which occur in measurements of weight, pressure, temperature, time etc.

For the charateristic of errors in a plane and in space, it is necessary, apart from the magnitude and sign, to consider also their direction. In the process of measurements of one and the same quantity by the same method, the errors of this category can change their direction, and each such change would reflect on the results of the measurement. These errors are therefore termed as those dependent on direction.

Let us show this by means of the following example.

Let us assume, that a target is marked off on a map according to the angle between the directions to the orientation point and the target, and the observation distance.

Let us assume, that the angle β_0 and the distance D_0 correspond absolutely to the exact position of the target on the map. Let us suppose, that in all measurements, the errors, in the marking off of the target on the map, are equal to each other in absolute magnitude and differ only in directions, i.e. let us suppose, that the distances on
95. the map between the point T_0, corresponding to the actual position of the target, and the points T_1, T_2 and T_3, obtained from separate results of the measurement, are the same in magnitude, but differing in directions (fig.52).

From the figure it is seen, that depending on the errors, admitted in the measurement of the angle β_0 and the distance D_0, the errors of individual results of the measurement, Δ_1, Δ_2 and Δ_3, have different directions. In other words, with specified directions of errors Δ_1, Δ_2 and Δ_3, equal in absolute magnitude, the results of the measurement will be different.

Fig.52.

Consequently, knowing only the magnitude, sign and direction of such errors, it is possible to determine the errors of individual results of the measurement.

Errors which, apart from the magnitude and sign, are also characterised by direction, are more conveniently expressed graphically, i.e. in the form of vectors.

An error, the value of which is determined by absolute magnitude and direction, is termed an error-vector.

Error-vectors can have any direction in a plane or in space. Therefore these errors, as opposed to linear errors, are called errors in a plane and in space.

Error-vectors, as all vectors generally, are expressed as a part of a straight line, having a length, a point of application and a direction.

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195. By direction is understood to be the following: in the first place, the direction (direction in a plane or in space) of the straight line along which lies the error-vector, and in the second place, the direction of the error-vector itself along the straight line (fig.53).

Fig.53.

In fig.53, AB is the value (size) of the error-vector, A - its point of application and MN - the direction of the straight line along which the error-vector lies; the arrow shows the direction of the error-vector along the straight line MN.

196. An error-vector is denoted by the conventional symbols, but with a dash above it, e.g. $\bar{\Delta}$ or by two letters also with a similar dash, with the first letter corresponding to the beginning (the point of application) of the error-vector, e.g. $\overline{AB}$.

The error-vector may be given analytically. In this case its value (size) is expressed in some linear measures, the point of application by coordinates, the straight line along which the vector lies, by an equation, and the arrow is substituted by the signs "+" and "-".

Considered separately, the error-vector is the result of a measurement in some definite direction. If, with a given measurement several individual error-vectors arise and each one of them has its own direction, then the error-vector of the result of the measurement represents the whole sum. Such an error-vector is the resultant of separate, component error-vectors. For a graphical example of this we can use the preparation of original settings for firing, where the error-vector (in determination of the direction of the mean trajectory with respect to the target) which represents the calculated settings, is the result of a combination of error-vectors in the determination of the direction of the target, the distance to the target and of the excess of the target.

To find the resultant error-vector, it is necessary to add the separate component error-vectors.

When adding the separate error-vectors, the following three cases can exist:

1. Separate error-vectors can have one common direction, i.e. they can lie along the same straight line.
2. Separate error-vectors can have different directions in the same plane.
3. Separate error-vectors can have different directions in space, i.e. they can lie in different planes.

1. Let us assume, that in determining Db by optical method, the error of measurement Dk ($\bar{\Delta}_1$) is equal to +100m and the error of measurement of departure ($\bar{\Delta}_2$) is +30m.

These two errors can be added algebraically, as well as geometrically.

In algebraical addition the resultant error will be equal to the algebraic sum of the component errors:

$$\bar{\Delta} = \bar{\Delta}_1 + \bar{\Delta}_2 = (+100)+(+30) = +130\text{m}.$$

To carry out geometrical addition of these errors, let us express them, to the same scale as two vectors and let us transfer these vectors on to the straight line OA (fig.54) which is parallel to them, at the same time making the starting point of the second vector to coincide with the end of the first vector.

197. Geometrically, this sum can be expressed by the intercept $\overline{OA}$.

Let us take the same example, but let us assume, that the error of measurement Dk $\bar{\Delta}_1 = +100\text{m}$, and the error of the measurement of departure $\bar{\Delta}_2 = -30\text{m}$.

In algebraic solution, the resultant error will be

$$\bar{\Delta} = (+100)+(-30) = +70\text{m}.$$

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197.

Fig.54.

Fig.55.

Geometrically, this error will be expressed by the intercept $\overline{OA}_1$ (fig.55) which is equal to

$$\bar{\Delta} = \bar{\Delta}_1 + \bar{\Delta}_2$$

The method of drawing these vectors is the same: the starting point of the second vector is made to coincide with the end of the first vector.

When adding the separate, component error-vectors, lying along the same line, the resultant error-vector obtained is an algebraic or geometrical sum of the component error-vectors. The direction of the resultant error-vector coincides with the direction of the component error-vectors.

For geometrical addition of separate component error-vectors, it is necessary to join the starting point of the second error-vector with the end of the first error-vector, the starting point of the third error-vector - with the end of the second etc. Then the starting point of the first error-vector will be the starting point of the resultant error-vector, and the end of the last error-vector - the end of the resultant error-vector.

EXAMPLE 1. In determining Db by means of a brief method, the following errors of distance were admitted:

In determination of the coordinates OP, +15m;
In plotting the coordinates OP on the map, -5m;
In plotting the target on the map, -80m;
In measuring the distance on the map, +10m.
Find the error in the determination of the distance.

Solution. We have 4 separate components of error-vectors which have direction along the same straight line. We will find the resultant error as the algebraic sum of the component errors:

$$\bar{\Delta} = (+15)+(-5)+(-80)+(+10) = -60\text{m}.$$

EXAMPLE 2. Three error-vectors $\bar{\Delta}_1\bar{\Delta}_2\bar{\Delta}_3$ are given, these vectors being expressed graphically as shown in fig.56. Find the resultant (total) error-vector.

Solution. We will use here the geometrical method of solving. For this purpose we join the starting point O_2 of the second error-vector to the end A_1 of the first error-vector, and the starting point O_3 of
198. the third error-vector to the end A_2 of the second error-vector. Then the point O_1 will be the starting point of the resultant error-vector, and the point A_3 - its end (fig.56); and the resultant error-vector will be

$$\bar{\Delta}_1 + \bar{\Delta}_2 + \bar{\Delta}_3 = \bar{\Delta} = \overline{O_1A_3}.$$

Fig.56.

2. When adding the error-vectors which have different directions, but which lie in the same plane, a geometrical method of addition is normally employed.

The method is based on that the vectors can not only be transferred along some line, but also that they can be moved parallel to this line, i.e. parallel to each other. In all other respects the resultant error-vector is drawn according to the same rule as for addition of error-vectors the direction of which is along the same straight line: the starting point of the second error-vector is made to coincide with the end of the first error-vector, the starting point of the third - with

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98. Fig.57 gives graphical examples of addition of several error-vectors, having different directions in the same plane*.

Fig.57.

A particular case of addition of error-vectors, having different directions in the same plane, is a case when the directions of error-vectors are perpendicular to each other. To this case of addition of error-vectors belongs, for example, the addition of errors in distance and in direction when preparing the initial data for firing.

99. Here, the resultant error-vector can be determined analytically as a hypotenuse of a right-angle triangle the sides of which are formed by the error-vectors, from the formula

$$\bar{\Delta} = \sqrt{\bar{\Delta}_1^2 + \bar{\Delta}_2^2}$$

and the angle contained by the direction of the resultant error-vector and the direction of one of the other error-vectors, - from the formulae:

$$\tan \alpha_1 = \frac{\bar{\Delta}_2}{\bar{\Delta}_1} \quad \text{or} \quad \tan \alpha_2 = \frac{\bar{\Delta}_1}{\bar{\Delta}_2}.$$

EXAMPLE. In preparation of original data, the following errors were admitted: in distance +100m and in direction - 60m (to the left). Find the resultant error-vector and the angle, which this error makes with the plane of fire.

Solution. From the formulae just shown, the size of the resultant error-vector

$$\bar{\Delta} = \sqrt{100^2 + 60^2} = 117\text{m},$$

and the tangent of the angle made by the resultant error-vector and the plane of fire

$$\tan \alpha = \frac{60}{100} = 0.6,$$

from which the angle taken from the table of natural trigonometric values

$$\alpha = 31^\circ = 5\text{-}17.$$

The graphical solution of this example is shown in fig.58.

Fig.58.

3. Addition of error-vectors, having different directions in space is not discussed in this Course on Artillery, because cases dealing with addition of such error-vectors are not, as a rule, met with in land artillery.

42. Vectorial errors.

Let us suppose, that a measurement of some quantity has error-vectors lying in the same straight line. If each individual result of the measurement gives its own error-vector, then in repeating the measurement of the same quantity many times, we will obtain a sum of error-vectors or, in other words, a system of error-vectors.

200. A system (aggregate) of error-vectors directed in space along the same definite straight line, is termed a system of vectorial errors or, briefly a vectorial system.

This system is conventionally called a vectorial system, and by this we understand to be the whole aggregate of errors and not an individual error.

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* Unaltered force of the vector along the straight line is only one case of its parallel transposition.

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200. Graphically, for this system of vectorial errors which lies on the line MN, the vectorial error is expressed by an intercept equal in value to the median error of the system.

Thus, for example, if the aggregate of the error-vectors follows the Gauss law and has a direction MN (fig.59) then the vectorial error which characterises this aggregate can be graphically shown as an intercept OB (or OC) applied to the point O and lying along the line MN. Here the point O - the beginning of the intercept - represents the true value of the measured quantity, i.e. the result, the error of which is zero.

Fig.59.

The size of the intercept OB, being the value of the median error of the aggregate of the error-vectors, shown in fig.59, can be easily found if these error-vectors are expressed numerically:

$\Delta_1 = +70m$; $\Delta_4 = +450m$; $\Delta_7 = +260m$; $\Delta_{10} = +200m$;
$\Delta_2 = -350m$; $\Delta_5 = +140m$; $\Delta_8 = -50m$; $\Delta_{11} = -180m$;
$\Delta_3 = -120m$; $\Delta_6 = -400m$; $\Delta_9 = -100m$; $\Delta_{12} = +80m$.

Then, arranging these errors in their absolute values in an increasing order:

50, 70, 80, 100, 120, 140, 180, 200, 260, 350, 400, 450

we find, that

$$OB = \frac{140+180}{2} = 160m,$$

or, to the scale in fig.59, - 16mm.

201. In the example shown, the intercept OB expresses the vectorial error of the aggregate of the error-vectors, lying along the line MN; at the same time, this intercept OB is equal in value (numerically) to the median error of the given aggregate. Therefore, when speaking of the vectorial error which characterises the aggregate of error-vectors, lying along the same common straight line, we normally understand by this to be the median error of the given aggregate, remembering, that here we are dealing with only a numerical equality. Understood in this way, the vectorial error has the same properties as the median error.

If we arrange (draw) all the error-vectors according to their absolute values in an increasing or a decreasing order, without taking into consideration their direction, then the vectorial error will occupy in this row the middle position, i.e. it will be greater than any of the error-vectors of one half of the row and less than any of the error-vectors of the other half.

The median error has no sign, and therefore the vectorial error has no direction along the common straight line, i.e. it is shown in the form of an intercept of the straight line and not by a vector.

The vectorial error is commonly denoted by the letter R, although the conventional notation for the median error E can be used. If the vectorial error is shown as an intercept of the straight line, then the letters standing for this error have a dash with two arrows over them, for example: $\overleftrightarrow{OB}$.

In fig.60 is shown the position of the vectorial error R in the vectorial system, as the aggregate of the error vectors.

Fig.60.

In a measurement of some quantity, there can be several vectorial systems, lying along the same straight line; for example, the vectorial systems which arise when determining different corrections during complete preparation of the original settings for firing.

If these vetorial systems follow the Gauss law, - and in the future we will be dealing only with such vectorial systems, - then the addition of the vectorial errors, corresponding to these systems, is carried out from the formula for addition of the Gauss laws, with which we are acquainted:

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201. $E = \sqrt{E'^2+E''^2+E'''^2+\ldots+E^{S2}}$

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202. in which for E we can substitute the total (resultant) vectorial error R, and instead of E', E'', E'''...E^S - the vectorial errors R_1, R_2, R_3...R_S, which characterise the individual vectorial systems. And then

$$R = \sqrt{R_1^2+R_2^2+R_3^2+\ldots+R_S^2}.$$

We can draw this conclusion from the analogy which was established between the vectorial and the median errors. At the same time the conclusion shows that the vectorial errors do not come within the law of vectorial addition. here we are adding the systems of error-vectors (median errors) and not individual error-vectors.

EXAMPLE. When determining the calculated distance, the vectorial errors in determination of meteorological and ballistic conditions of firing are as follows:

In measurement of air temperature.	30m.
" " " barometric pressure.	10m.
" " " temperature of charge.	20m.
" " " speed and direction of wind. . .	30m.
" " " loss of initial velocity	40m.

Vectorial error in determination of distance in topographical preparation of the data, is 60m.

Find the total vectorial error, i.e. the vectorial error in determination of the distance for the whole compete preparation of the data.

Solution. Substituting in the general formula, the numerical values which compose the vectorial errors, we obtain the numerical value of the total (resultant) error

$$R = \sqrt{30^2+10^2+20^2+30^2+40^2+60^2} = \sqrt{7500} \approx 85.44\text{m}.$$

The obtained result denotes, that in 50 cases out of 100, the errors, which occur due to inaccurate calculations of the conditions of firing, do not exceed 85.44m.

Sometimes, in measurements, the error-vectors arising in some direction affect another direction with which the solution of the basic problem is connected.

In sending the indication of the target from one observation point to another, the errors, in the determination of the position of the target by the sender providing the information regarding the target, affect the search of the target by the receiver of the information regarding the target.

Let us discuss this case in a general form.

Let us assume, that a vectorial error $\overline{OR}$ is given and that its direction is along the line KL (fig.61). With this direction as such we are not concerned, but we are substantially interested in some other direction MN, as each error-vector, arising in the vectorial system $\overline{OR}$, causes a deviation from the direction MN.

203. Thus, if in the system $\overline{OR}$ an error-vector $\overline{OA}$ is obtained, then this error will cause a deviation $\overline{A''A}$ from the direction MN, the error-vector $\overline{OB}$ of the same system will cause a deviation $\overline{B''B}$ etc. We will measure all these deviations along the line M'N' which is perpendicular to the direction MN and which makes an angle α with the direction KL, i.e. we will project all these deviations on the perpendicular to MN.

Fig.61.

Solving the right-angle triangles OA'A, OB'B etc, we obtain

$$\overline{A''A} = \overline{OA'} = \overline{OA} \cdot \cos\alpha;$$

$$\overline{B''B} = \overline{OB'} = \overline{OB} \cdot \cos\alpha \text{ etc.}$$

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Thus, each error-vector of a given vectorial system causes a

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203. deviation from some direction which is equal to the projection of the error-vector on the perpendicular to that direction. The relationship, however, between the error-vector and the deviation caused by it, is expressed by the equation

$$\bar{\Delta}_i' = \bar{\Delta}_i \cdot \cos\alpha,$$

where $\bar{\Delta}_i$ - error-vector;
$\bar{\Delta}_i'$ - the deviation from some direction caused by this error-vector;
α - angle between the direction of the vectorial error (system) and the perpendicular to some direction.

If the error-vectors follow the Gauss law, then the deviations caused by these error-vectors also follow this law, because the error-vectors differ from their corresponding deviations by the fact that the expression for the deviations contains the multiplier $\cos\alpha$, which is a constant quantity for a given condition. A new vectorial system is
204. obtained which, in the given example with a specified vectorial error OR, can have the following relationship:

$$\overleftrightarrow{OR'} = \overleftrightarrow{OR} \cdot \cos\alpha.$$

This new vectorial system differs substantially from the usual vectorial systems.

In the first place, this new vectorial system can not occur by itself - it is caused by some other vectorial system and depends on it.

In the second place, this new vectorial system represents a system of deviations, and not errors, and these deviations never coincide with the line, in the direction of which they lie, - they are always parallel to the line and lie away from it the further, the greater are their corresponding error-vectors (deviations A''A and B''B in the figure).

Therefore the term "vectorial error" is not applicable here; it is more true to refer to: vectorial or middle deviation. Commonly, however, the term used is "deviation from some direction", this term being understood to be the deviation caused by vectorial error and equal to the projection of this vectorial error on the perpendicular to some direction.

Denoting this deviation by d (in contrast to the vectorial error R which causes it) we obtain

$$d = R \cdot \cos\alpha.$$

EXAMPLE. Vectorial error in distance, when determining the position of the target from the division commander's observation point, is 50m, the direction (side) errors in determination of the position of the target being negligibly small. What is the probability that the battery commander, in setting the stereoscope according to target information provided by the division commander, will see the target within the limits of the stereoscope graticule, if the angle, formed by the observation lines of the division commander and the battery commander, is 5-00, and the distance of observation of the battery commander is 2500m?

Solution. The error in distance, admitted by the division commander, will cause a deviation of target position from the observation line of the battery commander. The angle between the observation lines, according to the condition, is equal to 5-00, or 30°. Hence, the angle between the observation line of the division commander (direction of the vectorial error) and the perpendicular to the observation line of the battery commander, being a complementary angle to 90°, is equal to (fig.62)

$$\alpha = 90° - 30° = 60°.$$

Fig.62.
(1) - DC - Division commander;
(2) - BC - Battery commander.

Let us project the given vectorial error on to the perpendicular to the observation line of the battery commander (BC), and then the

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204. middle deviation from this line

$$d = 50 \times \cos 60^0 = 50 \times 0.5 = 25\text{m},$$

or in goniometer divisions,

$$d = 25:2,5 = 0.10.$$

205. The probability of obtaining a deviation which does not come outside the limits of the graticule of the stereoscope ($\pm 0{-}30$):

a) from the table of values $\Phi(\beta)$

$$P = \Phi\left(\frac{30}{10}\right) = \Phi(3.0) = 0.95698;$$

b) from the scale of errors

$$P = 2(0.07+0.16+0.25) = 0.96.$$

In the same way as the vectorial error was projected in two directions, we can <u>resolve</u> the vectorial error into two directions.

Let us examine the case of resolving the vectorial error into two directions perpendicular to each other. Let us assume, that we have a vectorial error $\overleftrightarrow{OB}$ and two directions OX and OY perpendicular to each other. The direction OL of this vectorial error makes an angle α with the direction OX (fig.63).

Fig.63.

Projecting the vectorial error onto these directions, we obtain two projections, seemingly two vectorial errors:

$$\overleftrightarrow{OB'} = \overleftrightarrow{OB_x} = \overleftrightarrow{OB} \cdot \cos\alpha \quad \text{and} \quad \overleftrightarrow{OB''} = \overleftrightarrow{OB_y} = \overleftrightarrow{OB} \cdot \sin\alpha .$$

These two vectorial errors are peculiar in that in comparison with any other pair of vectorial errors they are, in the first place, dependent on each other, i.e. they arise simultaneously with the vectorial error $\overleftrightarrow{OB}$, and in the second place, when added they reproduce the vectorial error $\overleftrightarrow{OB}$.

This is easily confirmed, if we project $\overleftrightarrow{OB_x}$ and $\overleftrightarrow{OB_y}$ on to the direction OL.

The projection $\overleftrightarrow{OB_x}$ on to this direction is equal to

$$\overleftrightarrow{OC} = \overleftrightarrow{OB_x} \cdot \cos\alpha = \overleftrightarrow{OB} \cdot \cos^2\alpha ;$$

the projection $\overleftrightarrow{OB_y}$ on to the same direction, is equal to

$$\overleftrightarrow{OA} = \overleftrightarrow{OB_y} \cdot \sin\alpha = \overleftrightarrow{OB} \cdot \sin^2\alpha ,$$

and the sum of the projection

$$\overleftrightarrow{OC}+\overleftrightarrow{OA} = \overleftrightarrow{OB} \cdot \cos^2\alpha + \overleftrightarrow{OB} \cdot \sin^2\alpha = \overleftrightarrow{OB}(\cos^2\alpha + \sin^2\alpha) = \overleftrightarrow{OB}.$$

This means, that the projections, which we obtain when resolving the vectorial error, can be added geometrically, i.e. as vectors and not as vectorial errors.

206. ## 43. Elliptical and ellipsoidal errors.

If the error-vectors, arising in the process of the same measurement, do not coincide in direction, then also the vectorial errors, corresponding to such error-vectors, will not coincide in direction.

Let us examine two vectorial errors, directed along lines perpendicular to each other

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206. For further examination, we will take a particular example, when for opening fire onto the same target and by the same method, the original data is prepared by a group of people, for example, by a group of trainees (students).

Each one of the individuals preparing the data can admit an error in direction (an error in the goniometer setting) and an error in distance (an error in the sight setting)*. Two systems of error-vectors are obtained, which are directed along two lines perpendicular to each other (fig.64): one - in determination of the position of the target in distance (along the line DD) and the other - in determination of the position of the target in direction (along the line MN). Let us suppose, that the first system is characterised by vectorial error R_D, and the second system - by vectorial error R_N. Let us also suppose, that the point T_0 corresponds to the actual position of the target, i.e. to the absolutely precise original data.

Fig.64.

We will assume, that the first student has prepared the data for opening fire, with the errors: $\bar{\Delta}_1'$ - in distance, $\bar{\Delta}_1''$ - in direction. As the causes, which gave rise to these errors, were acting simultaneously, these errors will give a resultant error-vector $\bar{\Delta}_1$, and the target marked off by this student will be the point T_1. If the second student admitted errors: in distance $\bar{\Delta}_2'$ and in direction $\bar{\Delta}_2''$, then the resultant error-vector in this case will be $\bar{\Delta}_2$ and the target marked off will be at the point T_2. The errors in the data, prepared by the third student, $\bar{\Delta}_3'$
207. and $\bar{\Delta}_3''$ will give a resultant error-vector $\bar{\Delta}_3$ and the target will be at the point T_3 etc. In this way, in the preparation of the original data by each student, two error-vectors arise: each such pair will give its own resultant error-vector, and to each resultant error-vector will correspond its own target position.

Let us see, how these resultant error-vectors are added, in other words, let us establish, what is represented by the addition of all the target positions marked off.

The errors in the preparation of the original data follow the Gauss law, and consequently, both in distance and in direction there can be obtained separate error-vectors of any size and any sign, i.e. within practically the limits from T_0 to $\pm 4R_D$ in distance, and from T_0 to $\pm 4R_N$ in direction. Therefore, the target can be marked off at any point in a plane, in an area within these limits (fig.65).

Fig.65.

This means that the error in the preparation of the original data, in the conditions of the given example, will not be expressed by any one vector, but a combination of error-vectors, distributed in one plane and having different directions commencing from one common point and limited by some area. Such a combination (summation) of all the possible error-vectors in a plane, is termed an <u>error in a plane</u>.

Developing this statement for any pair of mutually perpendicular vectorial systems, it is possible to make the following deduction.

<u>Two mutually perpendicular vectorial systems or two mutually perpendicular vectorial errors, when added, produce an error in a plane</u>.
208.

If we add any number of vectorial errors, directed along two mutually perpendicular lines, the same deduction will be reached.

Let us assume, that vectorial errors R_1', R_2', R_3' etc act in some direction, and that in another direction, perpendicular to the first, we have vectorial errors R_1'', R_2'', R_3'' etc. Applying the rule for addition of vectorial errors, directed along one straight line, we obtain two resultant vectorial errors:

$$R' = \sqrt{R_1'^2 + R_2'^2 + R_3'^2 + \dots} \quad \text{and} \quad R'' = \sqrt{R_1''^2 + R_2''^2 + R_3''^2 + \dots},$$

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* We neglect the errors in level setting.

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208. the addition of which will give the error in a plane.

Thus, when adding the vectorial errors (systems), directed along two mutually perpendicular lines, we obtain the summation of the error-vectors the ends of which do not exceed the limits of a certain area.

In order to determine the shape (contour) of this area, we will calculate the distribution of probabilities of occurrence of an error within the limits of the area with which we are concerned, using the same example which deals with the preparation of the original data.

We will draw two scales of errors on mutually perpendicular lines, along which lie (are directed) the vectorial errors and we will mark the divisions on these scales in values of vectorial errors R_D and R_N (fig.66), and we will assume that the vectorial errors follow the Gauss law.

Fig.66.

The probability of occurrence of an error within the limits of the rectangle Oabc, or the probability that the target will be marked off within the limits of this rectangle, can be found from the theorem of multiplication as the probability of coincidence of the error in distance within the limits from 0 to $+R_D$, with the error in direction
209. within the limits from 0 to $+R_N$:

$$P = 0.25 \times 0.25 = 0.0625 = 6.25\%.$$

The probability of occurrence of an error within the limits of the rectangle defg, or the probability that the target will be marked off within the limits of this rectangle, can be found as the probability of coincidence of the error in distance within the limits from $+2R_D$ to $+3R_D$, with the error in direction within the limits from $-R_N$ to $-2R_N$:

$$P = 0.07 \times 0.16 = 0.0112 = 1.12\% \text{ etc.}$$

The probabilities, calculated in this way, will give the disposition of the errors in the plane when two mutually perpendicular vectorial errors are added.

If we neglect very small probabilities of occurrence of the error (less than 0.15%) and if we draw a broken line around the remaining rectangles, a stepped figure will be obtained, the limits of which - depending on the number of rectangles (when drawing the scales of errors in parts of R_D and R_N) - will be an ellipse.

Consequently, <u>when adding vectorial errors, directed along two mutually perpendicular lines and following the Gauss law, an error in a plane will be obtained within the limits of an ellipse. Such an error is termed an elliptical error</u>.

An elliptical error will also take place when vectorial errors are not mutually perpendicular. As an example of such an error, we can take an elliptical error which is obtained as a result of the intersection on the target (a fix) from two observation points (fig.67).

Fig.67.

Let us suppose, that the errors made by the observers, when making a fix on the target, follow the Gauss law and that the left observer carries out his measurements with absolute accuracy, and the right observer, with errors, and that the vectorial error of the right observer is R_R. It is obvious, that the target in this case will appear to be on the observation line of the left observer somewhere between the points T_1 and T_2 away from the point T_0 which corresponds to the true position of the target on $4R_R$.

If, however, we assume, that the right observer carries out his
210. measurements without errors, and the left observer

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210. the target will be marked off on the observation line of the right observer, somewhere between the points T_3 and T_4 which will lie away from the point T_o by $4R_L$, i.e. by the value of the limit errors of the left observer, along the observation line of the right observer.

As, in fixing the target both observers make errors in measurement, the target will be found marked within the limits of the quadrilateral ABCD. If now, we divide this quadrilateral into 64 quadrilaterals, in the same way as was done when adding two mutually perpendicular vectorial errors (see fig.66), and if we reject those quadrilaterals in which the probability of location of the target is very small, then also in this case a figure will be obtained which will be in shape close to an ellipse. Consequently, to sum up, when adding two vectorial errors, the directions of which form any angle with each other, an elliptical error is obtained.

Thus, by an elliptical error is termed an error in a plane which is obtained as a result of addition of vectorial errors, having different directions in the same plane and following the Gauss law.

In order to be able to judge the accuracy of a measurement, the result of which is a matter of obtaining an elliptical error, it is necessary to know the characteristic of this elliptical error. In the same way as for errors in other categories, which were examined previously, so also in the case of the elliptical error such a characteristic must be some middle (median) error.

It is obvious, that such a median error must have a shape of an ellipse which is associated with those vectorial errors, which when added will produce the given elliptical error.

For the characteristic of an elliptical error, is taken the so called primary ellipse, the main half-axes of which are additive vectorial errors, in the condition that these errors are mutually perpendicular.

Fig.68.

The primary ellipse (fig.68) is denoted by the letter $\mathcal{E}$, and its main half-axes - by letters "a" (major half-axis) and "b" (minor half-axis).

211. The primary ellipse determines the vectorial (median) errors in any direction. In other words, any semi-diameter of the primary ellipse is at the same time the vectorial error in the direction of this semi-diameter. Thus, for example, the semi-diameter OC is the vectorial error in the direction MN, and the semi-diameter OD - the vectorial error in the direction KL etc.

From this property of the primary ellipse, it is easy to pass on to the limit (in value), elliptical error - to the complete ellipse ($\mathcal{E}_{max}$). For this purpose, it is sufficient, to mark off along several directions, which pass through the centre of the primary ellipse, the limit error-vectors, in magnitude, taking them to be equal to 4 or 5 vectorial errors (semi-diameters of the primary ellipse) which correspond to these directions, and then to draw the complete ellipse (fig.69). On any diameter of this limit ellipse, it is possible to mark off a scale of errors on which the probabilities of occurrence of errors within specified limits along the direction of this diameter, can be determined.

Fig.69.

Let us return to fig.66 in which was shown the distribution of errors within the limits of elliptical error. From this figure it is seen, that the probability of occurrence of the error within the limits of the rectangle, the limits of which in distance is $\pm R_D$ and in direction $\pm R_N$, is equal to

$$6.25\%+6.25\%\cdot 6.25\%+6.25\% = 25\%;$$

and the area of this rectangle is known as a primary rectangle, and is

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211. The primary ellipse is located within the limits of this primary rectangle. Taking the ratio of the area of the primary ellipse to the area of the primary rectangle, we will find an approximate value of the probability of occurrence of the error within the limits of the primary ellipse:

$$\varepsilon = \frac{\pi ab}{4ab} \times 25 = 19.6\% \approx 20\%.$$

212. If we take into consideration, that the errors within the limits of the primary rectangle are distributed uniformly, then this probability will be equal to 20.3%.

Apart from the primary ellipse, we have, in some cases, to make use of the so called semi-ellipse (half an ellipse), the main half-axes of which are equal to the halves of the additive vectorial errors. The probability of occurrence of the error, within the limits of the semi-ellipse, is 5.5%.

In the particular case of addition of vectorial errors, when the vectorial errors are added, and these errors are equal to each other in magnitude, and lie along two mutually perpendicular lines in the same plane, a so called circular error is obtained. The circular error arises, for example, when points are marked off on a map or a drawing board from coordinates or when taking the coordinates of such points.

The question regarding deviations, caused by an elliptical error, is of great practical significance.

Let us assume, that there is a primary ellipse ε, the main half-axes (forming vectorial errors) of which are a and b, where the main half-axis a lies in the direction of the line KL. Let us also assume, that we have some other direction MN, the perpendicular to which is M'N' and the direction KL make an angle α (fig.70).

Fig.70.

Let us see, what deviation (middle deviation) will be caused by the primary ellipse or by the elliptical error, from the specified direction MN.

The projection of the major half-axis "a" onto the perpendicular M'N', i.e. the deviation, caused by the vectorial error $\overleftrightarrow{a}$ from the direction MN, is, from the well known formula:

$$a' = a \cdot \cos\alpha;$$

the projection of the minor half-axis "b" onto the same perpendicular M'N' (the deviation caused by the vectorial error $\overleftrightarrow{b}$ from the direction MN)

$$b' = b \cdot \cos(90^\circ - \alpha) = b \sin\alpha.$$

We have obtained two independent deviations (middle deviations), directed along the same straight line M'N'. According to the rule for addition of vectorial errors, having a common direction, the total
213. deviation, i.e. the deviation caused by the elliptical error ε,

$$D' = \sqrt{a^2 \cdot \cos^2\alpha + b^2 \cdot \sin^2\alpha}.$$

We have found a deviation from the direction MN (caused by an elliptical error) along the perpendicular to this direction M'N'. It is quite obvious, that we can find the deviation, caused by the same elliptical error, also from the direction M'N' along the perpendicular to it, i.e. along the direction MN.

In this case the deviations a'' and b'', caused by respectively the same vectorial errors $\overleftrightarrow{a}$ and $\overleftrightarrow{b}$, will be

$$a'' = a \cdot \cos(90^\circ - \alpha) = a \cdot \sin\alpha \quad \text{and} \quad b'' = b \cdot \cos\alpha.$$

Then the total deviation

$$D'' = \sqrt{a^2 \cdot \sin^2\alpha + b^2 \cdot \cos^2\alpha}$$

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213. The total deviation D, in contrast to D' and D'', is frequently referred to as the deviation along some direction.

We will use these two formulae when determining deviations, caused by an elliptical error, from some direction and lying in some direction; and by an elliptical error we will understand to be the primary ellipse*.

EXAMPLE 1. In an elliptical error ε, the main half-axes are 50m and 20m. What are the deviations, caused by this elliptical error, from the direction MN and in the direction MN, if this direction makes an angle of 18° with the major half-axis of the elliptical error?

Solution. Let us denote the main half-axes of the elliptical error by

$$a = 50\text{m} \quad \text{and} \quad b = 20\text{m}.$$

We will first find the total deviation (middle deviation) from the direction MN.

The deviation caused by the vectorial error $\vec{a}$,

$$a' = a\cos\alpha = 50\cos 18^\circ = 50\times 0.951 = 47.55\text{m}.$$

The deviation caused by the vectorial error $\vec{b}$,

$$b' = b\sin\alpha = 20\sin 18^\circ = 20\times 0.309 = 6.18\text{m}.$$

Then the total deviation caused by the elliptical error ε,

$$D' = \sqrt{a'^2+b'^2} = \sqrt{47.55^2+6.18^2} \approx 47.95\text{m}.$$

The total deviation caused by the specified elliptical error in the direction MN,

$$D'' = \sqrt{50^2\sin^2 18^\circ+20^2\cos^2 18^\circ} = \sqrt{15.45^2+19.02^2} \approx 24.5\text{m}.$$

214. EXAMPLE 2. An elliptical error which obtains in determination of the target position from the observation point of the battery commander, has the following characteristics: vectorial error in distance R_D = 80m, and vectorial error in direction R_N = 20m.

Determine the deviations caused, from the observation line and along the observation line of the side observation point, if the angle, contained by the observation lines of both observation points, is 4-00.

Solution. $a = R_D = 80\text{m}$; $b = R_N = 20\text{m}$; $\alpha = 4\text{-}00 = 24^\circ$.

a) The deviation caused by the elliptical error, from the observation line of the side observation point:

$$D'' = \sqrt{80^2\cos^2 24^\circ+20^2\sin^2 24^\circ} \approx 73.35\text{m}.$$

b) The deviation caused by the elliptical error along the observation line of the side observation point:

$$D'' = \sqrt{80^2\sin^2 24^\circ+20^2\cos^2 24^\circ} \approx 37.34\text{m}.$$

So far we have been adding vectorial errors, lying in the same plane. In artillery practice we have to encounter simultaneous action of several vectorial errors (deviations) which do not lie in the same plane. In the first place this refers to the deviations of distant bursts.

Let us examine a simpler and at the same time a more typical example - the addition of three vectorial errors lying along three mutually perpendicular lines.

We will assume that there are three vectorial errors R', R'' and R''', the directions of which coincide with the directions of the axes of a rectangular system of coordinates (fig.71).

* Because of its length, the derivation of the rule of addition of vectorial errors in a plane, is not given. A particular case of addition of vectorial errors in a plane, which is frequently used in artillery practice, is given in the 10th volume (book) of the Course of Artillery.

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Fig.71.

Let us add consecutively the pairs of the given vectorial errors, remembering that when adding two vectorial errors, an elliptiwal error is always obtained.

Addition of vectorial errors R' and R'', lying in the plane ZXZ_1X_1 will give us an elliptical error ε'. Addition of vectorial errors R'' and R''', lying in the plane XYX_1Y_1, gives an elliptical error ε''. Addition of vectorial errors R' and R''', lying in the plane ZYZ_1Y_1,
215. gives an elliptical error ε'''. Here the planes of all the three elliptical errors will be mutually perpendicular.

When given vectorial errors (systems) act simultaneously, three separate error-vectors are always obtained, these error-vectors when added giving a total error-vector.

If we look along the line X_1X, then the individual error-vectors with respect to the point O can have the following directions: in the system R' - above and below; in the system R'' - forward and back; and, finally, in the system R''' - to the right and to the left. The total error-vector can occur as a result of any combination of separate error-vectors, and hence it can occupy any position in space (in volume), limited by some surface.

We have established, that the cross-sections of this volume made by three mutually perpendicular planes represent ellipses. It is obvious, that also any other cross-sections of this volume will give ellipses, as these cross-sections may be regarded as projections of ellipses. This means, that such a volume represents an ellipsoid.

When adding three vectorial errors, which do not lie in the same plane, an ellipsoidal error is obtained.

An ellipsoidal error is denoted by the letter Э *.

Analagously to the elliptical error, the ellipsoidal error is characterised by a primary ellipsoid, the main half-axes of which are equal to the vectorial errors which form it (in the condition that these vectorial errors are mutually perpendicular).

If, along the directions of the corresponding axes of a system of coordinates we mark off 4 vectorial errors and we draw an ellipsoid, we will obtain an ellipsoid $Э_{max}$, the surface of which can be practically considered as the limit of the largest of all the possible error-vectors in a given system.

The projection of $Э_{max}$ on to any plane gives an ellipse, in which the projections of individual error-vectors are disposed according to the law of the elliptical error, as has been shown in fig.66.**

44. Problems on errors in a plane and in space.

PROBLEM 110. In a complete preparation of the original data for firing, correction in respect of wind was not taken into account, the wind causing a diminution of the distance of the flight of the shell and the deviation of the latter to the left. Determine the value and the direction of the admitted error, if it is known that the correction for the longitudinal component of the wind, is 80m, and for the side component, 26m.

Answer. 84m; the angle with the plane of fire is 18^0.

PROBLEM 111. In carrying out a command, the gunner made an error by setting of the goniometer on 7-35 instead of 7-55, and the sight on
216. 96 instead of 93. Determine the value and the direction of the deviation of the shell, which will be caused by the gunner's errors ($\Delta X = 50$m).

Answer. 178m; angle with plane of fire is $32^041'$.

* Russian alphabet.

** The ellipsoidal error will be discussed in greater detail in section 50 when examining the dispersion.

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216. PROBLEM 112. During firing, measurements are made of deviations in distance of the bursts, away from the target. The median error of the instrument is 30m and the middle deviation in distance Bδ = 25m. Find the median error of the determination of the position of the mean trajectory with respect to the target, in distance, if the deviation of individual bursts away from the target, is taken as the deviation of the mean trajectory.

Answer. 39m.

PROBLEM 113. Fire for adjustment is made from the measured deviations. The median error in determination of deviation in distance, is 20m; middle deviation in distance, Bδ = 30m. What is the median error in the correction of the distance from a group of four shots?

Answer. 18m.

PROBLEM 114. The median errors in distance when considering the meteorological conditions of fire when all the data is used (is prepared), are:

for longitudinal component of wind. 20m;
for barometrical pressure 8m;
for air temperature . . . . . 15m.

What is the median error of the correction in distance for the deviation due to meterorological conditions of firing?

Answer. 26.25m.

PROBLEM 115. In determination of the calculated distance, the median errors are as follows:

- when meterorological conditions of firing are considered - 0.6% of distance;
- when ballistic conditions of firing are considered - 0.4% of distance;
- when determining the topographical distance - 0.7% of distance.

What is the probability that the calculated distance will have an error not exceeding 2% of distance?

Answer. R ≈ 1% of distance; P = 0.82266.

PROBLEM 116. Fire for adjustment is made according to measured deviations. The median error in determination of the deviation in distance, is 25m; the middle deviation in distance Bδ = 36m. 6 shots are fired. The mean point is marked off on the drawing board with the median error of 9m. What is the probability that the correction of the distance will be made with an error not exceeding 25m?

Answer. R ≈ 20m; P = 0.60083.

PROBLEM 117. Indication of the target is sent from the right-hand observation point to the left-hand observation point. The vectorial error in determination of the target position in distance, made by the sender of the target indication, is 80m. What deviation will be caused by this error, from the observation line of the recipient, if the angle of the intersection of the target, is 2-00?

Answer. 16.64m.

PROBLEM 118. The median error in direction, in determining the target position from the division commander's point, is 20m. What deviation will be caused by this error from the observation line of the battery commander, when receiving the target indication, if the angle, contained by the observation lines of the division commander and the battery commander, is 4-00?

Answer. 18.28m.

PROBLEM 119. The elliptical error given by the main half-axes: a = 30m and b = 12m. What are the deviations caused by this elliptical error, from a direction AN and in direction AN, if this direction makes an angle of 36^0 with the direction of the half-axis a?

Answer. D' = 25.2m; D'' = 20.1m.

PROBLEM 120. The median errors in determination of the target position from the observation point of the division commander, are: in distance 150m and in direction 50m. Determine the deviation, caused by this elliptical error, from the observation line of the battery commander in receiving the target indication, if the angle between the observation lines of the division commander and the battery commander, is 1-50.

Answer. 54.7m.

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DISPERSION IN FIRING.

Chapter 9.

DISPERSION OF TRAJECTORIES AND POINTS OF BURSTS.

45. Phenomenon of dispersion of trajectories and its causes.

If shells of a given calibre and type were to have the same weight and shape, if the combat charges for them were completely identical in weight and ballistic properties of gunpowder, if the sighting of the gun were carried out in completely identical manner and the firing were made under unchangeable meteorological conditions, then in firing on the same sights, all the shells would describe the same trajectory and the point of arrival of all the shells would be the same.

In actuality, as a result of the presence of causes about which will be mentioned later, it is not possible to obtain a complete similarity when carrying out separate shots. Therefore each shell will have its own trajectory and its own point of arrival, regardless of the fact that the shell, the charge and the settings of the sight equipments, when conducting the firing, are not changing. Such a phenomenon is termed dispersion of shells, and more accurately - dispersion of trajectories.

All the causes, which produce dispersion of trajectories, can be reduced to the following three groups:

A. Causes, which give rise to a difference in initial velocities.

B. Causes, which give rise to a difference in the position of the bore axis of the gun barrel, at the instant of fire - a difference in the angles of departure, and positions of the plane of firing.

C. Causes, which give rise to a difference in conditions of flight of the shell after leaving the gun.

Let us examine individually each one of these groups, noting the more substantial causes.

218. A. Causes, which give rise to variation in initial velocities.

1. Variation in weights of combat charges, being the result of errors (inaccuracies), admitted in weighing of combat charges. Difference in weights of combat charges leads to a difference in initial velocities - a heavier combat charge will impart a higher initial velocity to the shell.

2. Variation in ballistic properties of powder of combat charges, due to the nature of the smokeless gunpowders themselves. Despite all care in preparation, sorting and storage of the smokeless gunpowders, the ballistic properties of the combat charges, prepared from a powder of the same kind and even of the same lot, will be different. This difference reflects on the process of combustion of the combat charge in the bore of the weapon and produces a difference in initial velocities.

3. Variation in temperatures of combat charges; conditions of supply of combat charges to the firing positions are different (type of transport, packing, weather conditions, etc); the conditions of storage of combat charges at the firing positions are different (in package or without, in housings, magazines or in open air, protected from the sun or otherwise, etc); finally, differences exist between the time intervals for separate shots, from the instant of loading to the instant of firing; consequently, the effect on the combat charges due to the heating of the barrel, which with intensive firing may become heated to a temperature measurable in tens and even hundreds of degrees, will be different. All this causes the combat charges to have a different temperature at the instant of firing, and the combat charge which has, at the instant of ignition, a higher temperature, will impart to the shell a higher initial velocity.

4. Variation in the compression of the charge. The fundamental

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218. [illegible] of the leading collar in different shells. As a result of these causes the capacity of the charge chamber and hence the compression of the charge* will vary with every shot. A change in the compression of the charge affects the initial velocity - with greater tightness of charging, the combustion of the combat charge is more intense and the initial velocity will be greater.

5. Variation in the weights of shells as a result of errors (inaccuracies, tolerances) in manufacture of shells and fuzes. A shell having a greater weight will have a smaller initial velocity.

219. A change in initial velocity is accompanied by a change in the flight distance of a shell - to a higher initial velocity corresponds a greater flight distance of the shell.

In the Rules for firing by land artillery, 1945 (appendix 4) is given a table for approximate reckoning of conditions of firing. This table shows, that a change in initial velocity of 1% causes a change in the distance of flight from 1 to 2%.

For example, in firing from a 152mm howitzer (model 1937), with the third loading and on a range of 7000m, a change in the initial velocity of 1%, changes the distance by 91m, which corresponds to 1.3% of distance.

Thus, even negligible changes in the initial velocity lead to large deviations of shells, and as these changes attain, in individual cases, 1 - 1.5% the variation in initial velocities is therefore the basic cause of dispersion of trajectories in distance.

B. Causes, which give rise to variation in angles of departure and in positions of the plane of fire.

1. Variation in the settings of the goniometer, levelling and sight due to different positions of the indicators with respect to the divisions on the corresponding scales of the sighting equipments, for different shots.

2. Variation in laying of the gun in horizontal and vertical planes as a result of inaccurate and different coincidences of the cross on the panorama with the bead of the sight (laying), different positioning of the bubble levels against the centre, as well as due to the back-lash of the mechanisms.

3. Variation in angles of departure and lateral displacements of the gun in firing, which is fundamentally determined by the design peculiarities of the gun itself.

This group of causes leads to an increase in dispersion in direction and in distance (in height).

Variation in angles of departure is more pronounced with small angles of departure and especially with high initial velocities. With angles of departure, close to 45°, the influence of the variation in the angles of departure becomes zero.

C. Causes, which give rise to variation in conditions of the flight of the shell.

1. Variation in atmospheric conditions. As the atmosphere is in a constant state of flux, the atmospheric conditions of flight for different shells, are different. This in itself leads to an increase in dispersion in direction and in distance. Changes in direction and the speed of the wind, which can take place at short intervals of time, produce an especially strong effect.

220. 2. Variations in ballistic coeficients. By the variation in ballistic coefficients we understand to be the variations in weight, shape and dimensions of different shells, and the positions of their centre of gravity, as well as the different dimensions and positioning of their leading collar. All these variations produce changes in the force due to the wind resistance, and as a result, in the distance of flight (range) of the shell.

* By the compression of the charge is meant the ratio of the weight of the charge to the capacity of the charge chamber.

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220. 3. Variation in greasing of the outer surface of the shells which likewise produces a change in the wind resistance.

From the foregoing, it follows, that dispersion of trajectories is inevitable.

However, if the dispersion of trajectories in firing is inevitable, nonetheless it is possible and essential to take steps to reduce the dispersion, as the smaller the dispersion of trajectories, the smaller is the required number of shells for the performance of the firing task.

It is possible to reduce the dispersion, in the first place, by raising the standard of manufacturing processes (reduction in tolerances, care in machining, accuracy in all measurements etc), and, in the second place, by means of corresponding measures in the process of use of the store, by the artillery units and sub-divisions themselves.

Measures taken for the reduction of dispersion when using the store:

1. Selection of combat charges of one and the same prepared consignment and their storage in conditions ensuring uniform charge temperature.

2. Selection of shells in batches and in weight marks, thorough cleaning of shells of excess grease, and mainly - of adhering particles of earth, sand etc.

3. Similar tightening extent of shells in charging.

4. Similarity and care in setting of the goniometer, levelling, and sight; similarity and care in laying.

5. Reduction in effect of back-lash in mechanisms by always turning the flywheels in the same direction*.

6. Setting up of the gun platform and the arrangement of it so that it eliminates jumping of the gun and, especially, the lateral displacement of the gun in the process of firing.

7. Selection of a sighting point which is sufficiently far removed, clearly visible and stable.

221. 46. Sheaf of trajectories.

Summation of all trajectories which obtain in firing from the same gun, in given conditions, with a sufficiently large number of shots, fired within a time interval of short duration, is termed a sheaf of trajectories.

All the causes, giving rise to dispersion of trajectories, have been resolved into three groups. Let us see, how each of these groups of causes affects the position of individual trajectories in the sheaf of trajectories.

Let us commence with the group of causes which give rise to variation of angles of departure, assuming that other causes, which give rise to dispersion of trajectories, are absent (fig.72).

Fig.72.

If the firing is conducted with small angles of departure, it can be considered that with small changes in the angle of departure the trajectories will be deviating up or down, retaining their shape, i.e. a bunch of trajectories will obtain, which will have the same curvature and which diverge at the point of departure O. We will assume that the trajectory 1 corresponds to the case when all the causes, giving rise to dispersion of trajectories, are absent.

If we assumed that only the group of causes which gives rise to variation in initial velocities is active, we would have obtained a bunch of trajectories originating from the point of departure, but all these trajectories would have progressed from the common tangent at the point of departure and would have had different curvatures (fig.73).

* In superimposing the panorama cross and the sighting point; positioning the sight and the gun pointers to coincide; in adjusting the level bubbles, etc.

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221. Approved For Release 1999/09/08 : CIA-RDP78-04861A000100010001-1 trajectory 1 corresponds to the case when causes, giving rise to dispersion of trajectories, are completely absent.

Fig.73.

Let us combine the actions of these two groups of causes. By superimposing fig.72 on fig.73 so, that the trajectories 1 of both diagrams coincide (fig.74), we will obtain a bunch of mutually crossing
222. trajectories, originating at the point of departure.

We would have obtained more cross-over points if we had taken into consideration the effect of the third group of causes - causes giving rise to variation in conditions of flight of the shell.

Fig.74.

In some cases even one and the same cause may produce intersection of trajectories. In fig.75 are shown trajectories of two shells of different weight. A lighter shell would depart at a higher initial velocity, and therefore its trajectory 1, at some distance from the point of departure, will proceed above the trajectory 2 of the heavier shell. But as a heavier shell has a higher transverse loading*, then, at some instant, it will have a greater flight velocity and will proceed further, and its trajectory will cross the trajectory of the lighter shell (in fig.75 - at point K).

Fig.75.

With a simultaneous action of several causes, there can be interlacing of trajectories as well as intersections. Let us take two shells, released at different angles of departure (fig.76): the heavier shell at

Fig.76.

a greater angle of departure and the lighter - at a smaller angle. At
223. first, the lighter shell will fly below the heavier one, but as the lighter shell has a higher initial velocity, it will, so to say, overtake the heavier shell and will cross its trajectory 2 at some point K_1. Further, the heavier transverse loading of the heavier shell will tell, and a second point of intersection of the trajectory K_2 will occur.

We will obtain the same intersections and interlacings of trajectories, if we examine the effect of causes giving rise to lateral deviations (lateral dispersion) of shells. Thus, the variation in directions of the flight of shells will produce a bunch of trajectories, which separate from the point of departure, this bunch being similar to the one shown in fig.72, but only in the horizontal plane, and the difference in the conditions of flight of shells will produce a bunch of trajectories, proceeding from the common tangent. Superimposition of these two bunches will produce mutual cross-overs and interlacings of the trajectories.

Thus, the sheaf of trajectories represents a bunch of intersecting and interlacing curves originating from a common point - the point of departure.

* By transverse loading is termed the ratio of the weight of the shell to its maximum cross-section. The force due to the wind resistance is inversely proportional to the transverse loading of the shell. A shell, having a greater transverse loading retains its speed in flight, to a greater extent.

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223. Overlapping of trajectories, shown in figs.72-76, is illustrated, for better representation, in an exagerated form. In order to show the mutual intersections and interlacings of the trajectories, the deviations of the ends of trajectories are drawn to a considerably bigger scale in comparison with the distances of the flight of the shells.

Actually, these deviations in comparison with distance are so small, that with the sheaf of trajectories drawn to the same scale, even as large as 1:500, it is not possible to show these intersections and interlacings. Therefore, when considering a small part of the trajectory curvature, intercepts of trajectories over small sections of the sheaf of trajectories are taken as a series of parallel straight lines.

An imaginary trajectory, passing through the middle of the sheaf of trajectories, is termed the mean trajectory (fig.77).

Fig.77.
(1) - Mean trajectory.

The mean trajectory is an imaginary trajectory which would obtain in the absence of causes, giving rise to dispersion of trajectories (trajectory 1 in the illustrations given above). The mean trajectory determines the position of the sheaf of trajectories, in space. The
224. point of impact, corresponding to the mean trajectory, is termed the centre of dispersion and denoted C_y.

47. Law of dispersion of trajectories.

When all the causes of dispersion are absent, one and the same trajectory will correspond to the shells released in identical conditions. But the presence of at least one of these causes leads to deviations of points of impact of individual shells, from the centre of dispersion; moreover, these deviations have a chance character.

Let us assume, that in the given conditions of firing, there is only one cause which gives rise to the dispersion of trajectories - variation of the weights of combat charges.

As a result of errors admitted in weighing, the combat charges will differ in their weights. The combat charge of exact weight will correspond to the centre of dispersion. Discrepancies of different value and sign, in the weights of individual combat charges, will lead to deviations, different in value and sign, of impact points of individual shells from the centre of dispersion. And as discrepancies in weights of combat charges, being the result of chance errors of measurement, are of chance character, therefore the deviations of individual impact points will likewise be of a chance character.

Examining the causes of dispersion we can note that by far the greatest number of these causes is associated in one or other degree, with chance errors of various types of measurements. There are also causes of dispersion the origin of which has no connection with the errors of measurements.

But also in this case it may be considered, that these causes are associated with deviations of a chance order (variation in temperatures of combat charges, variation in tightening of shells in charging, difference in condition of outer surface of shells etc).

If the chance deviations, produced by the action of separate causes of dispersion, do not follow the Gauss law, then the deviations of the points of impact, being the result of the mutual action of all the causes of dispersion, can be taken, without undue discrepancies, as deviations which follow the Gauss law or a law very close to this law. This is quite obvious from the properties of the Gauss law (see page 156).

The law of dispersion of trajectories (of points of impact) is a Gauss law. Experimental data confirm this deduction.

The law of dispersion can be graphically expressed by the Gauss curve, which shows the relationship between the values of deviations of impact points of shells from the centre of dispersion, and the probabilities of their occurrence (fig.78).

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225. Thus, for example, the probability of deviation within the limits from a to b, is expressed by the area abcd; the probability of occurrence of a deviation within the limits from k to l - by the area klmn etc.

Fig.78.

When applied to dispersion, the Gauss law can be stated thus:

1. Deviations of individual points of impact from the centre of dispersion, which are small in value, have a higher probability, than the large deviations - with the increase in the deviation, the probability of its occurrence decreases.

2. The deviations of individual points of impact from the centre of dispersion, which come within the limits equal in absolute magnitude, but different in sign, are equally probable.

3. Deviations of individual points of impact from the centre of dispersion, which exceed some quite definite value in given conditions, have such a small probability that they can quite reasonably be ignored.

Briefly, dispersion is non-uniform, symmetrical and finite.

These statements of the law of dispersion are dealt with in detail in volume 1 of the Course on artillery.

Let us turn to the consideration of the dispersion of trajectories.

Let us imagine a horizontal platform and a vertical screen positioned on it and let us assume, that through the centres of the screen and the platform passes the mean trajectory, which has a corresponding centre of dispersion Cy (fig.79).

Fig.79.

226. Let us draw through the centre of the screen a vertical (perpendicular) line BB and a line perpendicular to it, VV.

If two chance trajectories 1 and 2 pass through this screen, then two holes will be made in the screen (two points of contact) C_1 and C_2. The positions of these holes with respect to the centre of dispersion will be determined by the corresponding deviations: from the line VV by deviations in height, and from the line BB - by lateral deviations. Therefore in the vertical plane, the line VV is taken as the axis of dispersion in height, and the line BB - as the axis of lateral dispersion.

Let us now draw through the centre of the horizontal platform, two mutually perpendicular lines: B'B' and DD, and at the same time let us arrange the line B'B' so that it lies in the same vertical plane as the line BB on the vertical screen. Then the positions of the chance points of impact (craters) C_1 and C_2, with respect to the centre of the trajectory, can be determined: in distance - by the deviation from the line DD, and in direction - by the deviation from the line B'B', i.e. in this case the line DD is taken as the axis of dispersion in distance in the horizontal plane, and the line B'B' - as the axis of lateral dispersion in the same plane.

Thus, the dispersion of the points falling on the vertical plane is made up of the dispersion of these points in height and in their lateral dispersion, and the dispersion of the points falling on the horizontal plane is made up of the dispersion of these points in distance and in their lateral dispersion. Taking the ends of trajectories as being parallel, it is not difficult to establish (see fig.79), that for the same trajectories the lateral deviations in the vertical plane are equal, in magnitude and sign, to the lateral deviations in the horizontal plane. This means, that the lateral dispersion in the vertical plane is equal to the lateral dispersion in the horizontal plane.

Examining the dispersion of individual trajectories in one direction only, and taking into consideration that the dispersion in distance (in height) does not depend on the lateral dispersion, it can be reckoned that this dispersion follows the Gauss law in the same way as does the

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226. linear errors. Hence [illegible] measure of dispersion, as a measure of dispersion in a given (one) direction, we take the middle deviation, i.e. such a deviation which in absolute magnitude is greater than each of the deviations of one half of all the deviations, and less than each of the deviations of the other half. Consequently, the middle deviation is determined by, that the probability of occurrence of the deviation, which in absolute value is greater or less than the middle deviation, is equal to a half (0.5). Deviations, which in absolute magnitude are less than the middle deviation, are termed as the best half of the deviations; deviations, which in absolute magnitude are more than the middle deviation, are termed as the worst half of the deviations.

227. Quite frequently, instead of the term <u>middle</u> deviations, a term, <u>probable</u> deviation is used.

Because the dispersion of trajectories can be considered in three directions, the following designations for the middle deviations are used:

Bд* - middle deviation in distance,
Bб* - middle deviation in height, and
Bз* - middle lateral deviation.

Numerically, the law of dispersion in one direction is expressed by a <u>scale of dispersion</u>, which basically does not differ from the scale of errors. From the scale of dispersion it is possible to solve the same problems as from the scale of errors, but only as applied to the deviations of trajectories. In appearance, the scale of dispersion differs from the scale of errors by that on the scale of dispersion the mean trajectory is always drawn. In fig.80 is shown a scale of dispersion in distance with Bд = 25m.

Fig.80.

From this scale, it is possible, for example, to determine the probability of occurrence of a deviation within the limits from +25 to +50m. This probability is equal to 0.16. More will be said in the following chapter regarding the use of the scale of dispersion.

Assuming that the end of a trajectory over a short length can be taken as a straight line, we can easily establish the relationship between the middle deviations in distance and the middle deviations in height (fig.81).

Fig.81.

From the right-angle triangle ACB, we have:

$$B\text{б} = B\text{д} \cdot \tan \theta_c,$$

or

$$B\text{д} = \frac{B\text{б}}{\tan \theta_c} = B\text{б} \cdot \cot \theta_c.$$

228. This relationship allows, from the angle of impact and the value of one of these middle deviations, to determine the value of another middle deviation.

EXAMPLE. From the experimental data it is established, that Bб = 8m. Determine the value of Bд, if $\angle\theta_c = 22^\circ$.

<u>Solution</u>. We find the value of $\tan \theta_c$. From the table of natural logarithmic values $\tan 22^\circ = 0.404$. Then

$$B\text{д} = \frac{B\text{б}}{\tan \theta_c} = \frac{8}{0.404} \approx 19.8\text{m}.$$

In fig.81 it is seen, that between the change of trajectory in distance (ΔX) and the change of trajectory in height (ΔY) when the sight

* These are letters from the Russian alphabet, the smaller letters corresponding to "d" and "z" and "lateral" deviations respectively.

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228. [illegible] as between Bδ and Bϕ. From the right-angle triangle DCF, we have:

$$\frac{\Delta Y}{\Delta X} = \tan \theta_c,$$

whence

$$\frac{B\phi}{B\delta} = \frac{\Delta Y}{\Delta X}$$

Within the region of arrival of shells, the inclination of the ground in the direction of the firing gun and in the opposite direction, affects the value of dispersion.

As the lateral dispersion in the vertical and horizontal planes is the same, the lateral dispersion does not depend on the inclination of the ground in the direction of the firing gun or in the opposite direction. It is also obvious that the dispersion in height does not depend on the inclination of the ground, if the target plane occupies a strictly vertical position.

As regards the dispersion in distance: when firing onto slopes facing the direction of the firing gun, the dispersion in distance decreases, but when firing onto slopes facing away, this dispersion increases.

Fig.82.

In fig.82 is shown the influence of the inclination of the ground on the value Bδ. Thus, if the ground in the region of the arrival of shells is inclined towards the firing gun (AA), then the middle deviation
229. in distance $B\delta_1$ will be less than the table value Bδ, i.e. given in the Tables for firing in horizontal terrain. When the inclination of the ground is in the reverse direction (BB), the middle deviation in distance $B\delta_2$ will be less than that in the table.

Let us find the relationship between the value of the middle deviation in distance and the value of the angle of inclination of the ground.

Let us assume, that the terrain has an inclination in the direction of the firing gun and that the angle of this inclination is α (fig.83).

Fig.83.

Dropping a perpendicular from the point C, on to the continuation of the line DB, we obtain two right-angle triangles: CAB and CAD.

From these triangles, we have:

$$CA = B\delta_1 \cdot \sin(\theta_c + \alpha) \quad \text{and} \quad CA = B\delta \cdot \sin \theta_c,$$

because the angles CDB and θ_c are equal, being alternately opposite angles, and the angle CBA is equal to the sum of the angles $(\theta_c + \alpha)$, being the external angle (complementary) to the triangle CDB. Consequently,

$$B\delta_1 \cdot \sin(\theta_c + \alpha) = B\delta \cdot \sin \theta_c,$$

whence

$$B\delta_1 = \frac{B\delta \cdot \sin \theta_c}{\sin(\theta_c + \alpha)}.$$

Let us now suppose, that the ground has an inclination in the opposite direction - away from the gun - and that the angle of inclination is equal to α (fig.84).

Fig.84.

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230. Dropping a perpendicular from the point C on to the continuation BD, we obtain two right-angle triangles CAB and CAD, from which we have:

$$CA = B\partial_2 \cdot \sin ABC \quad \text{and} \quad CA = B\partial \cdot \sin \theta_c.$$

We will find what is the value of the angle ABC:

$$\angle ABC = 90^\circ - [\alpha + \angle ACD] = 90^\circ - [\alpha + (90^\circ - \theta_c)] = 90^\circ - \alpha - 90^\circ + \theta_c = \theta_c - \alpha .$$

Then

$$B\partial_2 \cdot \sin (\theta_c - \alpha) = B\partial \cdot \sin \theta_c;$$

$$B\partial_2 = \frac{B\partial \cdot \sin \theta_c}{\sin (\theta_c - \alpha)} .$$

EXAMPLE 1. The target is situated on the near slope, the angle of inclination being 3°. Determine the value of the middle deviation in distance $B\partial_1$, if $B\partial = 24$m and $\theta_c = 31^\circ$.

Solution. Substituting the numerical values of $B\partial$, α and θ_c in the general formula, we obtain

$$B\partial_1 = \frac{B\partial \cdot \sin \theta_c}{\sin (\theta_c + \alpha)} = \frac{24 \cdot \sin 31^\circ}{\sin (31^\circ + 3^\circ)} = \frac{24 \times 0.515}{0.559} \approx 22\text{m}.$$

EXAMPLE 2. Firing is conducted on a target situated on an opposite gradient, the angle of inclination of which is 4°. What is the value of the middle deviation in distance $B\partial_2$, if $B\partial = 30$m and $\theta_c = 32^\circ$?

Solution.

$$B\partial_2 = \frac{B\partial \cdot \sin \theta_c}{\sin (\theta_c - \alpha)} = \frac{30 \cdot \sin 32^\circ}{\sin (32^\circ - 4^\circ)} = \frac{30 \times 0.530}{0.469} \approx 34\text{m}.$$

So far we have been examining the dispersion of trajectories in one direction only. In actuality, the dispersion takes place in a plane.

Fig.85.

231. As the dispersion of trajectories in one (given) direction follows the Gauss law, then, according to the general rule of addition of vectorial errors in a plane, the dispersion of trajectories in a plane, being the result of the joint action of the dispersion in two directions, must have the shape of an elliptical dispersion, i.e. it must give ellipses of dispersion of points of arrival (hits) in the horizontal and vertical planes (fig.85).

The distribution of points of arrival (hits) in the form of an elliptical dispersion, follows the law governing an elliptical error (see fig.66). The distribution of points of arrival (hits), calculated on the basis of 0.5 middle deviation, is given in the Appendix 2, "Grid of circular dispersion". The rules explaining the use of this grid, are given in the following chapter.

Dispersion of the trajectories (points of arrival) in a plane is characterised, as any elliptical error, by the primary ellipse of dispersion $\mathcal{E}$, distibuted inside the primary rectangle of dispersion, each side of which is equal to two middle deviations. The probability of a hit inside the primary ellipse of dispersion, is 0.203.

Apart from the primary ellipse of dispersion, we have to make use of the semi-ellipse of dispersion, the probability of a hit in which is 0.055.

Causes giving rise to dispersion of trajectories, and hence also the middle deviations, being quantities which characterise this dispersion, do not remain constant in all cases of firing from a given gun, with a shell of a definite type, with a given charge and on a specified range. On the contrary, the dispersion can vary from one firing to another due to changes in the conditions of the process of the firing, made at different times and in different circumstances, as well as in the

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231. For example a batch of combat charges during one firing possesses a large variety of ballistic properties of gunpowder as compared to a batch of combat charges during another firing. It is obvious, that the dispersion in distance during the first firing will be greater than during the second. Firing conducted on different days produces conditions of greater or lesser variation in temperatures of combat charges etc.

Dispersion may change also during one firing, if this firing is of long duration or is conducted at short intervals. Changes in the value of dispersion occur likewise with sharp variations in meteorological and ballistic factors.

Therefore the middle deviations at a given instant will differ from their tabular values, i.e. the values corresponding to some mean conditions of firing. A dispersion at a given instant is characterised by the middle deviations at a given instant.

232. By a middle deviation at a given instant, is termed such a middle deviation which will characterise the dispersion of trajectories, if in the given instant a large number of shots were to be made.

In order to show how the middle deviations at a given instant can vary in values, we will take numerical values of middle deviations, obtained during firing from a 76mm gun using a long range shell OF-350 (table 19).

Table 19.

Angle of departure.				Angle of departure.			
10°	20°	30°	40°	10°	20°	30°	40°
Bδ in m				Bδ in m			

From table 19 it can be seen, that when repeating the firing for every angle of departure, different values of middle deviations were obtained, the variations in these values being quite considerable.

Consequently, it is necessary to take into consideration the possibility of such variations in the values of middle deviations, when deriving the rules for firing and the standards for the expenditure of amunition to effect destruction of different targets.

48. Working out of results of firing.

If we omit to consider firing which is conducted for the purposes of scientific research and experiment, then the working out of the results of firing can be resolved to the determination of the position of the centre of dispersion and to the determination of the values of the middle deviations at a given instant.

A. Determination of the position of the centre of dispersion.

The true position of the centre of dispersion, from the analogy with the true value of any measured quantity, can only be found from the results of a very great number of shots. With a limited number of shots only an approximate position of the centre of dispersion can be determined, i.e. the adequate values of its coordinates.

With a limited number of shots, the centre of the grouping of points of arrival (hits), is taken as the centre of dispersion. i.e. the mean of all these points of arrival (hits), which were obtained
233. during a given firing. To such a mean point will correspond the approximate position of the mean trajectory.

The position of the centre of grouping can be determined by two methods: graphical or by calculation. Both these methods and the manner of their application are treated exhaustively in volume 1 of the Course on artillery.

It is only necessary to discuss the question on the accuracy of determination of the position of the centre of dispersion, if instead of this centre we take the centre of grouping.

The adequate values of the coordinates of the centre of dispersion are found as the mean results from the coordinates of individual points of arrival (hits). Consequently, the accuracy (precision) of determination of the position of the centre of dispersion in a given direction, is

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233. found from the formula for the median error of the mean result

$$R = \frac{E}{\sqrt{S}},$$

in which the median error of the method of measurement E will be substituded by the corresponding middle deviation:

$$R_\beta = \frac{B\beta}{\sqrt{S}}; \qquad R_\sigma = \frac{B\sigma}{\sqrt{S}} \qquad \text{and} \qquad R_\delta = \frac{B\delta}{\sqrt{S}}.$$

EXAMPLE 1. 16 shots were fired on the same sight settings. From the coordinates of the craters of these 16 shots, the coordinates of the centre of grouping were found. What is the median error in determination of the centre of dispersion in distance, if we know that $B\beta = 20$m?

Solution. The median error in determination of the position of the centre of dispersion being the median error of the mean result

$$R_\beta = \frac{B\beta}{\sqrt{S}} = \frac{20}{\sqrt{16}} = 5\text{m}.$$

EXAMPLE 2. The position of the centre of grouping is determined from the results of 12 shots. What is the probability that, in taking the centre of grouping as the centre of dispersion, we admit an error in distance not exceeding 10m, and an error in direction not exceeding 2m, if $B\beta = 25$m and $B\sigma = 4$m?

Solution. The median errors in determination of the position of the centre of dispersion:
in distance

$$R_\beta = \frac{25}{\sqrt{12}} \approx 7.22\text{m};$$

in direction

$$R_\sigma = \frac{4}{\sqrt{12}} \approx 1.15\text{m}.$$

Then, the required probabilities from the table of values $\Phi(\beta)$:

$$P_\beta = \Phi\left(\frac{10}{7.22}\right) = \Phi(1.385) = 0.64978 \approx 0.65;$$

$$P_\sigma = \Phi\left(\frac{2}{1.15}\right) = \Phi(1.739) = 0.75918 \approx 0.76.$$

234. B. Determination of the adequate values of middle deviations.

The adequate values of the middle deviations can be determined likewise by two methods: graphical and by calculation.

The graphical method is quite fully outlined in volume 1 of the Course on artillery.

The method of calculation for the determination of the adequate values of the middle deviations is the same as when determining the adequate values of the median errors, i.e. by the position in the series of absolute values of deviations, by the mean arithmetical deviation, by the mean quadratic deviation and by the differences between adjacent deviations. In the working out of the results of firing, the last two methods are used. Let us examine these, by means of the following examples.

EXAMPLE 1. 20 shots were fired on to a vertical screen, on the same sight settings. All the shells hit the screen. In measuring, the following deviations of individual shell holes with respect to the centre of the screen, were found (in metres).

In height:
-1.6,+0.6,+2.4,+1.2,-0.6,-1.6,+1.0,+2.7,-0.3,-1.7,+1.4,
+2.4,+0.6,-1.6,+1.9,-1.3,+2.3,+0.8,+1.4 and +2.0.

In direction:
-1.0,-2.3,+0.8,-1.0,-2.1,-1.5,-0.4,-1.0,-1.9,-1.6,-0.4,
+0.1,-2.0,-1.2,-1.3,-1.0,-2.5,-1.3,-2.7 and +0.3.

Determine the adequate values of $B\delta$, $B\sigma$ and $B\beta$, if it is known that the angle of arrival $\theta = 2^\circ 30'$.

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234. Solution. We will first determine the coordinates of the centre of grouping by taking, for this purpose, the algebraic sums of deviations for each direction separately and by dividing these sums by the number of deviations. We then find the deviations of individual shell holes from the centre of grouping as algebraic differences between coordinates of individual shell holes and the coordinates of the centre of grouping. And finally, we find the values $B\phi$ and $B\delta$ from the corresponding mean quadratic deviations. The value $B\vartheta$ will be determined from $B\phi$ and $\tan\theta_c$. We will enter all the calculations in a table.

No. in sequence	In height (Y)			In direction (Z)		
	Deviation from the centre of the screen Y_i	Deviation from the centre of grouping λ_{Y_i}	$\lambda^2_{Y_i}$	Deviation from the centre of the screen Z_i	Deviation from the centre of grouping λ_{Z_i}	$\lambda^2_{Z_i}$
235.						

The coordinates of the centre of grouping:
in height

$$\xi_\phi = +\frac{12.0}{20} = +0.6\text{m};$$

in direction

$$\xi_\delta = \frac{24.0}{20} = -1.2\text{m}.$$

Therefore the required middle deviations:

$$B\phi = 0.67449\sqrt{\frac{44.94}{19}} \approx 1.027\text{m};$$

$$B\delta = 0.67449\sqrt{\frac{16.74}{19}} \approx 0.594\text{m};$$

$$B\vartheta = \frac{1.027}{\tan 2^\circ 30'} \approx 23.27\text{m}.$$

EXAMPLE 2. In firing on a datum, 16 shells were released on the same sight settings. The ground in the region of the position of the datum is level and without incline. By measuring the deviations of individual craters with respect to the datum, the following values were obtained in metres:

In distance:

-41, -64, +7, -31, -12, -53, -68, -40, -69, -60, -50, -91, -36, -63, -35 and -92.

236. In direction:

+5, +10, +6, +10, +9, +6, +6, +8, +6, +5, +8, +5, +9, +8, +5 and +5.

Determine the adequate values of $B\vartheta$, $B\delta$ and $B\phi$, using the method of differences between the adjacent deviations, if the angle of arrival $\theta_c = 12^\circ$.

Solution. We find the algebraic differences between two adjacent deviations, and then from the sum of the squares of these differences we determine the adequate values of the corresponding middle deviations.

No. in sequence	In distance (Y)	In direction (Z)

$$B\vartheta = 0.47694\sqrt{\frac{20555}{14}} \approx 18.25\text{m};$$

$$B\phi = B\vartheta \tan\theta_c = 18.28 \times 0.213 \approx 3.89\text{m};$$

$$B\delta = 0.47694\sqrt{\frac{120}{14}} \approx 1.4\text{m}.$$

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236. During process of manufacture of guns, a series of errors, inaccuracies, discrepancies etc are unavoidably admitted. Therefore, even the new guns of the same calibre, type and model will be varying, to a degree, in their ballistic properties. It is not possible to verify with absolute accuracy, the sighting gear of individual guns. Finally, uneven loading of the guns during firing leads to uneven wear
237. of the bores of the barrels. Hence, to the causes of dispersion, discussed above (see section 45) when conducting firing from batteries, one more cause is added - the variation of guns.

The variation in firing performance (firing power) of battery guns consists in that in firing on the same sight settings, the centres of grouping of individual guns do not coincide. By this it is understood that the variation in the firing performance of a gun is only the variation of this performance in respect of distance.*

By the variation in firing performance of guns is meant the difference in distances of the centres of grouping when firing from different battery guns with the same angle of elevation.

In theoretical basis of firing, we commence from the point that the variation in firing performance of guns is absent. The variation in the firing performance of guns when firing from a battery is an inevitable phenomenon. It is therefore necessary to establish: in the first place, to what extent it is practicable to remove this variation and, in the second place, what variation in firing performance should be considered as admissible.

Let us commence with the possibility of elimination of this variation in firing.

The fundamental cause of the variation in the firing performance is the uneven wear of the bores of the gun barrels, which leads to the variation in initial velocities in different battery guns when firing on the same settings.

Determination of the variation in the firing performance of guns is made by two methods: by means of measurement of the length of the breech chambers of guns and by firing onto a datum. Both these methods are fully described in the Rules for firing with land artillery, 1945 (Appendices 2 and 3).

Let us examine the method of firing off a number of rounds coupled with observation, and let us determine the magnitude of the error which is obtained in this case when determining the variation of the firing performance.

The major half-axis of the elliptical error of the intersection (fix) a is found from the formula**

$$a = \frac{\delta D}{1414 \sin \frac{\gamma}{2}}.$$

Let us assume, that the median error of the apparatus $\delta = 1.5$ of goniometer divisions, the observation distance $D = 3000$m, the angle of intersection $\gamma = 3\text{-}00 = 18^{\circ}$ and $B\partial = 25$m. Then, from the above formula

$$a = \frac{1.5 \times 3000}{1414 \sin 9^{\circ}} \approx 20\text{m};$$

238. the median error in distance of the intersetion of one burst (taking into consideration the dispersion) is, according to the rule of addition of vectorial (median) errors

* In the firing of batteries, there is also a variation in firing performance of guns in direction. But this variation in the firing performance, being fundamentally due to errors which are unavoidably admitted in the construction of a parallel sheaf, can not be accounted for beforehand (before firing), and is eliminated in the process of every firing by firing off a sheaf of a required width.

** See volume 10 of the Course on artillery, section 3.

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238. $R_1 = \sqrt{a^2+b^2} = \sqrt{20^2+29^2} \approx 32m.$

According to the Rules for firing, when carrying out a test shoot of guns (firing off a number of rounds) by a given method, in order to determine the position of the centre of grouping of one gun, it is necessary to make 6 - 8 shots from this gun. This means, that the accuracy of such a determination (with 8 shots) will be characterised by the median error

$$R = \frac{32}{\sqrt{8}} = 11.3m,$$

and then the median error which characterises the accuracy of determination of the variation in the firing performance of two guns,

$$R_p = \sqrt{11.3^2+11.3^2} = 16m \approx 0.64B\partial.$$

This calculation shows, that the determination of the value of the variation in the firing performance of guns using the method of a test shoot, is inevitably accompanied by an error, this error being comparitively large.

The correction for the variation in the firing performance of guns is measurable on the level indicator. In conversion of the corrections for the variation in the firing performance, expressed in metres (difference between the corrections of individual battery guns), into the correction expressed in divisions of the level indicator, an error is likewise inevitably admitted - an error of rounding and, finally, when introducing such a calculated correction in the setting of the level indicator, another error is added - the error of rounding of the setting of the level indicator itself. All this leads to even greater increase in the error when taking into consideration the variation in the firing performance of battery guns.

From this we can make the following deductions: in the first place, it is not possible entirely to eliminate the variation in the firing performance, when determining this variation by the method of a test shoot; in the second place, the value of this unavoidable (residual) variation in the firing performance is quite substantial.

We will arrive at the same deductions when we examine the method of determination of the variation in the firing performance by the measurement of the length of the breech chambers of guns. Here, there will be errors: when determining the length of the breech chambers; when determining from the elongation of the chambers, the deviations in the initial velocities of individual battery guns; when converting these deviations into corrections on the level indicator; and, finally, when introducing these corrections into the settings of the level indicator.

It has been established, that with careful determination, by one or the other method, the variation in the firing performance of guns and then making allowance for it in firing, the remaining (residual) variation in the firing performance does not exceed 1.5 - 2B∂. Such a residual variation in the firing performance of guns must be taken into consideration when firing from a battery.

Let us now turn to the question regarding the admissible variation in the firing performance of the guns during battery firing, i.e. such a variation in the firing performance, which, during battery fire, will not call for application of some special rules for firing as compared
239. to the rules worked out for one gun. Having obtained an answer to this question, we will establish, to what degree the residual variation in the firing performance of the guns will be manifest in battery firing.

Let us assume, that the variation in the firing performance of two guns is equal to 2B∂ and that B∂ = 20m. Let us see, what will be represented by the general scale of dispersion of these guns, i.e. the scale of battery dispersion.

If from each gun, 100 shots be fired on the same settings, and if we assume, that the points of arrival are disposed in each gun ellipse in accordance with the law of dispersion (see 1st and 2nd lines in fig. 85), then the disposition of the points of arrival in a battery salvo can

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239. easily be found by simple addition (see numerator in the 3rd line). In conversion to 100 shots, this disposition will be a half (see denominator in the 3rd line).

Fig.86.

The battery ellipse was found to be 10Bϑ, i.e. 200m. If we calculate the disposition of the points of arrival in the battery ellipse, for 8 middle deviations, then this disposition will differ very little from the disposition in the gun ellipse (see 4th line), and only the value of the middle deviation of the battery will increase to 25m.

The battery dispersion which obtains with such a variation in the firing performance of the guns, is demonstrated graphically in fig.87.

Fig.87.

Having calculated the battery dispersion for the case of a battery of 4 guns, and with the condition* that the variation in the firing performance of these guns is small,* we will obtain an almost similar result.

From this, we draw the conclusion, that with the variation in the firing performance of the guns, not exceeding 2Bϑ:

1) the law of dispersion in battery firing remains practically the same as in the case of firing from one gun;

240. 2) a change in the value Bϑ in comparison with the value of the variation in the firing performance, is insignificant;

3) the rules for firing, evolved for one gun, are applicable also for battery firing.

Therefore, if in a battery, the variation in the firing performance of the guns has been determined and is allowed for in firing, then the remaining (residual) variation in the firing performance will produce a very small effect on the battery dispersion.

If, however, in the battery firing, we do not allow for the variation in the firing performance of guns, then the battery dispersion will differ sharply from that of the gun and, which is most important, will cease to follow the Gauss law.

Let us assume, that the variation in the firing performance of two guns is equal to 4Bϑ. We will find for this case also, the disposition of the points of arrival in battery dispersion, by applying the same method as in the case of the variation in the firing performance of 2Bϑ (fig.88 and 89).

Fig.88.

Fig.89.

From figs.88 and 89 it is seen, that with the variation in the firing performance of guns of 4Bϑ, the disposition of the points of arrival of the battery dispersion does not follow the Gauss law, and the rules for firing, established for one gun, are not applicable in this case.

Finally, this means, that the firing from batteries according to the general rules is permissable only when allowances for the variation in the firing performance of the guns has been made. If, however, this variation of the battery guns is not allowed for and exceeds 2Bϑ, then,

* That is the variation [illegible]

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240. it is necessary to correct the firing data separately, when conducting a battery fire. This is called for in the Rules for firing with land artillery.

241. 50. Dispersion of the points of burst in time-fire.

By time-fire is termed the firing with shells capable of bursting in air at any desirable range (distance) from the gun, before coming into contact with obstruction, for example, with the ground surface.

Such shells are supplied with special time fuzes (tubes) the construction of which permits the setting of the time of the action of the shell, i.e. it permits to cause the shell to burst at a desired instant, at the desired point on the trajectory.

This burst can occur at any chance trajectory in the sheaf of trajectories. This means, that the dispersion of the points of bursts in time-fire is determined, in the first place, by the dispersion of trajectories themselves.

Besides this, due to the variation of the action in time, of the time fuzes, the burst may occur at different points of a given trajectory.

Thus, the dispersion of the points of bursts in time-fire is composed of: the dispersion of trajectories and the dispersion of the action of the time fuzes.

To the fundamental causes which give rise to dispersion of the action of time fuzes, or more accurately - which give rise to dispersion in time of the action of time fuzes - we can attribute the following:

- variation in the settings of fuzes, being the result of dissimilar register of the indicator against the divisions on the time ring of the fuze;

- variation in combustion of the time (of powder) compound being the result of dissimilar properties of this compound in different fuzes;

- variation in ballistic and meteorological conditions of firing which influence the combustion of the time compound with respect to time.

Let us assume, that the causes, which give rise to dispersion of points of bursts in time-fire, are completely absent. Then all the shells, released with identical settings of the goniometer, levels, sight and fuze, must burst at some one point, lying on the mean trajectory - in the centre of dispersion of bursts, denoted by $C_в$.

When these causes are present and act together, the deviations of individual points of bursts with respect to the mean point of bursts (the centre of dispersion of bursts) can have any direction in space.

The dispersion of trajectories obeys the Gauss law. With no special discrepancies present, we can consider that the dispersion of the time fuzes also follows this law. Therefore, we can take, that

242. the dispersion of the points of bursts in time-fire follows the Gauss law.

This means, that the combination of all the points of bursts, which can be obtained in time-fire with the same settings and with a large number of shots, must represent an ellipsoid - an ellipsoid of bursts. Consequently, the dispersion of the points of bursts in time-fire will be ellipsoidal.

The dispersion of the points of bursts in space during time-fire, from the analogy with the ellipsoidal error, is characterised by the primary ellipsoid of bursts.

The dispersion of the points of bursts in distance, in direction and in height is characterised by respective middle deviations:

$Bp_д$ - middle deviation of bursts in distance;

$Bp_б$ - middle lateral deviation of bursts;

$Bp_ц$ - middle deviation of bursts in height.

As the lateral dispersion of bursts in time-fire depends only on the dispersion of trajectories, then $Bp_б = B_б$.

If we project the ellipsoid of bursts on to three mutually perpendicular planes: on to the plane of fire, on to the horizontal plane and on to the vertical plane, we will obtain three projections - three ellipses of dispersion of bursts (fig. 90).

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242.

Fig.90.

It is obvious, that the middle deviations, characterising the dispersion of bursts in height and in distance, do not coincide with the corresponding main half-axes of the ellipsoid of bursts. This is a peculiarity of the middle deviations in time-fire. Hence, also in the primary ellipsoid of bursts, its main half-axes are not equal to the middle deviations accepted in practice - the deviations Bpф and Bpλ.

From the three projections of the ellipsoid of bursts, shown above, two projections are discussed in the theory of firing: the projection on the vertical plane, perpendicular to the plane of fire, and the projection on the plane of fire.

243. A. Ellipse of bursts in a vertical plane, perpendicular to the plane of fire.

The ellipse of bursts in a vertical plane, which is perpendicular to the plane of fire, gives a picture of the dispersion of the points of bursts, if viewed in the direction of fire.

The construction of such an ellipse of bursts is simple and is made in the following order:

- two mutually perpendicular lines VV and BB are drawn, the point of intersection of which is taken as the centre of dispersion, and the lines themselves - as the axes of dispersion;
- from this centre point, along the line VV, and to the right and left of this point, are marked off 4Bσ, and along the line BB - above and below, 4Bpф; the numerical values Bσ and Bpф are taken from the firing table;
- in the rectangle thus obtained, we draw the ellipse of bursts.

Fig.91.

In fig.91 is shown the ellipse of bursts obtained during firing with a high explosive fragmentation shell with fuzes Д-1 from a 122mm howitzer, model 1938 on second loading. The range of firing is 4000m; Bσ = 2.4m and Bpф = 7.4m. In order not to obscure the diagram, the disposition of the points of burst are given in height only.

In artillery calculations, such an ellipse is used to determine the position of the mean point of bursts with respect to the ground surface, from the relationships between the impacts* and aerial explosions. As the lateral dispersion is not allowed for here, the disposition of the bursts is drawn only with respect to height.

EXAMPLE 1. In the conditions given for the construction of the ellipse of bursts in fig.91, determine the most probable position of the mean point of bursts with respect to the ground surface, if with two battery salvoes, on the same settings, 2 impacts and 6 aerial bursts were obtained.

Solution. 25% impacts and 75% aerial bursts are obtained, which corresponds to the position of the mean point of bursts at a height of 1Bpф above the surface of the ground, or, in convertion to metres, 7.4m.

The question can be set in a different way: from the specified height of the mean point of bursts we require to find the probability of occurrence of an impact or an aerial burst, or any combination of the two.

EXAMPLE 2. Fire is conducted from a 122mm howitzer, model 1938, with a high explosive fragmentation shell with fuze Д-1 on the fourth loading. The range of firing is 2500m. What is the probability of occurrence of 2 impacts and 2 aerial bursts in one battery salvo, if the mean point of bursts with respect to the ground surface is located at a height of 12m?

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* Translator's note. Points of contact on ground on which the shell falls, without bursting in air.

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244. Solution. From the firing tables No.1 and 1a/1941, for the given conditions, $Bp_4 = 6m$.

We draw the disposition of bursts (fig.92). As can be seen, the probability of an impact is 0.09 and the probability of an aerial burst 0.91. Then the required probability

$$P = \frac{4x3x2x1}{2x1x2x1} 0.09^2 x 0.91^2 \approx 0.04.$$

Fig.92.

The determination, from the results of firing, of the position of the centre of grouping of the points of bursts in the vertical plane and of the adequate value of the middle deviation in height, is conducted to the same rules as for impact firing.

B. Ellipse of bursts in the plane of fire.

We will confine our discussion to the primary ellipse of bursts in the plane of fire.

It is necessary to note that the major half-axis of the primary ellipse of bursts does not coincide with the direction of the mean trajectory, and makes some angle with it (fig.93).

Fig.93.

In order to explain the causes for this absence of coincidence, let us consider the dispersion of the time bursts in vacuo.

The causes for the dispersion in time-fire in vacuo, are: the variation in angles of departure, the variation in initial velocities and the variation in the time of operation of the time fuzes.

245. The deviations (errors) of individual points of bursts from the mean point of bursts C_9, beimg the result of combined action of the causes enumerated above, is characterised by the following vectorial errors:

- by the vectorial error Bp_θ, which is produced by errors (variation) in the angle of departure; because, with an error only in the angle of departure the burst may occur above or below the line of fire (more correctly - above or below the line parallel to the line of fire and passing through the point C_9), therefore the vectorial error Bp_θ will be directed along the perpendicular to the line of fire, i.e. at an angle of $90^\circ+\theta_0$ to the horizontal, where θ_0 - the angle of departure;

- by the vectorial error Bp_v, produced by the errors (variation) in initial velocity; this vectorial error will be directed along the line of fire, i.e. at an angle of θ_0 to the horizontal; the error in initial velocity will give rise to a deviation of the burst with respect to the point C_9 along the line, parallel to the line of fire;

- by the vectorial error Bp_t, produced by the errors (variation) in the time of operation of the time fuze; with the change in the time of operation of the time fuze, the burst will be displaced only along the trajectory, and therefore the vectorial error will be directed along the tangent to the trajectory at the point C_9, i.e. at an angle of $180^\circ-\theta_0$ to the horizontal.

Fig.94.

As these vectorial errors lie in the same plane (the plane of the target) and have different directions, they will, when added, produce

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245. The direction of the major half-axis of the elliptical error (the ellipse of dispersion) will be determined by the relationship between the values of the vectorial errors Bp_θ, Bp_ψ and Bp_t. Depending on this relationship, the major half-axis of the ellipse of dispersion may coincide with the mean trajectory, pass above it or pass below it.

In order to construct the ellipse of bursts on the plane of fire, it is necessary to find, from the specified tabular values of the middle deviations $Bp\partial$, $Bp\delta$ and $B\delta$, and the angle of arrival θ_c, the values of
246. the main half-axes of the primary ellipse of bursts and the angle formed by its major half-axis and the axis OX. The main half-axes of the primary ellipse a and b and the angle α, which is formed by the half-axis a and the axis OX, are determined from the following formulae*

$$a = \sqrt{\frac{Bp\partial^2\cos^2\alpha - Bp\delta^2\sin^2\alpha}{\cos 2\alpha}};$$

$$b = \sqrt{\frac{Bp\delta^2\cos^2\alpha - Bp\partial^2\sin^2\alpha}{\cos 2\alpha}};$$

$$\tan 2\alpha = \frac{B\delta^2 - Bp\delta^2 - Bp\partial^2\tan^2\theta_c}{(Bp\partial^2 - Bp\delta^2)\tan\theta_c}.$$

It is usual to take the angle α as positive, if the positive direction of the axis a of the primary ellipse is above the horizon, and as negative, if this direction is below the horizon (fig.95). The angle of arrival θ_c with respect to the horizon is always positive.

Fig.95.

EXAMPLE. Firing is conducted from a 122mm howitzer, model 1938 with high explosive fragmentation shells with fuzes Д-1 on the fifth setting. The range of firing is 3500m. Determine the basic characteristics of the primary ellipse of bursts in the plane of fire.

Solution. From the firing tables No.146 and 146/140 Д, for the given conditions $Bp\partial$ = 30m, $Bp\delta$ = 10m, $B\delta$ = 5.5m and $\angle\theta_c = 17^\circ 27'$.

We determine the angle α, i.e. the angle formed by the major half-axis of the primary ellipse of bursts with the horizon:

$$\tan 2\alpha = \frac{B\delta^2 - Bp\delta^2 - Bp\partial^2\tan^2\theta_c}{(Bp\partial^2 - Bp\delta^2)\tan\theta_c} = \frac{30.25 - 100 - 900\times 0.099}{(900-100)\ 0.314} \approx -0.6363,$$

then

$$2\alpha \approx -32^\circ 30' \quad \text{and} \quad \alpha = -16^\circ 15'.$$

247. We will find the main half-axes of the primary ellipse:

$$a = \sqrt{\frac{Bp\partial^2\cos^2\alpha - Bp\delta^2\sin^2\alpha}{\cos 2\alpha}} = \sqrt{\frac{900\times 0.92 - 100\times 0.08}{0.843}} \approx 31.3\text{m};$$

$$b = \sqrt{\frac{Bp\delta^2\cos^2\alpha - Bp\partial^2\sin^2\alpha}{\cos 2\alpha}} = \sqrt{\frac{100\times 0.92 - 900\times 0.08}{0.843}} \approx 5\text{m}.$$

Fig.96.

In fig.96 is shown the primary ellipse drawn from this data.

51. Problems on dispersion of trajectories and points of bursts.

PROBLEM 121. Determine the value of $B\partial$, if $B\delta$ is 1.4m and the angle of arrival is 5°.

Answer. 16m.

*Because of its length the derivation of the formulae is not given.

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247. PROBLEM 122. Determine the value of Bϑ, if Bϕ is 63m and the angle of arrival is 40°.

Answer. 63m.

PROBLEM 123. Determine the angle of arrival, if Bϑ is 15m and Bϕ is 2m.

Answer. 7°36'.

PROBLEM 124. Determine the value of Bϑ, if Bϕ is 6.3m, ΔY is 10m and ΔX is 50m.

Answer. 31.5m.

PROBLEM 125. Firing is conducted on to a target, situated on the facing slope. Determine the value $B\vartheta_1$, if Bϑ is 25m, the angle of arrival is 10°4' and the angle of inclination of the ground is 2°.

Answer. 20.9m.

PROBLEM 126. Firing is conducted on to a target, situated on the reverse slope. Determine the value $B\vartheta_2$, if Bϑ is 20m, the angle of arrival is 8°32' and the angle of inclination of the ground is 0-50.

Answer. 30.8m.

PROBLEM 127. Firing is conducted on to a target, situated on the reverse slope. Determine the value $B\vartheta_2$, if Bϑ is 15m, the angle of arrival is 9°03' and the angle of inclination of the ground is 0-35.

Answer. 19.5m.

PROBLEM 128. Firing is conducted on to a target, situated on the reverse slope. Determine the value $B\vartheta_2$, if Bϑ is 35m, ΔX is 50m, ΔY is 23m and the angle of inclination of the ground is 0-45.

Answer. 39m.

248. PROBLEM 129. In impact firing on identical settings, the following deviations of points of arrival, from the datum, in distance, were obtained in metres: 41, 62, 80, 60, 28, 96, 67, 53, 8 and 65. Determine the position of the centre of grouping with respect to the datum in distance, if all the bursts were undershoots.

Answer. -56m.

PROBLEM 130. In impact firing on identical settings, the following deviations of points of arrival, from the datum, in distance (y) and the lateral deviations (z), were obtained in metres:

No. of shot	y in m	z in m	No. of shot	y in m	z in m

Determine the position of the centre of grouping with respect to the datum, in direction and in distance.

Answer. ξ_ϑ = +68; ξ_e = -16m.

PROBLEM 131. Find the value of Bϑ, if the mean arithmetical deviation of points of arrival from the centre of grouping in distance, is 24m.

Answer. 20.3m.

PROBLEM 132. Determine the value of Bϑ, if the mean quadratic deviation of points of arrival, from the centre of grouping in distance, is 33m.

Answer. 22.3m.

PROBLEM 133. Find the value of the median error in determining the position of the centre of dispersion in distance, if Bϑ = 24m and if the position of the centre of grouping was determined from the results of 8 shots.

Answer. 8.48.

PROBLEM 134. The position of the centre of grouping in direction was determined from the results of 16 shots. What is the probability that, in taking the centre of grouping as the position of the centre of dispersion, we will obtain an error which will not exceed 1m, if Bϑ = 3.2m?

Answer. 0.60083.

PROBLEM 135. In impact firing on identical settings, the following deviations of individual points of arrival, from the datum, in distance (y) and the lateral deviations (z), were obtained:

No. of shot	y in m	z in m	No. of shot	y in m	z in m

Determine the position of the centre of grouping with respect to the datum, and the accuracy of determination of the position of the centre of dispersion.

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248. Answer. x_0 = 28m; y_0 = 2m; z_0 = 10.4m; h_0 = 1.64m.

249. PROBLEM 136. In impact firing with shells, on identical settings, the following deviations of individual points of arrival with respect to the datum, were obtained, in metres:
in distance (y):

+20, -35, -5, +15, 0, +45, -10, -25, +15, -60, -15 and -5;

in direction (z):

+6, +2, -4, +8, -2, +3, +11, -6, -5, +2, +3 and +6.

Determine the adequate values of B_∂ and B_σ from the mean quadratic deviations, if it is known that the axis OY of the rectangular system of coordinates coincides with the direction of the plane of fire.

Answer. B_∂ = 8.6m and B_σ = 3.6m.

PROBLEM 137. On identical settings, when firing with shells with time fuzes, the following heights of individual aerial bursts with respect to the ground surface were obtained: 0-10, 0-12, 0-11, 0-09, 0-08, 0-12, 0-07, 0-05, 0-08, 0-12, 0-07, 0-04, 0-09, 0-15 and 0-06, Find the adequate value of $B_{p\phi}$, if the range of fire is 3600m.

Answer. 7.3m.

PROBLEM 138. In testing two guns of the same calibre, but of different models, when firing on identical settings and with other conditions being equal, the following deviations of individual points of arrival with respect to the datum in distance (y and y_1) and in direction (z and z_1), were obtained, in metres:

No. of shot	Model A gun		Model A_1 gun	
	y	z	y_1	z_1

Determine the adequate values of B_∂ and B_σ for both guns, if the axes OY and OY_1 coincide with the direction of the plane of fire.

Answer. Model A gun: B_∂ = 16.2m; B_σ = 5.7m. Model A_1 gun: B_∂ = 10.9m; B_σ = 3.03m.

PROBLEM 139. In impact firing on identical settings the following lateral deviations of individual points of arrival with respect to the target, were obtained, in metres: -12, +2, +9, +2, -6, +5, -3, -10, +7, +10 and -4. Determine the adequate value B_σ from the differences between adjacent deviations.

Answer. 5.2m.

PROBLEM 140. In impact firing on identical settings the following deviations in distance with respect to the datum were obtained, in metres: +15, +35, +75, +55, +15, -45, -15, +5, -35, +40, -10, -75, -45, 0, -5, -35, -5, +15, -45 and -55. Find the adequate value of B_∂ from the differences between the adjacent deviations.

Answer. 20m.

250. PROBLEM 141. In impact firing on identical settings the following deviations of individual points of arrival with respect to the orientation point were obtained, in metres:

In distance: +46, +144, +57, +169, +219, +284, +145, +86, +240, +160, +16, +119, +238, +120, +150, +184, +215, +180, +111, +96;

In direction: -24, -18, +10, -4, -10, +15, -20, -34, -46, -47, -12, -6, -30, -37, -44, -52, -47, -48, -50 and -52.

It is required to determine, from the differences between the adjacent deviations, the adequate values of B_∂ and B_σ, if the plane of fire in the region of arrival of shells is marked out with sufficient accuracy.

Answer. B_∂ ≈ 43m; B_σ = 7.75m.

PROBLEM 142. Determine the probability of occurrence of an aerial burst when firing shells with time fuzes, if the mean point of bursts is at 6m above the ground surface and $B_{p\phi}$ = 5.5m.

Answer. 83.7% (83.2%).

PROBLEM 143. In firing shells with time fuzes, the mean point of bursts is at 2m above the ground surface. What is the probability of occurrence of an impact, if $B_{p\phi}$ = 3.6m?

Answer. 63.7% (63.2%).

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250 PROBLEM 144. Determine the most probable position of the mean point of bursts with respect to the ground surface, in time-fire, if, in three battery salvoes on identical settings, 5 impacts and 7 aerial bursts were obtained, and Bpℓ = 7m.

Answer. +2.2m.

PROBLEM 145. Determine the most probable position of the mean point of bursts with respect to the ground surface, in time-fire, if, in four battery salvoes, 10 impacts and 6 aerial bursts were obtained, and Bpℓ = 12m.

Answer. -5.8m.

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PROBABILITY OF A HIT.

52. Factors which determine the value of the probability of a hit.

The value of the probability of a hit is the basis of the most important artillery calculations.

The probability of a hit determines the reliability and efficiency in firing. From the numerical value of the probability of a hit, we find the most efficient methods of conducting the fire, particularly that of fire for adjustment. Knowing the probability of a hit, it is possible to work out the mean standards of expenditure of rounds for the destruction of various targets.

Before commencing to discuss the methods of determining the probability of a hit, it is necessary to establish those factors on which this probability depends.

The probability of a hit depends on the following: the size of the target, the value of dispersion, the position of the mean trajectory with respect to the target, and the direction of the firing.

1. Size of target. With the same value of dispersion, with the same position of the mean trajectory with respect to the target and with the same direction of the firing, but with different sizes of the target, the probability of a hit will be the greater, the greater the size of the target.

Fig.97.

In fig.97 is shown the manner in which the probability of a hit decreases with the decrease in the size of the target, with all other conditions being equal; here the dimensions of the target are shown as progressively decreasing by a half, i.e. the area of the target decreases
252. 4 times. The change in the probability of a hit is not proportional to the change in the dimensions of the target, which naturally, follows from the fact that the dispersion is uneven.

2. The value of dispersion. With identical dimensions of the target, the same position of the mean trajectory with respect to the target and with the same direction of the firing, but with different value of dispersion, the probability of a hit will be the greater, the less the value of dispersion (of the value of the middle deviations). For example, if the ellipse of dispersion is completely contained within the boundaries of the target, the probability of a hit will be 100%. If, however, the dimensions of the ellipse of dispersion are such, that it covers the target by only a part of its area, the probability of a hit will be less than 100%, and will be less, the smaller the part of the ellipse of dispersion which covers the target (fig.98).

Fig.98.

3. The position of the mean trajectory with respect to the target. From the unevenness of dispersion it follows that the probability of a hit will be greatest in the case when the mean trajectory passes through the centre of the target. As the mean trajectory moves away from the centre of the target, the probability of a hit will decrease, and if this distance from the centre will exceed 4 middle deviations, the probability of a hit will, for all practical purposes, be zero (fig.99).

Fig.99.

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252. Conclusions. The direction of the fire determines the position of the ellipse of dispersion over the target. The lateral dispersion is considerably less than the dispersion in distance. Therefore, with the same value of dispersion, with the same position of the mean trajectory with respect to the centre of the target and with the same dimensions of the target, the probability of a hit will be the greatest for the case when the axis of the lateral dispersion is directed along the greatest dimension of the target.

Thus, for example, if the axis of the lateral dispersion BB is directed along the target, the shape of which is an elongated rectangle (fig.100) and the dimensions of the ellipse of dispersion are such that the ellipse comes completely within the boundaries of the target, the probability of a hit will be 100%. If, however, the axis of the lateral
253. dispersion is directed perpendicularly to this target, the probability of a hit will be less than 100%. For all practical purposes, this means that, with all other conditions being equal, the probability of a hit will be greatest with a flank fire, and smallest with frontal fire.

Fig.100.

Summing up all that has been said about the factors which determine the value of the probability of a hit, we can make a series of important practical conclusions.

1. The value of dispersion depends on the range of firing - the smaller the range, the smaller the dispersion. Therefore, in order to obtain the smallest dispersion, and hence the greatest probability of a hit, the firing positions of the battery should be disposed as close as possible to the enemy. The experience of the Great National war has shown the expediency of a mass application of guns, firing on direct sights from the positions of our front lines. Here, apart from the accuracy of sighting, an exceptionally great value was also obtained from short ranges of firing, which ensures a high probability of a hit.

2. The fire for adjustment must be made so as to attain the highest possible accuracy of coincidence of the mean trajectory and the target, as in this case the firing tasks (problems) are solved with the minimum expenditure of rounds and time.

3. A flank and an oblique fire should be widely employed, by suitable dispersion of the firing positions of the batteries and allocation of the firing tasks between the batteries.

53. Probability of a hit in a zone of infinite length.

It is known, that numerically, the law of dispersion in one direction is expressed by a scale of dispersion, where this direction may or may not be coincident with the direction of one of the main diameters of the ellipse of dispersion. The above follows from the fact that the property of the elliptical error permits the construction of a scale of errors on any diameter of the limit (in magnitude) elliptical error ε_{max} (see page 211).

Therefore, if the target represents a zone (strip) of infinite length (a line of wire obstructions, a line of trenches, a road, defile etc), the determination of the probability of a hit onto such a target is resolved to the determination of the probability of occurrence of a deviation, within given limits, in one direction. Such a problem, from the analogy with determination of the probability of occurrence of an error within specified limits, can be solved from the scale of dispersion and from the table of values $\Phi(\rho)$.

254. The scale is superimposed on the target, taking into account the position of the mean trajectory with respect to the target; after this the probability of a hit, on that part of the ellipse of dispersion which covers the target, is determined. It must be emphasized, that the scale of dispersion is superimposed on the target, and not the target, on the scale of dispersion, because in the opposite case, when

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254. the probability is being solved graphically, an unnecessary part of the scale of dispersion is marked out on the drawing - the drawing then becomes huge, of small scale and not clear.

When using the table of values $\Phi(\beta)$, the departure of the mean trajectory from the boundaries of the target, is converted into units of the corresponding middle deviation; the probability of occurrence of a hit within these limits is then determined, i.e. with frontal fire - between the near and far boundaries of the target, and with fire along the line (zone) - between the left and right boundaries of the target.

Let us examine two cases: when the direction of one of the main half-axes of the primary ellipse of dispersion is parallel to the direction (length) of the target; and when the direction of one of the main half-axes of the primary ellipse of dispersion makes an angle α, not a right-angle, with the direction of the target.

The examples of determining the probability of a hit, in both of these cases, will be shown in individual examples, commencing with the case when the direction of one of the main half-axes of the primary ellipse is parallel to the direction of the target (zone).

EXAMPLE 1. A battery is firing on a zone of wire obstructions, of depth 30m. The fire is frontal, i.e. the plane of fire is perpendicular to the zone of wire obstructions. Let us determine the probability of a hit on this zone, if it is known that $B\delta$ = 22m and that the mean trajectory passes through the middle of the zone.

We will solve this problem with the aid of the scale of dispersion and the scale of values $\Phi(\beta)$.

1. Solution, using the scale of dispersion (fig.101).

Fig.101.

We draw two parallel lines PP and ZZ, the distance between which, taken to an arbitrary scale, is equal to the width of the target, i.e. 30m.

We mark a point C_y in the zone PPZZ so, as to make the mean trajectory pass through the centre of the target.

We draw a line DD, through the point C_y, parallel to the lines PP and ZZ, i.e. parallel to the direction (length) of the zone, and we draw the line BB, perpendicular to the line DD. We will take the
255. directions of the lines DD and BB as directions of the main diameters of the ellipse of dispersion, in other words - as the axes of dispersion in distance DD and in lateral dispersion BB.

Along the line BB from the point C_y, to the chosen scale, we mark off $B\delta$ in both directions (in the diagram - above and below; according to the conditions of firing - forward and back), i.e. we superimpose on the target that part of the scale of dispersion in distance which will cover the zone.

We establish the proportion:

22m-0.25;
15m-x;

whence

$$x = \frac{0.25 \times 15}{22} \approx 0.17.$$

The probability of a hit on a half of the zone has been obtained. From this the required probability:

$$P = 0.17 \times 2 = 0.34 = 34\%.$$

The same problem can be solved, using the scale of dispersion, drawn with the accuracy up to 0.5 $B\delta$.

In this case the target will be covered by zones containing 13% hits and by parts of zones containing 12% hits.

We establish the proportion for that part of the depth of the target which is covered by the zone containing 12% hits:

11m-0.12;

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$$x = \frac{0.12 \times 4}{11} \approx 0.044.$$

Then the required probability:

$$P = (0.13+0.044)2 = 0.348 \approx 35\%.$$

2. Solution, using the table of values $\Phi(\beta)$.

From the condition, the mean trajectory passes through the middle of the zone of wire obstructions. Therefore, the probability of a hit on this target can be found as a probability of occurrence of a deviation within the limits from 0 to ±15m.

We will convert 15m, i.e. one half of the zone width, into units of the middle deviation in distance:

$$\beta = \frac{15}{22} \approx 0.682.$$

Then from the table of values $\Phi(\beta)$, by interpolation, we obtain:

$$P = \Phi(0.682) = 0.35352+0.00097 = 0.35449 \approx 0.355 = 35.5\%.$$

Fig.102.

EXAMPLE 2. A battery is firing along a highway, the width of which is 6m. Determine the probability of a hit onto the highway, if the mean trajectory passes through its left-hand side, and $B\sigma = 4.2$m.

256. 1. Solution, using the scale of dispersion (fig.102).

We see, that the highway is covered by the zone, containing 25% hits, and by part of a zone, containing 16% hits.

We establish the proportion:

4.2m-0.16;
1.8m-x;

whence

$$x = \frac{0.16 \times 1.8}{4.2} \approx 0.069.$$

Then the probability of a hit on the highway:

$$P = 0.25+0.069 = 0.319 \approx 32\%.$$

More accurate calculations, using the scale of dispersion over 0.5 $B\sigma$, will give:

$$P = 0.13+0.12+\frac{0.09 \times 1.8}{2.1} \approx 0.327.$$

2. Solution, using the table of values $\Phi(\beta)$.

We determine the probability of a hit within the limits from 0 to -6m:

$$\beta = \frac{6}{4.2} = 1.429;$$

whence, by interpolation, we obtain:

$$\Phi(\beta) = \Phi(1.429) = 0.66521-0.00034 = 0.66487.$$

Then the required probability

$$P = \frac{1}{2}\Phi(\beta) = 0.33243.$$

EXAMPLE 3. A battery is conducting a frontal fire along a line (zone) of trenches, the average depth of which is 40m. Determine the probability of a hit onto this line, if the mean trajectory passes at 15m in front of the centre of the near side of the line (zone of trenches), and $B\sigma = 38$m.

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Fig.103.

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256 Here the target is covered by the part of the zone containing 25% hits, and by the part of the zone containing 16% hits. We establish two proportions

$$38m-0.25; \quad 38m-0.16;$$
$$23m-x_1; \quad 17m-x_2;$$

257. whence

$$x_1 = \frac{0.25x23}{38} \approx 0.151 \quad \text{and} \quad x_2 = \frac{0.16x17}{38} \approx 0.072.$$

Then, the probability of a hit on the line of trenches:

$$P = 0.151+0.072 = 0.223.$$

2. Solution, using the table of values $\Phi(\beta)$.
We find the probability of a hit within the limits from 0 to ±15m:

$$\beta_1 = \frac{15}{38} = 0.395,$$

whence

$$\Phi(\beta) = 0.20749+0.00260 = 0.21009.$$

Then, the required probability:

$$P = \frac{1}{2}\left[\Phi(\beta_2)-\Phi(\beta_1)\right] = \frac{1}{2}(0.67093-0.21009) = 0.23042.$$

EXAMPLE 4. A battery is conducting a frontal fire on a line of trenches situated on a reverse slope. The width of the line of trenches is 30m. Its near side, with respect to the apex, is 10m removed. Determine the probability of a hit on this line of trenches, if the mean trajectory passes through the concealing apex; the angle of the slope $\alpha = 5^0$; the angle of arrival $\theta_c = 22^0$; and $B\partial = 18m$ (fig.104).

Fig.104.

Solution. We will first determine the value of the middle deviation $B\partial_2$, in direction of the reverse slope, from the formula (see page 230).

$$B\partial_2 = \frac{B\partial \sin\theta_c}{\sin(\theta_c-\alpha)} = \frac{18 \sin 22^0}{\sin 17^0} \approx 23m.$$

258. Then the required probability from the scale of dispersion (fig.105)

$$P = \frac{0.25x13}{23}+\frac{0.16x17}{23} = 0.259 \approx 26\%.$$

Solution, using the table of values $\Phi(\beta)$

$$P = \frac{1}{2}\left[\Phi(\beta_2)-\Phi(\beta_1)\right] = \frac{1}{2}\left[\Phi(\tfrac{40}{23})-\Phi(\tfrac{10}{23})\right] = \frac{1}{2}\left[\Phi(1.74)-\Phi(0.435)\right] =$$
$$= \frac{1}{2}\left[0.75945-0.23087\right] = 0.26429 \approx 26.4\%.$$

Fig.105.

Solutions of examples 1, 2, 3 and 4 have shown the most typical cases of determining the probabilities of a hit, when one of the main diameters of the ellipse of dispersion is parallel to the direction of the zone.

Let us now turn to the case when the main diameters of the ellipse of dispersion form an angle α with the direction of the zone, this angle not being a right-angle.

Let us assume, that the ellipse of dispersion, with respect to the target which represents a zone of infinite length, occupies the position shown in fig.106.

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Fig.106.

It is obvious, that in this case the target will be hit by all the shells the deviations of which in the direction OY, perpendicular to the direction of the zone, will be greater than y_1 and less than y_2. We will therefore find the middle deviation, caused by the primary ellipse of dispersion in direction OY.

The deviation, caused by the elliptical error in a direction, is (see page 213)

$$D'' = \sqrt{a^2\sin^2\alpha + b^2\cos^2\alpha}\ .$$

In our case $a = B\partial$, $b = B\sigma$ and the angle α - the angle between the direction of the main major diameter of the ellipse of dispersion (the axis of the lateral dispersion BB) and the direction of the zone.

Then the above formula can be written as:

$$V = \sqrt{B\partial^2\sin^2\alpha + B\sigma^2\cos^2\alpha},$$

where V - the deviation (middle) caused by the primary ellipse of dispersion along the perpendicular (normal) to the direction of the zone.

If in the condition of the problem, we are given an angle α_1 instead of the angle α, the former being contained by the same main diameter and the normal to the direction of the target, then

$$V = \sqrt{B\partial^2\cos^2\alpha_1 + B\sigma^2\sin^2\alpha_1}.$$

As we are determining the probability of occurrence of a deviation along the line OY, it is necessary to convert the departure of the mean trajectory from the near side of the target (expressed by intercept OA), and the departure of the far side of the target (intercept OV), to the direction OY, i.e. to project on to this direction.

From the diagram, it is seen that these projections

$$OA_1 = y_1 = OA\sin\alpha$$

and

$$OV_1 = y_2 = OA_1 + L = y_1 + L,$$

where L - width of target.

If, however, the departure of the mean trajectory with respect to the side of the target is given along the normal, then there is no need for such a convertion.

In all other respects the methods of solving such problems are the same as for those discussed by us in the above examples.

EXAMPLE 5. When directing the fire, as shown in fig.106, determine the probability of a hit, if $B\partial = 20$m, $B\sigma = 3$m, $\alpha = 30^\circ$, OA = 12m and L = 20m.

Solution. Irrespective of the method of solving, it is first necessary to find the values V, OA_1 and OV_1:

$$V = \sqrt{20^2\sin^2 30^\circ + 3^2\cos^2 30^\circ} = \sqrt{400\times 0.25 + 9\times 0.75} \approx 10.3\text{m}.$$

$$OA_1 = 12\sin 30^\circ = 6\text{m} \quad \text{and} \quad OV_1 = 6\text{m} + 20\text{m} = 26\text{m}.$$

260. Then the required probability, from the scale of dispersion (fig.107):

$$P = \frac{0.25\times 4.3}{10.3} + 0.16 + \frac{0.07\times 5.4}{10.3} \approx 0.301 = 30.1\%,$$

and from the table of values $\Phi(\beta)$

$$P = \frac{1}{2}\left[\Phi(\beta_2) - \Phi(\beta_1)\right] = \frac{1}{2}\left[\Phi\left(\frac{26}{10.3}\right) - \Phi\left(\frac{6}{10.3}\right)\right] = \frac{1}{2}\left[\Phi(2.524) - \Phi(0.583)\right] =$$

$$= \frac{1}{2}(0.91382 - 0.30586) = 0.30398 \approx 30.4\%.$$

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Fig.107.

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$$V = \sqrt{B\partial^2 \sin^2\alpha + B\sigma^2 \cos^2\alpha}$$

gives the value of the middle deviation in a general form, i.e. for any direction of fire with respect to the target.

Thus, if the fire is conducted as frontal, i.e. $\alpha = 90^0$, then

$$V = \sqrt{B\partial^2 \sin^2 90^0 + B\sigma^2 \cos^2 90^0} = B\partial,$$

and with flank fire, i.e. with $\alpha = 0^0$;

$$V = \sqrt{B\partial^2 \sin^2 0^0 + B\sigma^2 \cos^2 0^0} = B\sigma.$$

54. Probability of a hit on a rectangle, the sides of which are parallel to the main half-axes of the ellipse of dispersion.

Let us suppose that the target represents a rectangle, the sides of which are parallel to the main half-axes of the ellipse of dispersion, and that this rectangle of the ellipse of dispersion occupies the position shown in fig.108.

The rectangle abcd obtained as a result of intersection of two strips: strip M, the sides of which are parallel to the major, main diameter BB of the ellipse of dispersion, and the strip N, the sides of which are parallel to the minor, main diameter DD of the same
261. ellipse. It is obvious, that this rectangle will be hit by those rounds which simultaneously hit (fall on) the strip M and the strip N.

Fig.108.

In the given case a combination of events takes place - a hit on the rectangle - being the coincidence of simple independent events - a hit on the strip M and a hit on the strip N. Denoting the probability of a hit on the strip M by p_M, and on the strip N - by p_N, we find the probability of a hit on the target from the theorem of multiplication of probabilities:

$$P = p_M p_N.$$

Consequently, the solution of the problem requiring determination of the probability of a hit on the rectangle, the sides of which are parallel to the main half-axes of the ellipse of dispersion, resolves to the solution of two problems dealing with determination of the probability of obtaining a hit on a strip (zone) of infinite length, and thereafter multiplying these probabilities.

EXAMPLE 1. Fire is conducted on a blindage having a shape of a rectangle with sides of 6 and 10m. The plane of fire is perpendicular to the smaller side of the blindage, and the mean trajectory passes through the middle of this side (fig.109). Determine the probability of a hit on the blindage, if $B\partial = 28$m and $B\sigma = 2.2$m.

1. Solution, using the scale of dispersion.

Fig.109.

We determine the probability of a hit in distance

$$p_\partial = \frac{0.25 \times 10}{28} \approx 0.09.$$

We determine the probability of a hit in direction

$$p_\sigma = 2\left(0.25 + \frac{0.16 \times 0.08}{2.2}\right) \approx 0.62.$$

Then the required probability, being the probability of a compound

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261. $$P = p_\beta p_\sigma = 0.09 \times 0.62 = 0.0558 \approx 5.6\%.$$

2. Solution, using the table of values $\Phi(\beta)$.
Probability of a hit in distance

$$p_\beta = \frac{1}{2}\Phi\left(\frac{10}{28}\right) = \frac{1}{2}\Phi(0.357) = 0.09514.$$

262. Probability of a hit in direction

$$p_\sigma = \Phi\left(\frac{3}{2.2}\right) = \Phi(1.364) = 0.64242.$$

Then

$$P = p_\beta p_\sigma = 0.09514 \times 0.64242 \approx 0.06112 \approx 6.1\%.$$

EXAMPLE 2. Frontal fire is conducted from a gun, against the embrasure of long duration firing points. The dimensions of the embrasure are 0.8m by 0.2m. Determine the probability of obtaining at least one hit on the embrasure with 6 shots, if $B_4 = 0.2$m, $B_\sigma = 0.3$m and if the mean trajectory passes 0.1m below the middle (centre) of the lower side of the embrasure (fig.110).

Fig.110.

Solution. We will first find the probability of a hit on the embrasure with one shot.

1. From the scale of dispersion (with accuracy up to $0.5B_4$ and $0.5B_\sigma$):

$$p_4 = 0.12 + 0.09 = 0.21;$$

$$p_\sigma = 2\left(0.25 + \frac{0.09 \times 0.1}{0.15}\right) \approx 0.62;$$

$$P = p_4 p_\sigma = 0.21 \times 0.62 \approx 0.13.$$

2. From the table of values $\Phi(\beta)$

$$p_4 = \frac{1}{2}\left[\Phi\left(\frac{0.3}{0.2}\right) - \Phi\left(\frac{0.1}{0.2}\right)\right] = \frac{1}{2}\left[\Phi(1.5) - \Phi(0.5)\right] = \frac{1}{2}(0.6883 - 0.26407) = 0.21213;$$

$$p_\sigma = \frac{1}{2}\Phi\left(\frac{0.4}{0.3}\right) = \frac{1}{2}\Phi(1.333) = 0.6314;$$

$$P = p_4 p_\sigma = 0.21213 \times 0.6314 \approx 0.134.$$

We will now determine the probability of a hit on the embrasure, at least once:

$$P_1 = 1 - (1 - 0.13)^6 = 1 - 0.87^6 = 0.566,$$

or

$$P_1 = 1 - (1 - 0.134)^6 = 1 - 0.866^6 = 0.576.$$

263. ## 55. Determination of the probability of a hit on a target of any shape.

In those cases, when the shape of the target is such that it is not possible to represent it (target) in the form of a rectangle, the sides of which are parallel to the main half-axes of the primary ellipse of dispersion, the determination of the probability of a hit is carried out graphically by means of a grid of circular dispersion.

The grid of circular dispersion is drawn and calculated under an assumption that the dispersion in the ellipse, along its main diameters, is the same in value. Instead of an ellipse, a circle is obtained, and hence the nomenclature of the grid.

To draw the grid of circular dispersion on a sheet of graph or millimetric paper, two mutually perpendicular lines are drawn (fig.111)

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263. the centre of dispersion C. The side of each square (of the graph) of the grid is taken as equal to some part of the middle deviation: 0.5; 0.25 etc, depending on the required accuracy.

Fig.111.

After the grid has been drawn, the probabilities of a hit on each square are worked out in the same way as for a rectangle, the sides of which are parallel to the axes of dispersion. For this calculation the table of values $\Phi(\beta)$ is used, and the calculations are made correct to 0.0001, i.e. to 0.01%.

In fig.111 is shown a part of the grid of circular dispersion, drawn to the scale of one square to 0.5 of middle deviation. The rectangle abcd was obtained as a result of intersection of the strips M and N.

The probability of a hit on the strip M

$$p_M = \frac{1}{2}\Phi(0.5) = \frac{0.26407}{2} = 0.13204,$$

264. and the probability of a hit on the strip N

$$p_N = \frac{1}{2}\left[\Phi(1.0)-\Phi(0.5)\right] = \frac{0.5-0.26407}{2} = 0.11797.$$

Then, from the theorem of multiplication, the probability of a hit on the rectangle abcd

$$P_1 = p_M p_N = 0.13204 \times 0.11797 = 0.0155767588 \approx 0.0156.$$

We will write this probability on the rectangle abcd.

The rectangle efgk was obtained as a result of intersection of the strip K and the strip L. Therefore, the probability of a hit on this rectangle

$$P_2 = \frac{1}{2}\left[\Phi(1.0)-\Phi(0.5)\right] \times \frac{1}{2}\left[\Phi(2.0)-\Phi(1.5)\right] =$$

$$= \frac{0.5-0.26407}{2} \times \frac{0.82266-0.68833}{2} \approx 0.0079 \text{ etc.}$$

Such a grid is given in full in the appendix 2.

The grid of circular dispersion is drawn in units of middle deviations. Therefore, in order to determine, from this grid, the probability of a hit on a specified target, it is necessary to express the coordinates, with respect to the axes of dispersion, of the more characteristic points of the target contour, likewise in units of the middle deviations. After this, from these coordinates, the target is marked off on the grid, and the probabilities of a hit on the squares which are found to be covered by the contours of the target, are added. The probabilities of a hit on the squares, covered only partially by the target, are here determined by visual interpolation.

When transferring the target contours on to the grid of circular dispersion, these contours, in comparison with the actual, will be distorted, being the result of the distortion of the scales of the middle deviations when drawing the grid itself. But this will not affect the results of calculations, because the grid has been drawn in units of middle deviations and not in metric units. This, therefore, represents the usefulness of the grid of circular dispersion, because from it the probability of a hit can be determined, in the first place, on a target of any shape and, in the second place, on a target situated in either a vertical or a horizontal plane.

Fig.112.

265. EXAMPLE. A target represents a horizontal hexagon, the position [illegible] with respect to the centre of dispersion, is shown in fig.112.

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265. The vertices of the hexagon, in metres, are as follows:

Vertices	A	B	C	D	E	F
Along the axis OX						
Along the axis OY						

It is required to determine the probability of a hit on the target, if it is known that $B_\partial = 40$m and $B_f = 5$m.

Solution. Converting the coordinates of the vertices of the target, along the axis OX, into units of B_∂, and along the axis OY into units of B_f, we obtain

Vertices	A	B	C	D	E	F
Along the axis OX						
Along the axis OY						

Along the axis OX, the depth of the target is from $-0,5B_\partial$ to $+1B_\partial$, and along the axis OY, the width of the target is from $-0.5B_f$ to $+2.5B_f$. We will therefore draw, on the millimetric paper, that part of the grid of circular dispersion which will cover the hexagon. We will make use, here, of the grid of circular dispersion given in the Appendix 2.

We will transfer, onto the grid (fig.113), the vertices of the hexagon, from the coordinates expressed in units of B_∂ and B_f, and we will join these vertices by straight lines. The shape of the target, obtained on the grid and its actual shape in fig.112, should be noted.

All that remains now is to add the probabilities of a hit on the squares covered by the target. If we consider, that one millimetre of a given square represents one hundredth of the probability of a hit on it, then, by adding the millimetres in the partially filled squares, we determine the probability of a hit on the target, with a very great accuracy, without making use of interpolation.

Fig.113.

Below are given the probabilities of a hit on individual squares; the disposition of the additive components corresponds to the disposition of the squares in fig.113 (from left to right and from top to bottom):

4+ 36+ 8 = 48
53+ 89+ 40 = 182
109+124+ 96 = 329
146+156+128 = 430
113+174+ 94 = 381
17+152+ 31 = 200

the sum of which will give the required probability

$$P = 0.157 = 15.7\%.$$

266.

56. Determination of the probability of a hit from the shadow diagram.

Up to the present we have been dealing with planary targets, i.e. with targets either only in a horizontal or only in a vertical plane. In the cases, when it is required to determine the probability of a hit on a solid target, use is made of the, so called, shadow diagram.

The principle of this method consists in the following (fig.114).

Fig.114.

Tangents, to the target are drawn parallel to the trajectory, and are produced to their point of intersection with the [illegible]

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266. As a result of this, a shadow projection (shadow diagram) of the target will appear on the horizontal plane. In our illustration, this shadow diagram takes the form of the rectangle EDCF.

It is obvious, that all the rounds falling on the rectangle ABCD will hit the target.

From the right-angle triangle KED, we have:

$$ED = \frac{EK}{\tan \theta_c}.$$

We will denote the length of the shadow diagram ED by d, the height of the target EK by h and the depth of the target AE by L. Then the length of the whole rectangle

$$AD = L+d = L+\frac{h}{\tan \theta_c}.$$

The probability of a hit on the rectangle ABCD is equal to the probability of a hit on the target; consequently, the solution of the problem resolves into determination of the probability of a hit on the rectangle, the sides of which are parallel to the main half-axes of the primary ellipse of dispersion. If, however, the shadow projection of the target is not such a rectangle, we have to use the grid of circular dispersion.

EXAMPLE 1. Let us suppose, that the target (fig.114) represents a small stone building with a flat roof, in which the height h is 4m, the length (front view) is 6m and the width L is 3m.

267. We will determine the probability of a hit on this target, if $B\beta = 25$m, $B\sigma = 2.4$m, angle $\theta_c = 21^\circ 48'$ and the mean trajectory passes 10m in front of the centre of the building foundation.

Solution. We will find the length of the rectangle on the horizontal plane, which corresponds to the probability of a hit on the target:

$$L+d = 3+\frac{4}{0.4} = 13\text{m}.$$

Then the probability of a hit on the target, from the table of values $\Phi(\beta)$

$$P = P_\beta P_\sigma = \frac{1}{2}\left[\Phi\left(\frac{23}{25}\right)-\Phi\left(\frac{10}{25}\right)\right]\Phi\left(\frac{3}{2.4}\right) = \frac{1}{2}\left[\Phi(0.92)-\Phi(0.40)\right]\Phi(1.25) =$$
$$= \frac{1}{2}(0.46509-0.21268)0.60083 = 0.0758.$$

From the table of dispersion, working to the accuracy of $0.5B\beta$, we have:

$$P = \left(\frac{0.13\times 2.5}{12.5}+\frac{0.12\times 10.5}{12.5}\right)\times 2\left(0.25+\frac{0.09\times 0.6}{1.2}\right) \approx 0.0749.$$

EXAMPLE 2. Determine the probability of a hit on a stationary tank, the dimensions of which are as follows: length - 7m, width - 3m and height - 2m, if $B\beta = 15$m, $B\sigma = 1.2$m and angle $\theta_c = 5^\circ$. Here, the mean trajectory coincides with the centre of the base of the tank; the movement of the tank, prior to coming to rest, was frontal.

Fig.115.

Solution. Taking the shape of the tank as a right-angle parallelepiped (fig.115), we determine the length of the horizontal rectangle corresponding to the probability of a hit on the tank:

$$L+d = 7+\frac{2}{\tan 5^\circ} = 30\text{m}.$$

from this, the required probability:

a) from the table of values $\Phi(\beta)$

$$P = \frac{1}{2}\Phi\left(\frac{30}{15}\right)\Phi\left(\frac{1.5}{1.2}\right) = \frac{1}{2}\Phi(2.00)\Phi(1.25) = \frac{1}{2}0.82266\times 0.60083 \approx 0.24714 \approx 24.7\%;$$

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$$P = (0.25+0.16)x2(0.25+\frac{0.09x0.3}{0.6}) = 0.41x0.59 \approx 0.2419 \approx 24.2\%.$$

268. **57. Determination of the probabilities of a hit by the method of comparison of areas.**

For small sections of the ellipse of dispersion, it can be assumed that the points of arrival (in such sections) are distributed uniformly. With such an assumption, it is possible to determine the probability of a hit on targets of small dimensions, by the method of comparison of areas, i.e. it is possible to assume that the probability of a hit on the target is related to the probability of a hit on the section of the ellipse of dispersion, in the same ratio as the area of the target is related to the area of the section of the ellipse of dispersion.

We will denote the area of the target by S, the probability of a hit on the target by P, the area of the section of the ellipse of dispersion by s and the probability of a hit on this section by p. Then, from the assumed ratio, we obtain:

$$P = p\frac{S}{s}.$$

It must be emphasized, that the method of comparison of areas provides only an approximate solution, where the answer will be the nearer to the true value, the nearer are the dimensions of the target to the dimensions of the compared section of the ellipse of dispersion.

In artillery practice, the method of comparison of the area of the target with the area of the primary rectangle of dispersion, is widely used.

We know, that the area of such a rectangle for the horizontal plane, is

$$2B\beta x2B\delta = 4B\beta xB\delta$$

and for the vertical plane

$$2B\phi x2B\delta = 4B\phi xB\delta,$$

and the probability of a hit on this rectangle - 0.25 (see page 211).

Then the formula given above will appear as:

$$P = 0.25\frac{S}{4B\beta xB\delta} = \frac{S}{16B\beta xB\delta},$$

or

$$P = 0.25\frac{S}{4B\phi xB\delta} = \frac{S}{16B\phi xB\delta}.$$

EXAMPLE 1. A blindage, having the appearance of a rectangle of dimensions 4x3m, is covered by the primary rectangle of dispersion. Determine the probability of a hit on the blindage, if $B\beta = 15m$ and $B\delta = 1.5m$.

Solution. Substituting the numerical values of S, $B\beta$ and $B\delta$ in the general formula, we obtain:

$$P = \frac{4.3}{16x15x1.5} = \frac{1}{30} \approx 3.3\%.$$

EXAMPLE 2. The vulnarable part of the ground wall of a long duration firing point, has a width of 2m and a height of 0.8m. Determine the probability of a hit on this part of the ground wall, if the mean trajectory passes through its centre, $B\phi = 1.1m$ and $B\delta = 1.2m$.

269. Solution.

$$P = \frac{2x0.8}{16x1.1x1.2} \approx 0.076 = 7.6\%.$$

The mean expenditure of rounds on firing is determined from the formula

$$S = \frac{a}{p},$$

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where a - the required number of hits,
p - the probability of a hit with one shot (see page 76).

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269. We will change the notation for the number of rounds to the letter N. Then for one hit

$$N = \frac{1}{p},$$

and after substituting the expressions for the probability of a hit on the primary rectangle for p, we have

$$N = \frac{16B\partial xB\sigma}{S},$$

or

$$N = \frac{16B\phi xB\sigma}{S}.$$

These formulae are included in the Rules for firing by land artillery, 1945, where on page 318 it is stated that the second formula may be used only when the height of the vulnarable area does not exceed $2B\phi$, and the width - $2B\sigma$. This requirement of the Rules for firing follows from the principle of the method of comparison of areas.

EXAMPLE 3. In the conditions of example 2, determine the average expenditure of rounds for obtaining one hit.

Solution.

$$N = \frac{16x1.1x1.2}{2x0.8} = 13 \text{ to } 14 \text{ rounds.}$$

We will obtain a somewhat closer result, if we substitute, the area of the primary rectangle by the area of the primary ellipse of dispersion, in the formula for the probability of a hit. Taking into consideration, that the area of the primary ellipse of dispersion is $\pi xB\partial xB\sigma$ or $\pi xB\phi xB\sigma$, and that the probability of a hit on the primary ellipse is 0.203, we obtain:

$$P = 0.203\frac{S}{\pi xB\partial xB\sigma} \approx \frac{0.065xS}{B\partial xB\sigma},$$

or

$$P = \frac{0.065xS}{B\phi xB\sigma}.$$

Taking the conditions of the same example 2, let us determine the probability of a hit on the vulnarable part of the ground wall, by comparing its area with the area of the primary ellipse of dispersion.

270. Carrying out corresponding substitutions in the formula derived above, we obtain

$$P = \frac{0.065x2x0.8}{1.1x1.2} = 0.079.$$

To check this result, we carry out a calculation from the table of values $\Phi(\beta)$:

$$P = \Phi\left(\frac{0.4}{1.1}\right)x\Phi\left(\frac{1.0}{1.2}\right) = \Phi(0.364)\Phi(0.833) = 0.19394x0.42578 \approx 0.08258.$$

It is quite apparent, that the comparison with the area of the primary ellipse of dispersion has provided an answer which is closer to the true answer.

In the case of targets of very small dimensions, it is possible to compare their areas with the area of the semi-ellipse of dispersion. Then

$$P = 0.055\frac{S}{\pi x0.5xB\phi x0.5B\sigma} = \frac{0.07xS}{B\phi xB\sigma}.$$

EXAMPLE 4. The embrasure of a long duration firing point (post) has a width of 0.6m and a height of 0.4m. It is required to determine the probability of a hit on the embrasure, if the mean trajectory passes through its centre, $B\phi = 0.6m$ and $B\sigma = 0.7m$.

Solution. As the outline of the embrasure does not come outside the limits of the semi-ellipse of dispersion, the required probability

$$P = \frac{0.07x0.6x0.4}{0.6x0.7} = 4\%.$$

If the dimensions (size) of the target are such, that they can be drawn sufficiently closely - with comparitively small "gaps" - into the [illegible] of the primary ellipse of dispersion, then the method of comparison of the

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270. irrespective of its distance from the centre of dispersion.

Fig.116.

EXAMPLE 5. The target is a circular blindage of radius 3m. The centre of the blindage C with respect to the centre of dispersion C_y is displaced as shown in fig.116. It is required to determine the probability of a hit on the blindage, if $B\partial = 24m$ and $B\delta = 4m$.

Solution. We draw the circle inside the rectangle, the sides of which are parallel to the main half-axes of the primary ellipse of dispersion, and we determine the probability of a hit on this rectangle: the probability of a hit, in distance

$$p'_\partial = \frac{1}{2}\left[\Phi\left(\frac{18}{24}\right)-\Phi\left(\frac{12}{24}\right)\right] = \frac{1}{2}\left[\Phi(0.75)-\Phi(0.50)\right] = \frac{1}{2}(0.38705-0.26407) \approx 0.0615;$$

271. the probability of a hit, in direction

$$p'_\delta = \frac{1}{2}\left[\Phi\left(\frac{8}{4}\right)-\Phi\left(\frac{2}{4}\right)\right] = \frac{1}{2}\left[\Phi(2.00)-\Phi(0.50)\right] = \frac{1}{2}(0.82266-0.26407) \approx 0.2793;$$

the probability of a hit on the rectangle

$$P' = p'_\partial p'_\delta = 0.0615 \times 0.2793 \approx 0.0172.$$

We substitute this probability in the formula of comparison of areas; then the probability of a hit on the blindage

$$P = p\frac{S}{s} = 0.0172\frac{\pi \times 9}{36} = 0.0107 \approx 1\%.$$

58. Probability of a hit, with an existing error in the determination of the target position.

Before changing over to the fire for the destruction of the target, the position of the latter with respect to the mean trajectory is judged from the results of the fire for adjustment. The result of the fire for adjustment does not provide any one definite position of the target, but a distribution of the target (see section 21). This means, that, in changing to the fire for destruction, on settings corresponding to the most probable position of the target, it is possible to admit an error because the actual position of the target may not correspond to the command settings.

As a greater or a smaller error in determination of the position of the target with respect to the mean trajectory is inevitable, a question arises: how should the probability of a hit on the target be found, when an error in the determination of the target position, is present?

The errors in determination of the position of the target obey the Gauss law. The same law governs the errors occurring due to dispersion. Hence the median error B', characterising the mutual action (combination) of these two systems of errors, from the general rule of addition of vectorial errors, will be equal to

$$B' = \sqrt{B^2+a^2},$$

where B - middle deviation in a given direction;
a - median error in the method of determination of the target position.

These median errors or middle deviations are expressed, depending on direction, by the following formulae:
in distance

$$B\partial' = \sqrt{B\partial^2+a^2};$$

in direction

$$B\delta' = \sqrt{B\delta^2+a^2};$$

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272.

$$B\delta' = \sqrt{B\delta^2+a^2}\ .$$

We will use these formulae when determining the probabilities of a hit. In all other respects the solution of the problem is carried out by one of the methods previously discussed. It must be emphasized, that these formulae are true only in calculations of the probability of a hit with one shot, and the mathematical expectation of the number of hits; they are not, however, applicable for determination of the probability of at least one, two, etc hits.

EXAMPLE 1. A battery is conducting a frontal fire, on a railway track of 6m in width. It is required to determine the probability of a hit on the track, if $B\delta = 20$m, the mean trajectory passes at 10m in front of the near side of the track, and the position of the mean trajectory with respect to the track is determined with a median error of 15m.

Solution. We find the value of the middle deviation, taking into account the median error in determination of the position of the target:

$$B\delta' = \sqrt{20^2+15^2} = 25\text{m}.$$

Using the obtained value of $B\delta'$, we determine the probability of a hit on the specified target:

$$P = \frac{1}{2}\left[\Phi\left(\frac{16}{25}\right)-\Phi\left(\frac{10}{25}\right)\right] = \frac{1}{2}\left[\Phi(0.64)-\Phi(0.40)\right] = \frac{1}{2}(0.33402-0.21268) =$$

$$= 0.06067 \approx 6.1\%.$$

EXAMPLE 2. A battery is conducting a flank fire along a highway of 5m in width. Judging from the results of fire for adjustment, it may be considered that the mean trajectory passes through the side (edge) of the highway. What is the probability of a hit on the highway, if $B\delta = 3.2$m and the median error, in determination of the position of the mean trajectory with respect to the target, is 2.4m?

Solution.

$$B\delta' = \sqrt{3.2^2+2.4^2} = 4\text{m}.$$

$$P = \frac{1}{2}\Phi\left(\frac{5}{4}\right) = \frac{1}{2}\Phi(1.25) = 0.30042 \approx 30\%.$$

59. Problems on probability of a hit.

PROBLEM 146. Determine the probability of a hit on a stone wall 4.5m in height when the fire is frontal, if the mean trajectory passes through the centre of the wall base and if $B\delta = 3$m.

Answer. 0.34417 (34%) *

PROBLEM 147. Determine the probability of a hit on a line of wire obstructions, 28m in depth, when the fire is frontal, if the mean trajectory passes through the centre of the near side of the line of obstructions and $B\delta = 23$m.

Answer. 0.29414 (29%).

273.

PROBLEM 148. Determine the probability of a hit on a line of wire obstructions of 35m in depth, if the mean trajectory passes through the centre of the line, the fire is conducted perpendicularly to the front of the target and $B\delta = 25$m.

Answer. 0.3617 (35%).

PROBLEM 149. Determine the mean expenditure of rounds, required to effect four hits on a line of trenches of 4.5m in average depth, if the mean trajectory passes through the centre of the line of trenches, $B\delta = 19$m and the fire is frontal.

Answer. $p = 0.5755$ (56%); $N = 7$ to 8 rounds.

PROBLEM 150. Determine the probability of a hit on a railway track of 12m in width, with fromtal fire, if the mean trajectory passes

* In this and the following problems, the values of probabilities

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273. 20m in front of the near side of the zone.

Answer. 0.09838 (9.7%).

PROBLEM 151. Firing is conducted along a railway track of 9m in width...It is required to determine the probability of a hit on the track, if the mean trajectory passes 3m to the left of its right side and $B\sigma = 3.4$m.

Answer. 0.60723 (59.8%).

PROBLEM 152. A line of wire obstructions is situated on the near slope. The depth of the line is 20m. The angle of inclination of the slope is 1-00. Determine the probability of a hit on the line, if $B\delta = 20$m, angle $\theta_c = 18^0$, the mean trajectory passes through the centre of the front side of the wire obstructions and the plane of fire is perpendicular to the direction of the line.

Answer. $B\delta_1 = 15.2$; $P = 0.31259$ (30.9%)

PROBLEM 153. A line of trenches, the average depth of which is 25m, is situated on the reverse slope and the foreward side of the line is 15m from the concealing ridge. The angle of inclination of the slope is 0-75. What is the probability of a hit on this line of trenches with a frontal fire, if $B\delta = 24$m, angle $\theta_c = 36^0$ and the mean trajectory passes through the concealing ridge?

Answer. $B\delta_2 = 27$m; $P = 0.19509$ (19.4%).

PROBLEM 154. A battery is conducting fire on a railway track, the width of which is 12m. The plane of fire makes an angle of 45^0 with the direction of the track. It is required to determine the probability of a hit on the track, if the mean trajectory passes through its near side, $B\delta = 30$m and $B\sigma = 3$m.

Answer. $B = 21.3$m; $P = 0.14803$ (14.5%).

PROBLEM 155. Determine the probability of a hit on a zone (strip) of infinite length and 5m in width, which makes an angle of 60^0 with the plane of fire, if $B\delta = 15$m, $B\sigma = 2$m and the mean trajectory passes 3m in front of the near side of the zone, reckoning in the direction of the plane of fire.

Answer. $B = 13$m; $P = 0.09977$ (9.8%).

PROBLEM 156. It is required to determine the probability of a hit on an observation point (post), having a shape of a rectangle of 10x5m, with frontal fire, if $B\delta = 16$m, $B\sigma = 1.4$m and the mean trajectory passes through the observation point.

Answer. 0.121 (12.1%).

PROBLEM 157. A blindage has a shape of a rectangle of dimensions 6x4m. Frontal fire is conducted. Determine the probability of a hit on the blindage, if $B\delta = 20$m, $B\sigma = 1.6$m and the mean trajectory passes through the centre of the smaller dimension of the blindage.

Answer. 0.04817 (4.6%).

PROBLEM 158. A battery is firing on a bridge, 22m in length and 6m in width. The plane of fire is directed along the bridge. What is the probability of a hit on the bridge, if $B\delta = 34$m, $B\sigma = 4.8$m and the mean trajectory passes 25m in front of the centre of the bridge?

Answer. 0.05004 (4.9%).

PROBLEM 159. A dugout is 12x8m and has a shape of a rectangle, extending along the plane of fire. It is required to determine the
274. probability of a hit on the dugout, if $B\delta = 18$m, $B\sigma = 2.4$m and the mean trajectory passes 5m in front of the left side of the dugout.

Answer. 0.07998 (8.1%).

PROBLEM 160. The coordinates of the vertices of a quadrilateral with respect to the centre of dispersion are as follows:

Vertices	A	B	C	D
Along the axis OX in m.				
Along the axis OY in m.				

Determine the probability of a hit on the quadrilateral, if $B\delta = 23$m, $B\sigma = 5.5$m and the axis OX is coincident with the direction of the fire.

Answer. 11%.

PROBLEM 161. Determine the probability of a hit on a stationary
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tank when fire is frontal (fig. 117), if the mean trajectory passes

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274. ... AM = 2m, MH = 5m, MO_1 = 2.6m, BM = 1.8m, BC = 1m, CD = 1.5m, DE = 4m, FK = 2m and O_1O = 0.9m

Fig.117.

Answer. 49%.

PROBLEM 162. A battery is conducting a frontal fire, in the form of a concentrated sheaf, against a stone wall 4m in height and 2.5m in thickness. What is the probability of a hit on this wall, if $B\beta$ = 23m, angle $\theta_c = 11^{\circ}20'$ and the mean trajectory passes 5m in front of the wall?

Answer. 20.1%.

PROBLEM 163. It is required to determine the probability of a hit on a stationary tank the dimensions of which are 5x2x1.7m, if the mean trajectory passes through the base of the tank, $B\beta$ = 13m, $B\delta$ = 1.2m and angle $\theta_c = 4^{\circ}24'$. The plane of fire is directed along the tank.

Answer. 17.3%.

PROBLEM 164. The embrasure of a long duration firing point, of 0.3x0.9m, is covered by the primary rectangle of dispersion. What is the probability of a hit on the embrasure, if $B\beta$ = 0.4m and $B\delta$ = 0.5m?

Answer. 8.4%

PROBLEM 165. The target is a circular blindage of radius 2m. It is required to determine the probability of a hit on the target, if $B\beta$ = 20m, $B\delta$ = 3m and the blindage is covered by the primary ellipse of dispersion.

Answer. 1.35%.

PROBLEM 166. The target is a trench extending 5m along the front and 10m in depth. The mean trajectory passes 15m in front of the near side of the trench. What is the probability of a hit on the trench, if the median error in determination of the position of the mean trajectory in distance is equal to 18m, $B\beta$ = 25m and $B\delta$ = 2.5m?

Answer. 4.1%.

PROBLEM 267. The target is a blindage, the dimensions of which are: 8m along the front and 12m in depth. Frontal fire is conducted. The mean trajectory passes through the centre of the near side of the blindage. It is known, that the median errors in determination of the target position with respect to the mean trajectory are as follows: in distance - $0.5B\beta$ and in direction - $0.8B\delta$. What is the probability of a hit on the blindage, if $B\beta$ = 30m and $B\delta$ = 5m?

Answer. 2.9%.

275. to 280. Appendix 1.

Table of values $\Phi(\beta)$.

Probability of occurrence of an error within the limits from 0 to $\pm\beta$.

280a. Appendix 2.

Grid of circular dispersion.

To scale in 0,5 of the middle deviation.

The numerals in the squares, the sides of which are equal to 0.5 of the middle deviations, give the probability in hundredths of a percent (0.01%).

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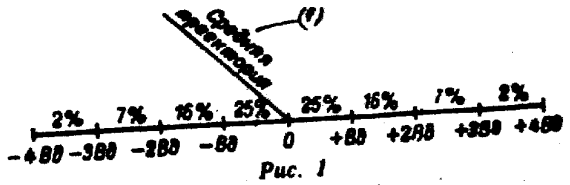

Рис. 1

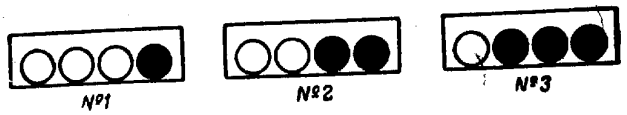

Рис. 2

...шаров в ящике
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пара,
лого
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№ 2
шары
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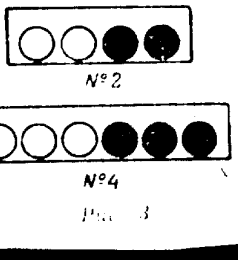

Рис. 3

2 красных и 3 белых, шары ... проиграть — выигрывает, если пую 1 шар и может при этом выиграть или проиграть — выигрывает, если вытянет шар с № 1 или красный шар. Какова вероятность выигрыша?

Рис. 4

Решение. Всех возможных случаев появления шара — 5. Случаев, благоприятствующих выигрышу, на первый взгляд 4: два шара с № 1 и два красных шара. Отсюда вероятность выигрыша как будто

$$P = \frac{4}{5}.$$

Но такое решение неверно, так как два случая, благоприятствующих

... лив для любой группы несовместных событий.

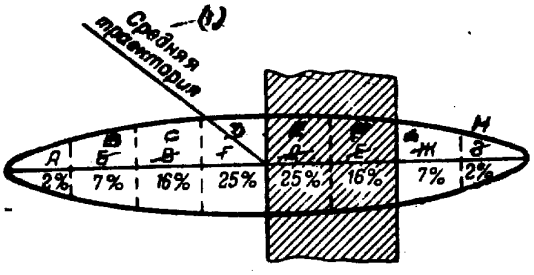

Рис. 5

Воспользуемся для этого следующим частным примером. В закрытом ящике имеется 50 шаров, отличающихся друг от друга только цветом: 8 белых, 10 зеленых, 5 синих, 12 черных

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точка, т. е. предполагается получение только недолетов и перелетов.

Решение. Недолеты дадут все те снаряды, которые придутся на полосы *А*, *Б* и *В* (рис. 6). Отсюда вероятность недолета при 1 выстреле сложится из вероятностей попадания в полосу *А*, полосу *Б* и полосу *В*, т. е. будет равна

$$P = p_A + p_Б + p_В = 0{,}02 + 0{,}07 + 0{,}16 = 0{,}25.$$

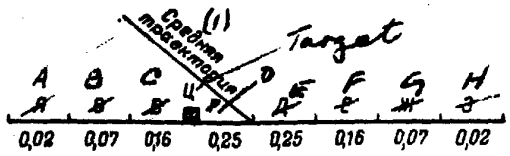

Рис. 6

шара. Для удобства дальнейших рассуждений занумеруем шары в обоих ящиках (рис. 7).

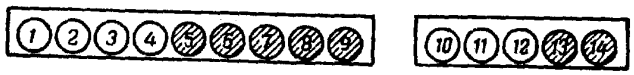

Рис. 7

Зададимся, исходя из этих условий, каким-либо сложным

шаров второго ящика может быть по

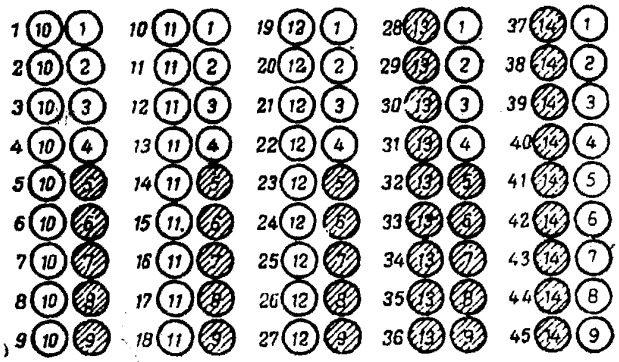

Рис. 8

Пример. Средние траектории, соответствующие прицелам 100 и 102, занимают положение, показанное на рис. 9 ($\Delta X = 50$ *м*, $Вд = 25$ *м*). Какова вероятность захватить цель в вилку, на ближнем пределе которой должно быть 2 недолета, а на дальнем — 2 перелета?

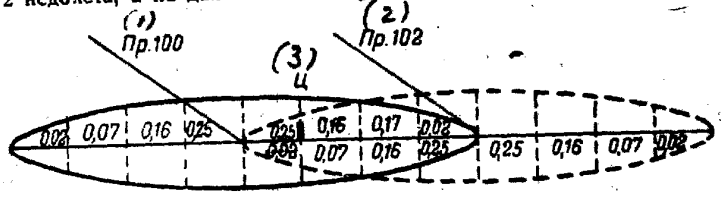

Рис. 9

Решение. Вилка, на ближнем пределе которой получено 2 недолета, а на дальнем — 2 перелета, — событие сложное, состоящее из двух сложных событий, каждое из которых состоит из двух простых событий.

Найдем сначала вероятность получения одного составляющего слож-

Если первым был вынут бе

вления затем красного шара бу

случаев, бл

ярного ш

тится толь

случаев.

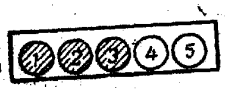

Рис. 10

Для удо

занумеруем

Рассуждая так же, как пр

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независимых

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Число случаев, благоприятствующих интересующего нас сложного события, должно быть равно числу совпадений двух красных шаров. Таких совпадений по нашей схеме—6: № 1 с № 2, № 1 с № 3, № 2 с № 1, № 2 с № 3, № 3 с № 1 и № 3 с № 2.

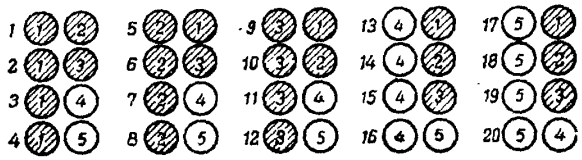


Рис. 11

Отсюда вероятность вынуть подряд 2 красных шара, не возвращая в ящик шар, вынутый первым, по общей формуле равна:

$$m \quad 6$$

10 *см*, на котором сделано 10 зарубок — по одной на каждый сантиметр. Допустим также, что при падении стержень ламывается на 2 части и этот перелом равновозможен на любой из этих зарубок. Определим вероятность перелома стержня на участке *CD* длиной 3 *см* (рис. 12).

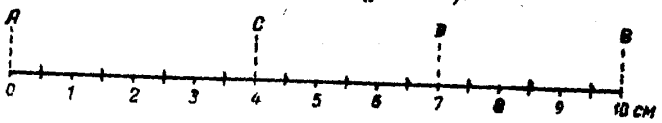


Рис. 12

Всех возможных (единственно возможных, несовместных равновозможных)

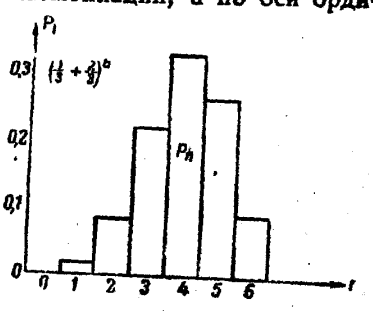
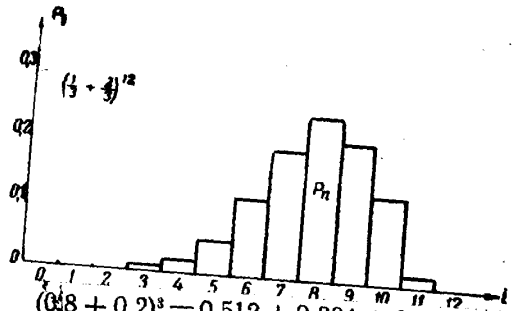


Рис. 13

малых величин, подсчетов и сравнения пренебрежимо



$$(0,8+0,2)^3 = 0,512 + 0,384 + 0,096 + 0,008 = 1,0;$$

$$(0,1+0,9)^4 = 0,0001 + 0,0036 + 0,0486 + 0,2916 + 0,6561 = 1,0.$$

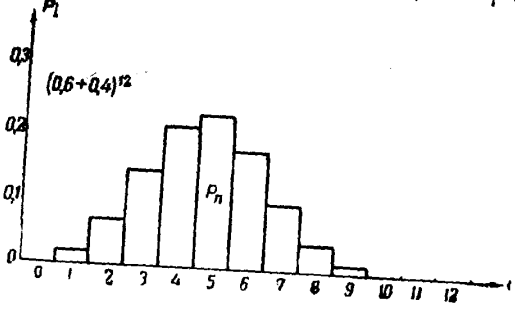


Рис. 15

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по 1 *см*, равное числу всех возможных комбинаций, а по оси

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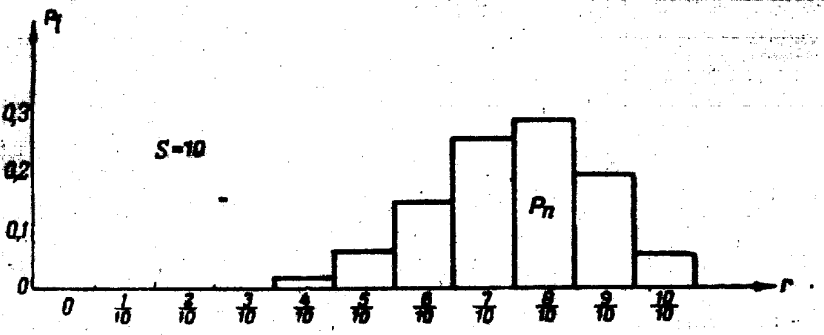


Рис. 16

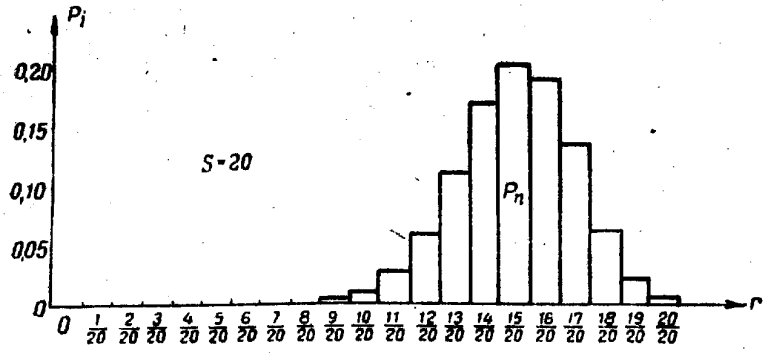


Рис. 17

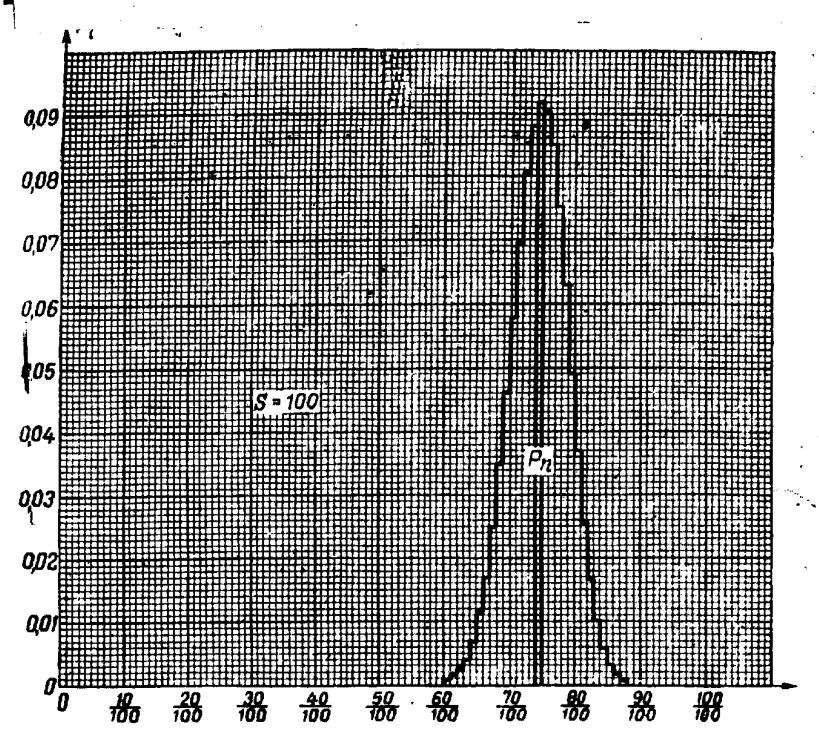
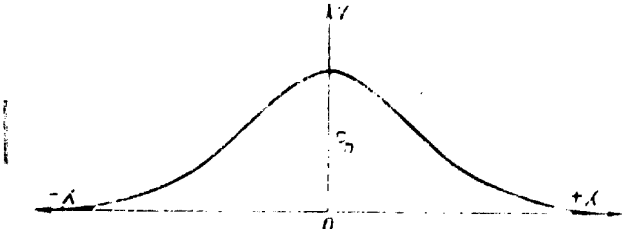


Рис. 18



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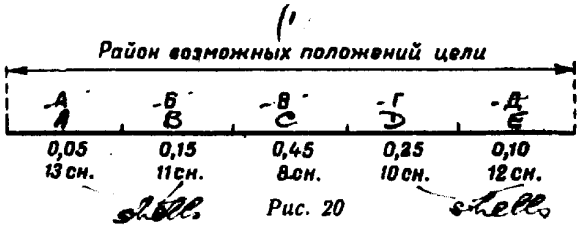


Рис. 20

Р е ш е н и е. Здесь переменная величина — количество снарядов, расходуемое на пристрелку. Частные значения этой величины 13, 11, 8, 10 и 12. чения в зависимости от того, где проходит средняя траектория

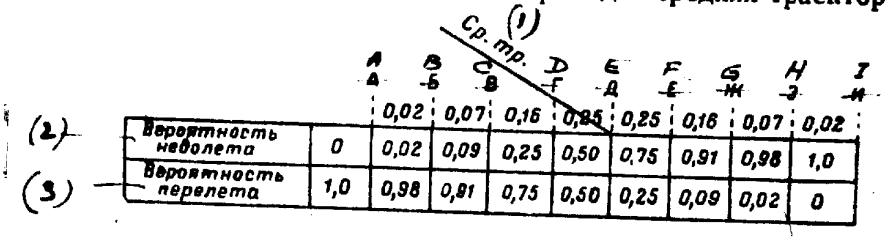


Рис. 21

относительно средней траектории или различные положения

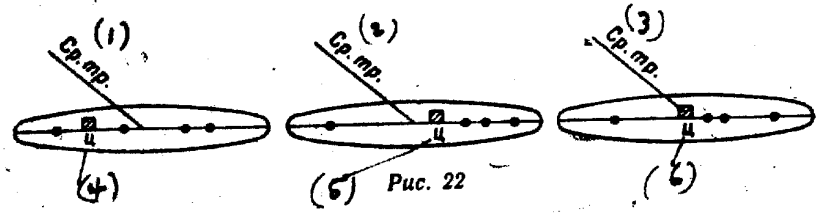


Рис. 22

цели. Так, если на некото-

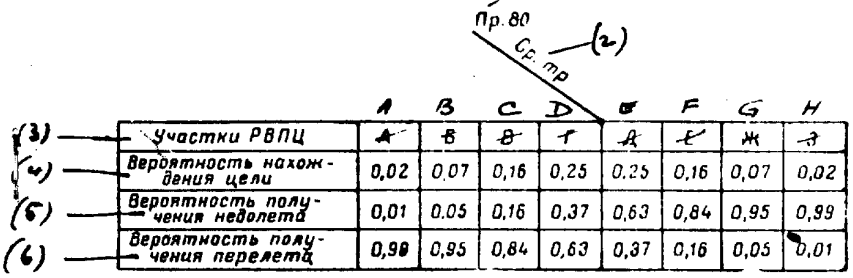
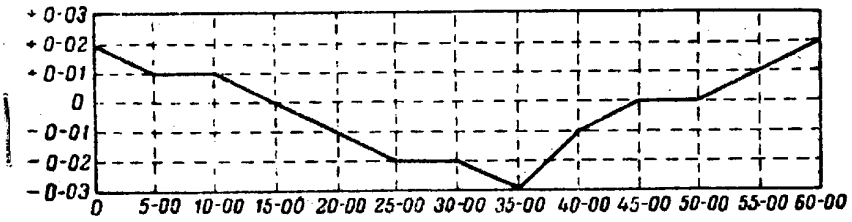

Пр. 80
Ср. тр
A B C D E F G H
Участки РВПЦ
Вероятность нахождения цели
Вероятность получения неполета
Вероятность получения перелета
0,02 0,07 0,16 0,25 0,25 0,16 0,07 0,02
0,01 0,05 0,16 0,37 0,63 0,84 0,95 0,99
0,98 0,95 0,84 0,63 0,37 0,16 0,05 0,01

Рис. 23

зуясь этим графиком, мы можем исключить ошибку данной стереотрубы



ном масштабе, а по оси ординат в другом условном масштабе частости получения ошибок в этих пределах (рис. 25).

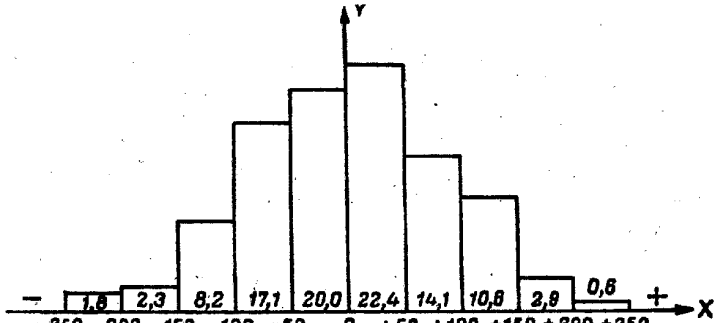


Рис. 25

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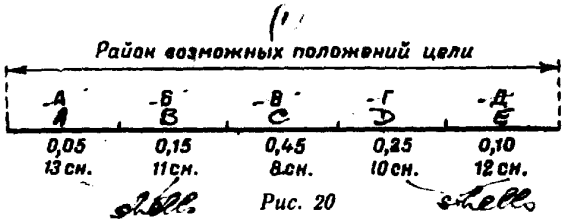

Рис. 20

Р е ш е н и е. Здесь переменная величина — количество снарядов, расходуемое на пристрелку. Частные значения этой величины 13, 11, 8, 10 и 12. По условию расход снарядов на пристрелку определяется нахождением цели

Ср. пр.	А А	В Б	С В	D Г	Е Д	F Е	G Ж	H З	I И
	0,02	0,07	0,16	0,25	0,25	0,16	0,07	0,02	
(2) Вероятность недолета	0	0,02	0,09	0,25	0,50	0,75	0,91	0,98	1,0
(3) Вероятность перелета	1,0	0,98	0,91	0,75	0,50	0,25	0,09	0,02	0

Рис. 21

относительно цели. Так, если цель находится в точке *А* (рис. 21), вероятность недолета равна 0, а вероятность пере-

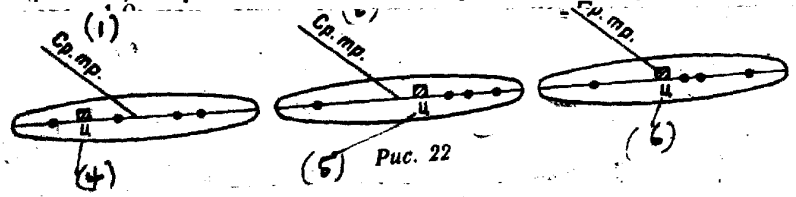

Рис. 22

Пр. 80 / Ср. пр.	А	В	С	D	Е	F	G	H
(3) Участки РВПЦ	А	Б	В	Г	Д	Е	Ж	З
(4) Вероятность нахождения цели	0,02	0,07	0,16	0,25	0,25	0,16	0,07	0,02
(5) Вероятность получения недолета	0,01	0,05	0,16	0,37	0,63	0,84	0,95	0,99
(6) Вероятность получения перелета	0,99	0,95	0,84	0,63	0,37	0,16	0,05	0,01

Рис. 23

зуясь этим графиком, мы можем исключить ошибку данной стереотрубы

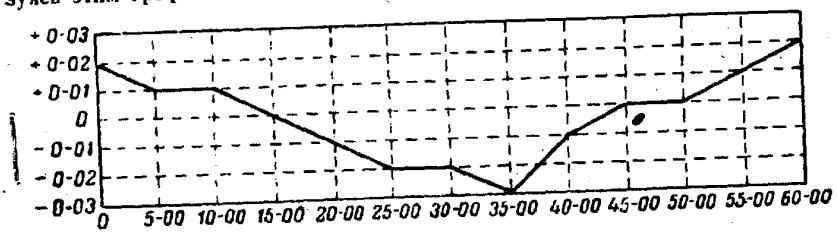

Рис. 24

из полученного отсчета. Так, если отсчет оказался равным 25-09, то

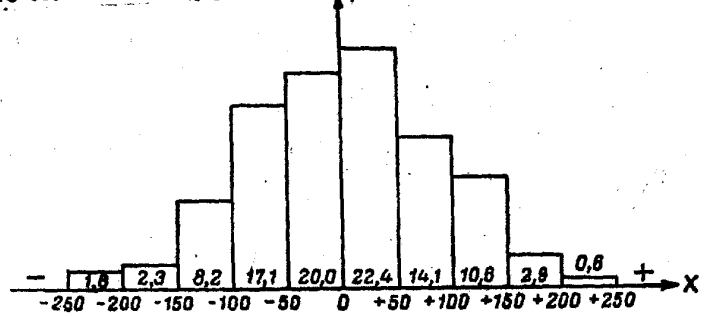

Рис. 25

OVER

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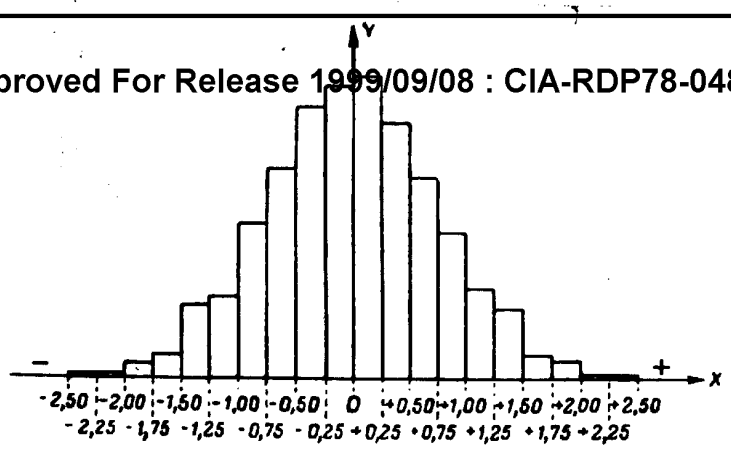

Рис. 26

абсолютной величине и разных по знаку ошибок должны быть равны между собой (рис. 27).

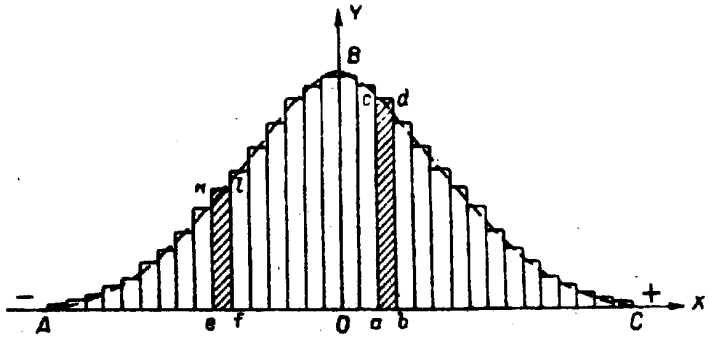

Рис. 27

некоторых пределах, [illegible] пример, о вероятности получения ошибки в пределах от 4,99999 и до 5,00001 *м* и т. п.

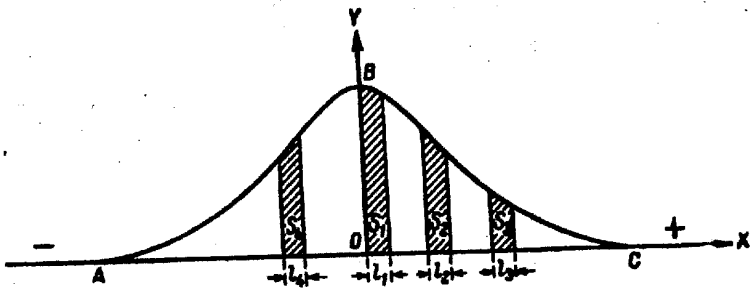

Рис. 28

Кривая закона Гаусса очень наглядно подтверждает все уже известные нам основные положения этого закона. Положим, что разности пределов ошибок, вероятности получения которых выражены площадями s_1, s_2, s_3 и s_4, равны между собой (рис. 28);

оснований другой, т. е. необходимо соблюсти примерно тот принцип построения, который ранее был применен при построении графиков вероятностей комбинаций.

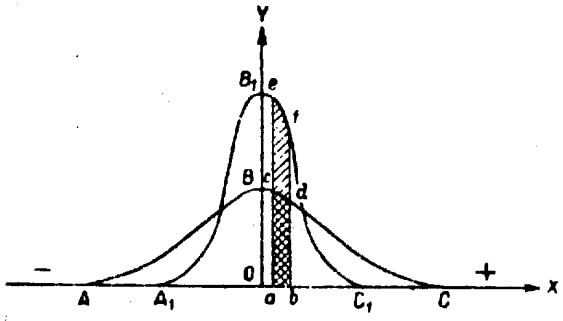

Рис. 29

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На рис. 29 показаны кривые закона Гаусса для двух различных способов (рядов) измерения.

Этот рисунок позволяет нам сделать следующие заклю-

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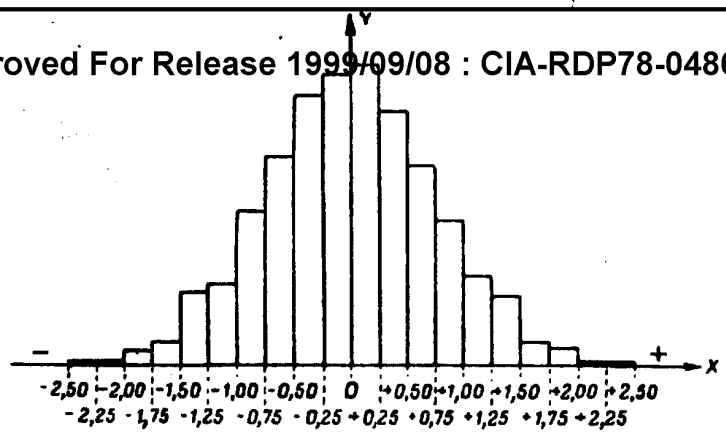

Рис. 26

Мы построили два графика по результатам большого и вместе с тем различного числа измерений, произведенных раз-

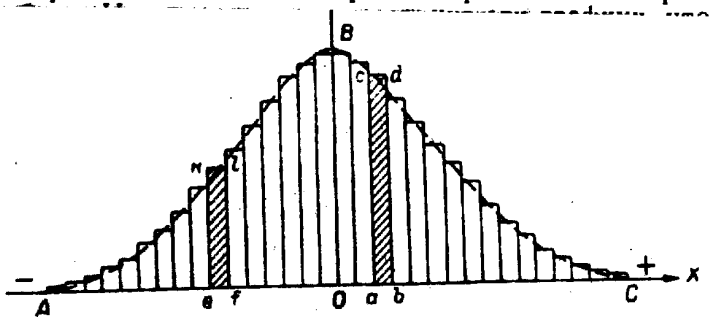

Рис. 27

3. *Для каждого способа измерения существует свой*

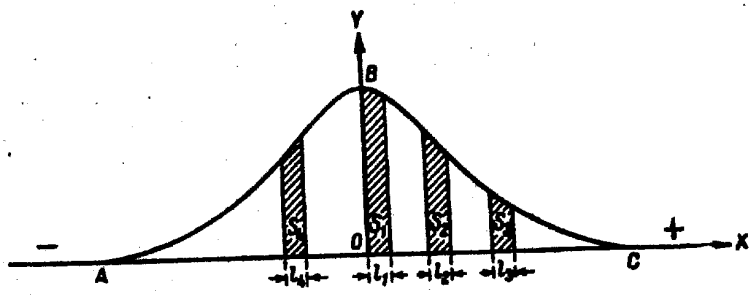

Рис. 28

Кривая закона Гаусса очень наглядно подтверждает все уже известные нам основные положения этого закона. Положим, что разности пределов ошибок, вероятности получения которых выражены площадями S_1, S_2, S_3 и S_4, равны между собой (рис. 28):

принцип построения, который ранее был применен при построении графиков вероятностей комбинаций.

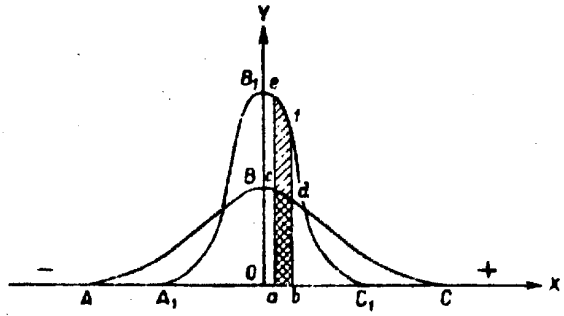

Рис. 29

На рис. 29 показаны кривые закона Гаусса для двух различных способов (рядов) измерения.

Этот рисунок позволяет нам сделать следующие заклю-

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другого способа, если, конечно, масштабы.

бок Гаусса дает так называемый

В
количе
Эти др
плоско
на сво
ней (б
дно яш
играю
ников
бинки,
ударе
ния, по
Так ка
и связ
нения
носят,
ракте
распр
кам,
нием
рения
закону Гаусса

знаку, равновероятны;

— ошибки по своей величине не выходят за пределы ±8.

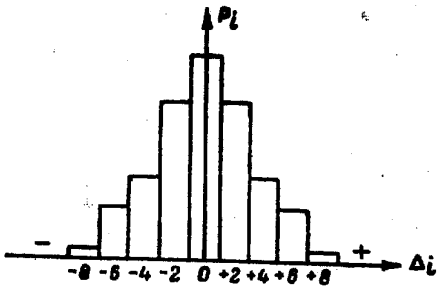

Рис. 31

А это значит, что мы пришли к тем же трем положениям закона случайных ошибок Гаусса, которые ранее установили опытным путем, — неравномерность, симметричность и небеспредельность.

Более наглядно эти же три положения выражает график, построенный по данным табл. 7 (рис. 31).

в виде графика (рис. 32).

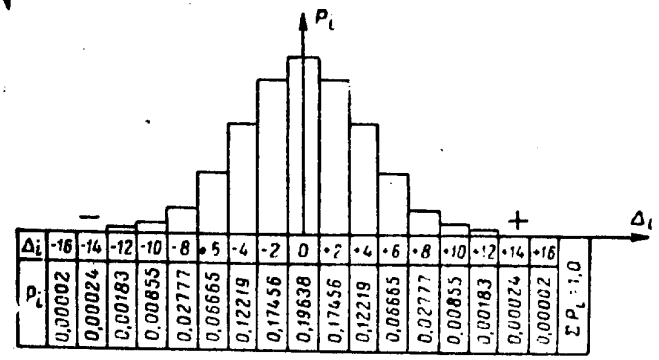

Рис. 32

Этот график еще полнее выражает положения закона

в табл. 8 добавить вертикальные графы для вероятностей

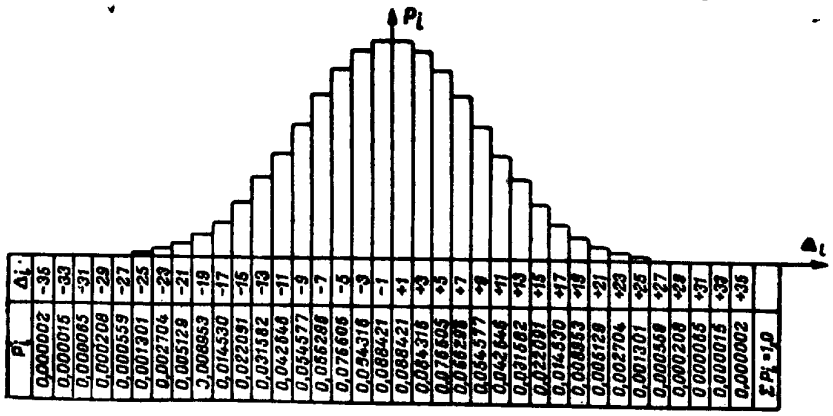

Рис. 33

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масштабы.

Очень наглядное представлени
Этот график дает так называемый п

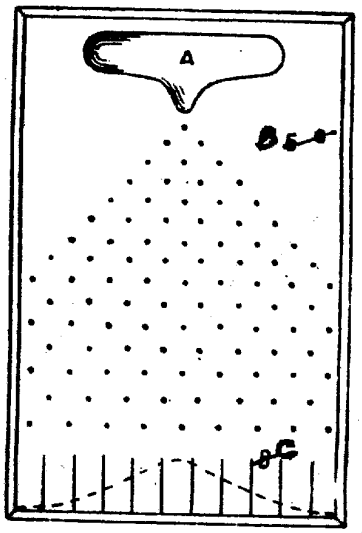


Рис. 30

В
количе
Эти др
плоско
на сво
ней (бу
дно ящ
играю
ников
бинки,
ударе
ния, по
Так ка
и связ
нения
носят,
ракте
распр
кам,
нием
рения

знаку, равновероятны;

— ошибки по своей величине не выхо

А эт
пришли
ложения
ошибок
ранее ус
путем, –
симметри
дельност

Боле ошибки следуют закону Гаусса
три положения выражает
график, построенный по дан-
ным табл. 7 (рис. 31).

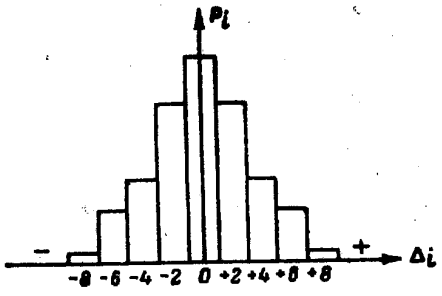
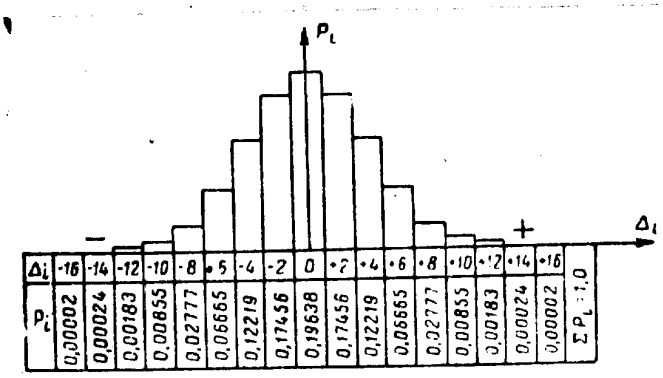


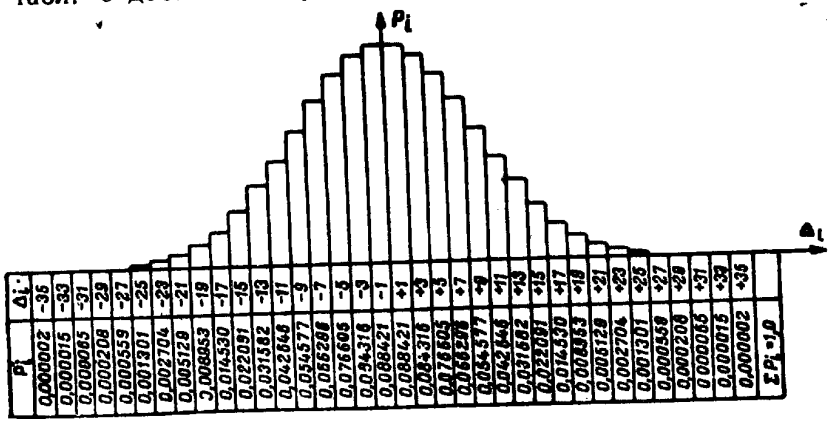
Рис. 31



Δ_i	-16	-14	-12	-10	-8	-6	-4	-2	0	+2	+4	+6	+8	+10	+12	+14	+16	
P_i	0,00002	0,00024	0,00183	0,00855	0,02777	0,06665	0,12219	0,17456	0,19638	0,17456	0,12219	0,06665	0,02777	0,00855	0,00183	0,00024	0,00002	$\Sigma P_i = 1,0$

Рис. 32

Этот график еще полнее выражает положения закона
в табл. 8 добавить вертикальные графы для вероятностей



Δ_i	-35	-33	-31	-29	-27	-25	-23	-21	-19	-17	-15	-13	-11	-9	-7	-5	-3	-1	+1	+3	+5	+7	+9	+11	+13	+15	+17	+19	+21	+23	+25	+27	+29	+31	+33	+35	
P_i	0,000002	0,000015	0,000085	0,000208	0,000559	0,001301	0,002704	0,005128	0,008853	0,014530	0,022091	0,031582	0,042646	0,054577	0,066286	0,076606	0,084316	0,088621	0,088621	0,084316	0,076606	0,066286	0,054577	0,042646	0,031582	0,022091	0,014530	0,008853	0,005128	0,002704	0,001301	0,000559	0,000208	0,000085	0,000015	0,000002	$\Sigma P_i = 1,0$

Рис. 33

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их величине порядке (рис. 34) таким средним (срединным) оказалось девятое яблоко.

Рис. 34

Занимая срединное место, это яблоко по своей величине больше каждого из яблок одной (правой) половины и меньше

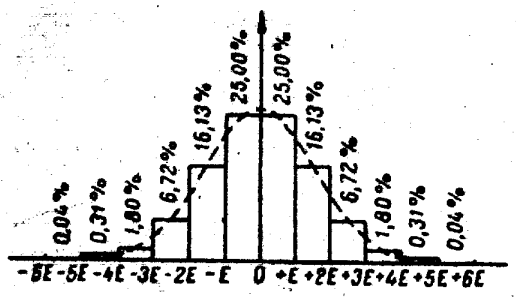

Рис. 35

может быть выражен шкалой, изображенной на рис. 36.

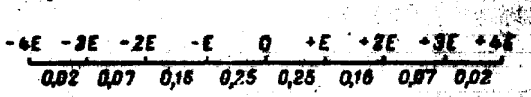

Рис. 36

Эта шкала называется *шкалой ошибок*. Такой шкалой ошибок, как правило, пользуются при расчетах, связанных шкалой ошибок [illegible] артиллерийской стрельбы. упрощает технику расчетов. [illegible]ерждение основ-

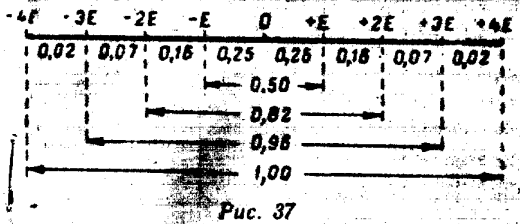

Рис. 37

ошибки (рис. 38).

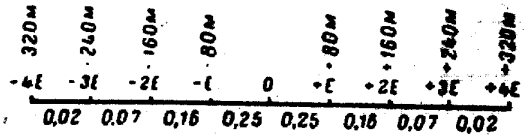

Рис. 38

При рассмотрении этой шкалы можно сделать следующие выводы.

[illegible] предела (рис. 39).

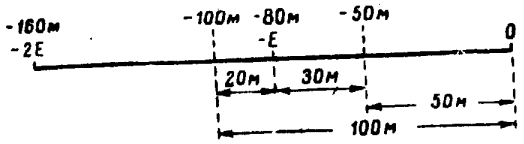

Рис. 39

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OVER

Для [illegible] величиной E ошибки распределе[illegible]

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их величине, порядке (рис. 34) таким средним (срединным) оказалось девятое яблоко.

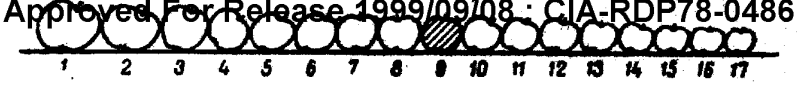

Рис. 34

Занимая срединное место, это яблоко по своей величине больше каждого из яблок одной (правой) половины и меньше каждого из яблок другой (левой) половины всей партии яблок. По аналогии, этим же свойством характеризуется и срединная ошибка.

Срединной ошибкой называется такая ошибка, которая по своей абсолютной величине больше каждой из ошибок

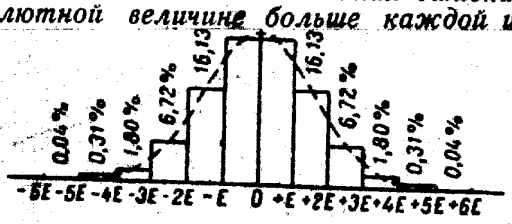

Рис. 35

может быть выражен шкалой, изображенной на рис. 36.

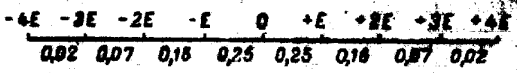

Рис. 36

Эта шкала называется *шкалой ошибок*. Такой шкалой ошибок, как правило, пользуются при расчетах, связанных с теоретическими обоснованиями артиллерийской стрельбы.

По шкале ошибок можно видеть подтверждение основных положений закона Гаусса. Действительно:

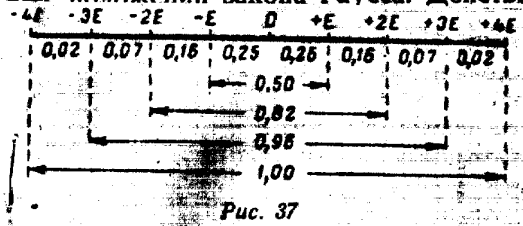

Рис. 37

Шкала ошибок, какой бы вид она ни имела, дает возможность решить сле-

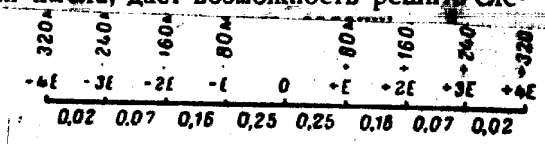

Рис. 38

При рассмотрении этой шкалы можно сделать следующие выводы.

предела (рис. 39).

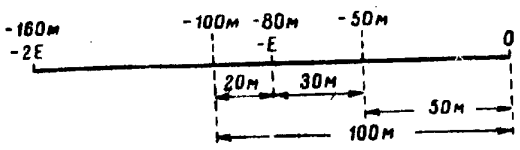

Рис. 39

Допустив, что в пределах каждой *E* ошибки распреде-

OVER

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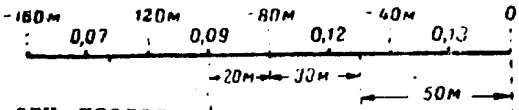
Несколько более точный ответ будет получен, если взять шкалу ошибок, выраженную в долях E, например в $0{,}5E$ (рис. 40).

$$p_1 = \frac{30 \cdot 0{,}12}{40} = 0{,}09; \qquad p_2 = \frac{20 \cdot 0{,}09}{40} = 0{,}045;$$

$$P = 0{,}09 + 0{,}045 = 0{,}135 = 13{,}5\%.$$



) до $\pm\beta$, причем эти пределы выражены в сотых долях E (через $0{,}01E$). Ход решения при пользовании таблицей значений $\Phi(\beta)$ таков (рис. 41):

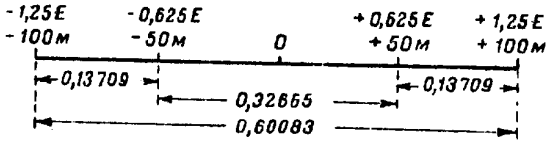
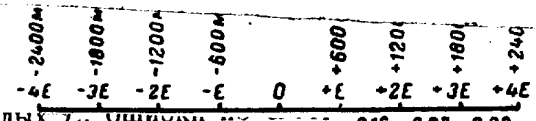


Рис. 41

аклю-



ыраженных в целых E, ошибки [illegible] 0,02
ляются *равномерно*, т. е. для этих пределов можно
ие значения ошибки (рис. 43), выраженные в доля

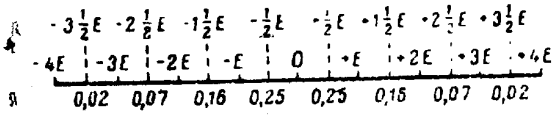


Рис. 43

$\left(-\frac{1}{2}\right)$

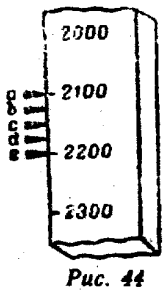


Рис. 44

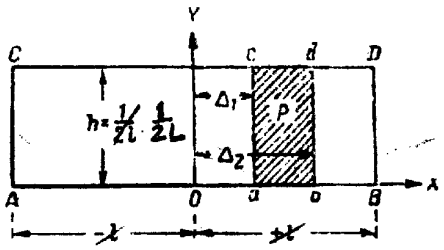


Рис. 45

Выражая ошибки Δ_1, Δ_2, [illegible] для этого одно деление шкалы ($2l$) на 8 равных частей — прета [illegible] ой [illegible] ки

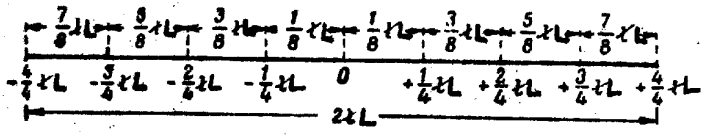


Рис. 46

асчетах при

елов (рис. 46) и возьмем для каждого такого предела ср [illegible] ошибки. Так, для предела от 0 до $\pm$

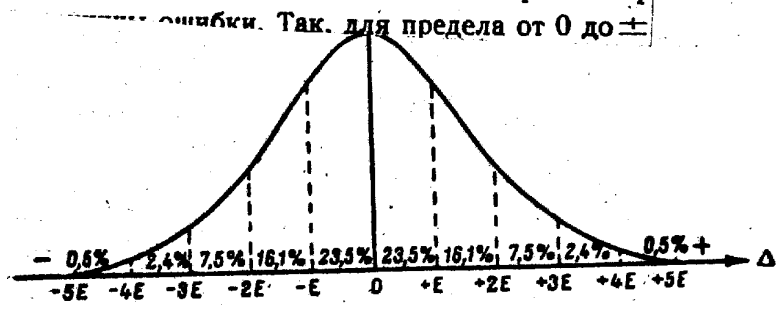


Рис. 47

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Как мы видим, распределение вероятностей [illegible] суммарном законе очень мало отличается от распределения, свойственного

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Рассмотрим, какой вопрос будет получен, если взять шкалу ошибок, выраженную в долях E, например в $0{,}5E$ (рис. 40).

$$p_1 = \frac{30 \cdot 0{,}12}{40} = 0{,}09; \qquad p_2 = \frac{20 \cdot 0{,}09}{40} = 0{,}045;$$

$$P = 0{,}09 + 0{,}045 = 0{,}135 = 13{,}5\%.$$

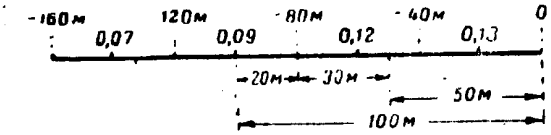
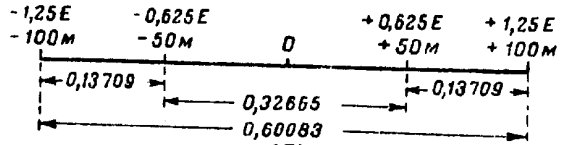


Рис. 40

чений $\Phi(\beta)$ таков (рис. 41).



срединной ошибке (рис. 42), можно сделать следующее заключение.

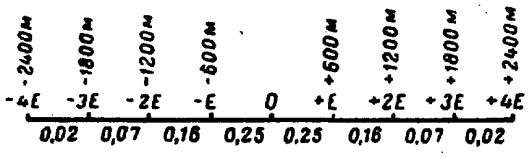


Рис. 42

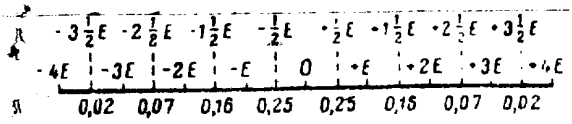


Рис. 43

ак, для пределов от 0 до $\pm E$ такое среднее зн
ки может быть принято равным $\pm \frac{1}{2}E$, для пределов
$0E$ может быть принято равным $\pm 1\frac{1}{2}E$ и т. д.

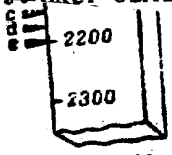


Рис. 44

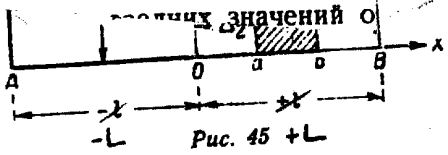


Рис. 45

Совершенно очевидно, что указатель при снятии отсчета *с одинаковой вероятностью* может оказаться против любой точки шкалы в пределах ее деления. Следовательно, *ошибки*

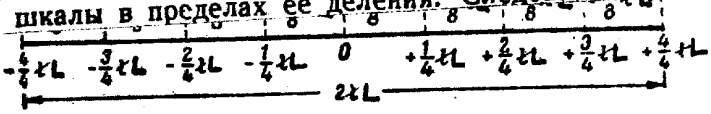


Рис. 46

Произведя расчеты, аналогичные расчетам при $l = \frac{E}{2}$, получим график такого суммарного закона (рис. 47).

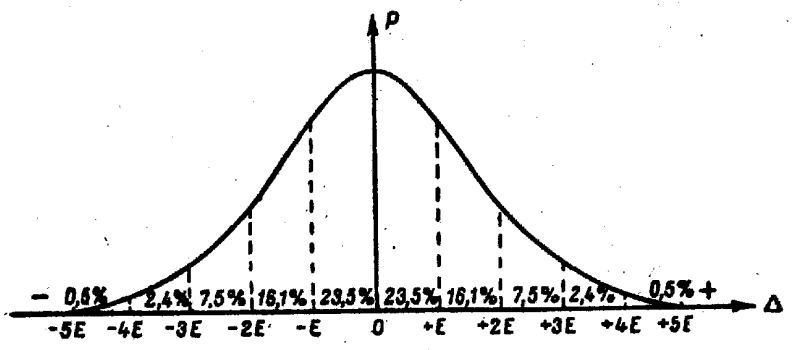


Рис. 47

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Как мы видим, распределение вероятностей получения ошибки в этом суммарном законе очень мало отличается от распределения, свойственного

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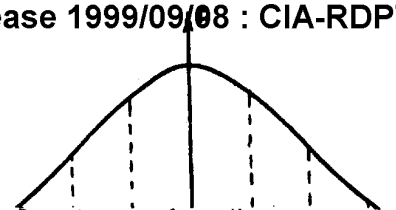
может быть принят за закон Гаусса.

При $l > E$ суммарный закон уже не будет законом Гаусса.



при $l = 4E$, суммарный закон даже в графическом выражении не напоминает закон Гаусса (рис. 49).

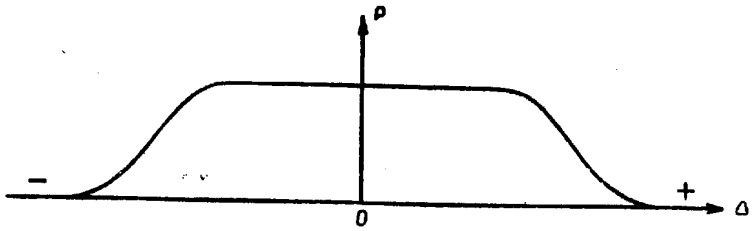


Рис. 49

ее в. К=10 м (рис. При сложении закона Гаусса с законом равной вероятности средняя

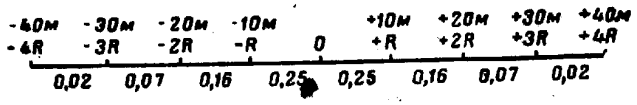


Рис. 50

и за-
ения

При рассмотрении этой шкалы можно сделать следующие выводы:

1. Практически предельной ошибкой среднего результата можно считать

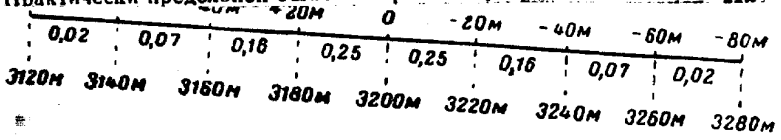


Рис. 51

ения

б) вероятность того, что истинное значение дальности находится

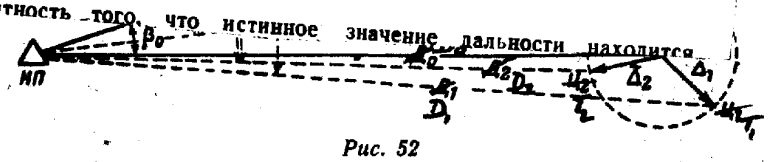


Рис. 52

Следовательно, только зная величину, знак и направление

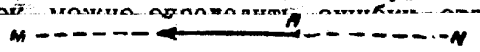


Рис. 53

На рис. 53 AB есть величина ошибки-вектора, A — точка ее приложения и MN — направление прямой, на которой лежит

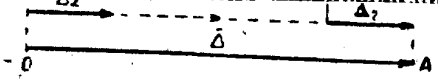


Рис. 54

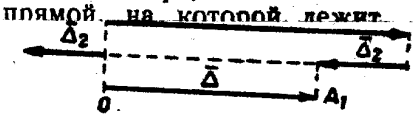


Рис. 55

Геометрически эта ошибка будет выражена отрезком $\overline{OA_1}$ (рис. 55), равным

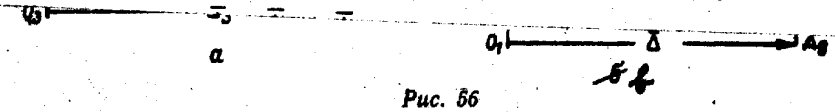


Рис. 56

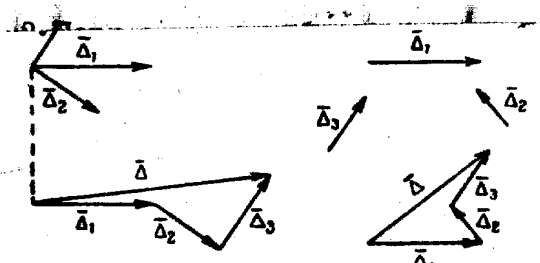


Рис. 57

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может быть принят за закон Гаусса.

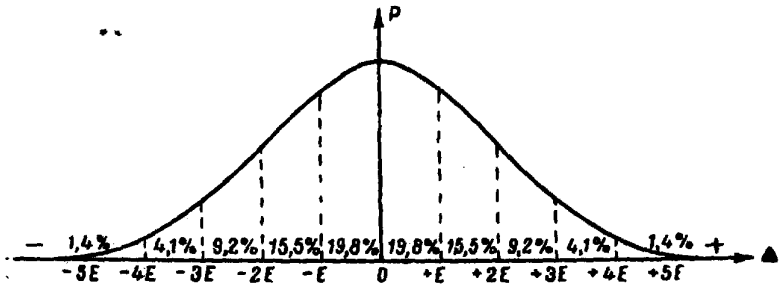

Рис. 48

Так, например, при $l = 2E$ графически суммарный закон хотя и выражается

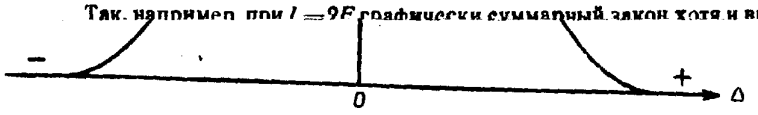

Рис. 49

При сложении закона Гаусса с законом равной вероятности средняя

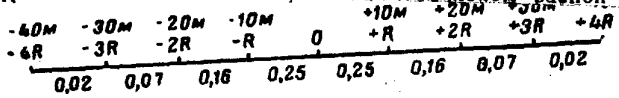

Рис. 50

При рассмотрении этой шкалы можно сделать следующие выводы:

1. Практически предельной ошибкой среднего результата можно считать

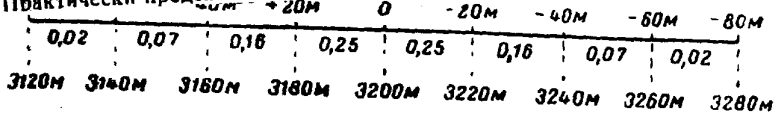

Рис. 51

2) вероятность того, что истинное значение дальности находится

Рис. 52

Следовательно, только зная величину, знак и направление

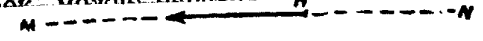

Рис. 53

На рис. 53 AB есть величина ошибки-вектора, A — точка ее приложения и MN — направление прямой, на которой лежит

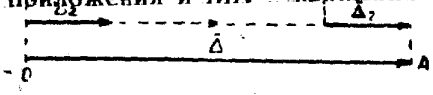

Рис. 54

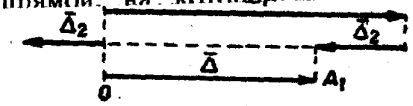

Рис. 55

Геометрически эта ошибка будет выражена отрезком $\overline{OA_1}$ (рис. 55), равным

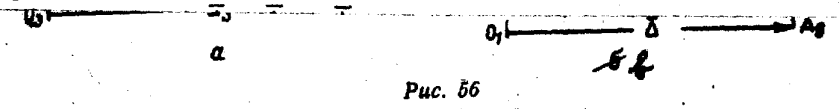

Рис. 56

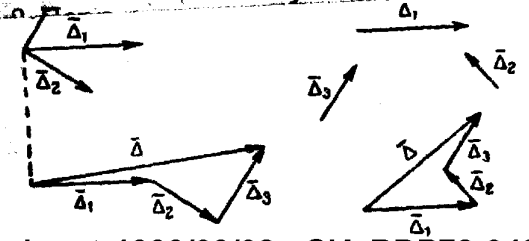

Рис. 57

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мой величины, т. е. результату, ошибка которого равна нулю.

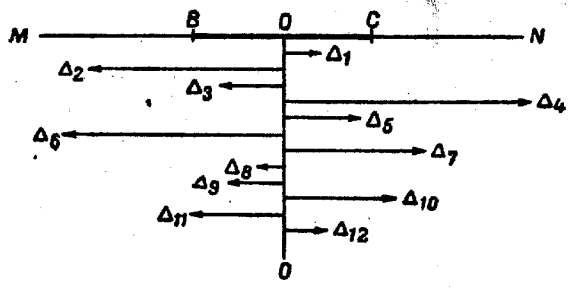

Рис. 59

Величина отрезка *OB* как величина срединной ошибки совокупности ошибок векторов, изображенных на рис. 59, может

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ми си-

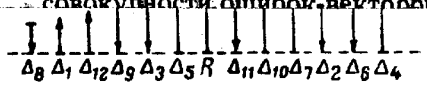


Рис. 60

будем иметь дело, — то сложение векто-

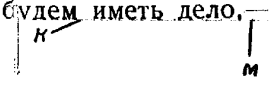


Рис. 61

прямоугольные треугольники *OA'A*, *OB'B* и

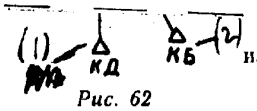


Рис. 62

или в деле

одна в определенной по-

получим
ториаль-

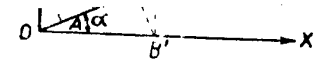


Рис. 63

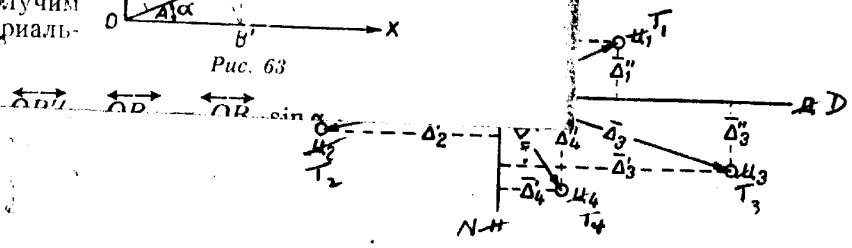


Рис. 64

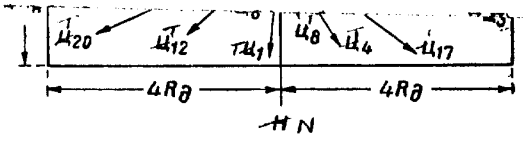
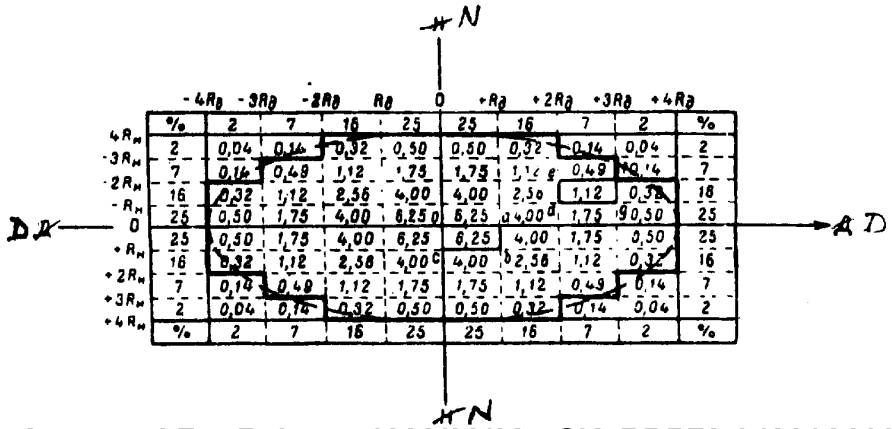


Рис. 65

следуют закону Гаусса.



	-4Rд	-3Rд	-2Rд	Rд	0	+Rд	+2Rд	+3Rд	+4Rд	
+4Rн	%	2	7	16	25	25	16	7	2	%
+3Rн	2	0,04	0,14	0,32	0,50	0,50	0,32	0,14	0,04	2
+2Rн	7	0,14	0,49	1,12	1,75	1,75	1,12	0,49	0,14	7
+Rн	16	0,32	1,12	2,56	4,00	4,00	2,56	1,12	0,32	16
0	25	0,50	1,75	4,00	6,25	6,25	4,00	1,75	0,50	25
-Rн	25	0,50	1,75	4,00	6,25	6,25	4,00	1,75	0,50	25
-2Rн	16	0,32	1,12	2,56	4,00	4,00	2,56	1,12	0,32	16
-3Rн	7	0,14	0,49	1,12	1,75	1,75	1,12	0,49	0,14	7
-4Rн	2	0,04	0,14	0,32	0,50	0,50	0,32	0,14	0,04	2
	%	2	7	16	25	25	16	7	2	%

Рис. 66

OVER

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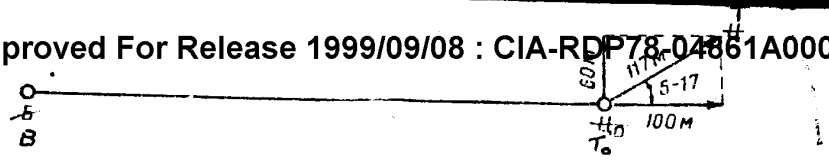

Рис. 58

3. Сложение ошибок-векторов, имеющих различные направ-

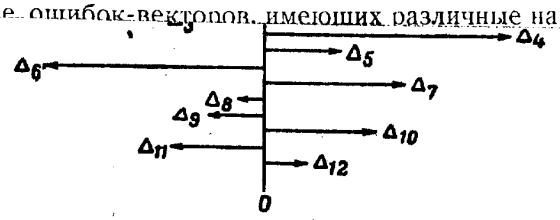

Рис. 59

Величина отрезка *OB* как величина срединной ошибки

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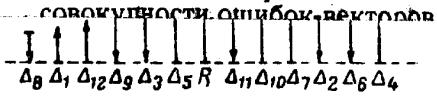


Рис. 60

будем иметь дело, — то сложение векто-

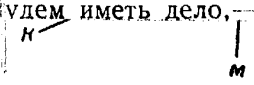
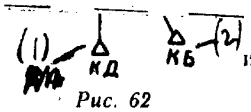


Рис. 61

прямоугольные треугольники *OA'A*, *OB'B* и



или в деле

Рис. 62

одна в определении по-

олучим
ториаль-

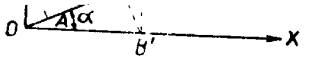


Рис. 63

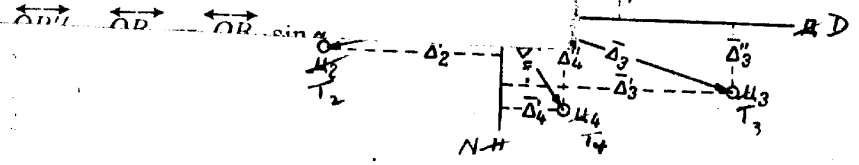


Рис. 64

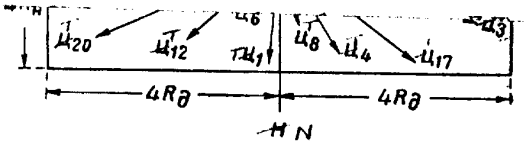


Рис. 65

Это означает, что ошибка подготовки исходных данных

	-4Rд	-3Rд	-2Rд	Rд	0	+Rд	+2Rд	+3Rд	+4Rд	
	%	2	7	16	25	25	16	7	2	%
4Rн	2	0,04	0,14	0,32	0,50	0,50	0,32	0,14	0,04	2
3Rн	7	0,14	0,49	1,12	1,75	1,75	1,12	0,49	0,14	7
2Rн	16	0,32	1,12	2,56	4,00	4,00	2,56	1,12	0,32	16
Rн	25	0,50	1,75	4,00	6,25	6,25	4,00	1,75	0,50	25
0	25	0,50	1,75	4,00	6,25	6,25	4,00	1,75	0,50	25
+Rн	16	0,32	1,12	2,56	4,00	4,00	2,56	1,12	0,32	16
+2Rн	7	0,14	0,49	1,12	1,75	1,75	1,12	0,49	0,14	7
+3Rн	2	0,04	0,14	0,32	0,50	0,50	0,32	0,14	0,04	2
+4Rн	%	2	7	16	25	25	16	7	2	%

Рис. 66

OVER

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...ксении векторциальных ошибок, ...го являются ... эти ошибки взаимно ... перпендикулярны. Если ... получается ошибка на плос... Такая ошибка называется эл...

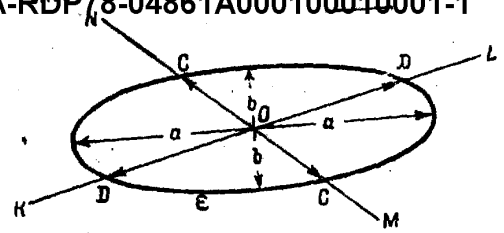
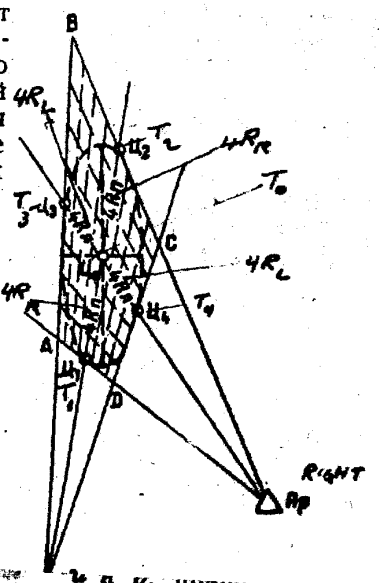


Рис. 68

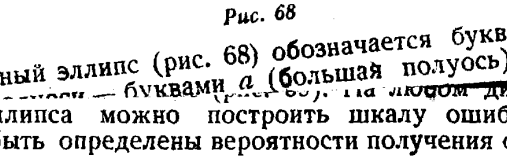
...будет ...ектори... ...взаимно ... такой ...ическая ...ультате ...гельных ...



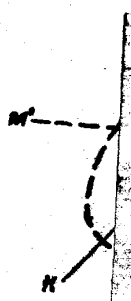
...аблюда... ...следуют ...аблюда... ...олютно ... причем ... наблю... ...го цель ...есенной ... наблю... ...и U_1 и ...оответ... ...ю цели,

...ри... ...а и b, причем главная полу...

...ер...



...немся к рис. 66, на котором было показано распред... ...прямоугольной системы координат (рис. 71).



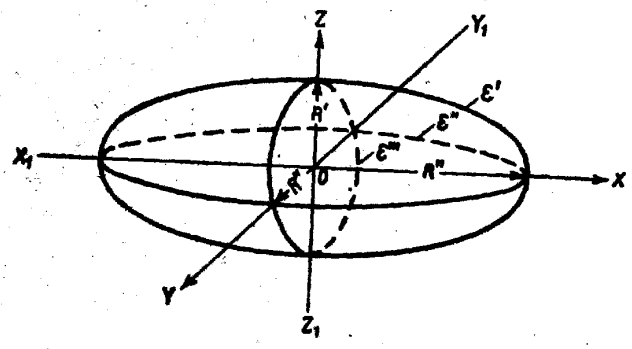


Рис. 71

...вание траек...

Сложим последовательно и попарно заданные вектори...

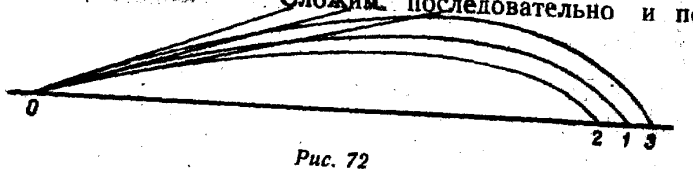


Рис. 72

Если стрельбу ведут при малых углах бросания, то можно ... изменении угла бросания траек...

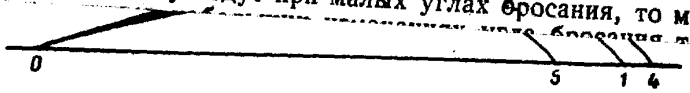


Рис. 73

...имели бы различную кривизну (рис. 73). Условимся и здесь, что траектория 1 соответствует случаю полного отсутствия ...

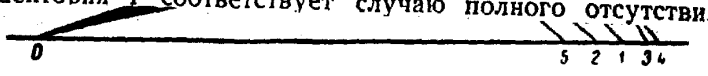
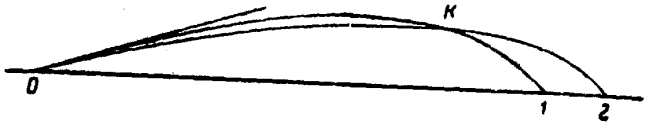


Рис. 74

...грузку, то, начиная с некоторого момента, он будет иметь ...



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CPYRGHT Approved For Release 1999/09/08 : CIA-RDP78-04861A000100010001-1

жении векториальных ошибок, мно перпендикулярным линиям получается ошибка на плот- Такая ошибка называется эл-

будет ектори- взаимно такой ическая ультате гельных

аблюда- следуют аблюда- олютно причем наблю- го цель есенной наблю- и $Ц_1$ и оответ- ю цели,

правый ерения

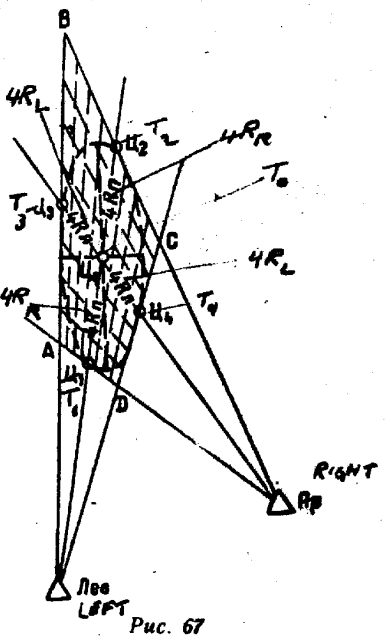

Рис. 67

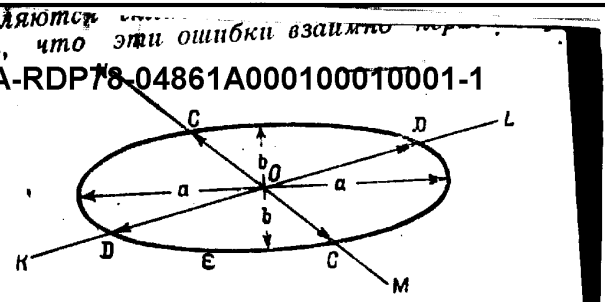

Рис. 68

иничный эллипс (рис. 68) обозначается буквой e, строить полный эллипс (рис. 69). На любом диаметр ого эллипса можно построить шкалу ошибок, п могут быть определены вероятности получения ошибо ых пределах по направлению этого диаметра.

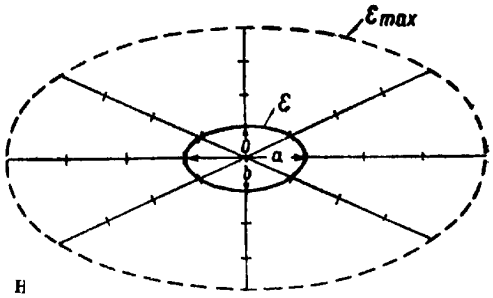

Рис. 69

ь a н
Допус
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ерпенд рис. 66, на котором было показано распред
$M'N'$ к координат (рис. 71).

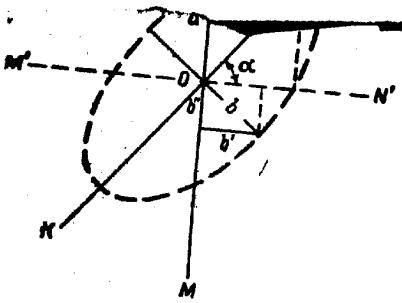

Рис. 70

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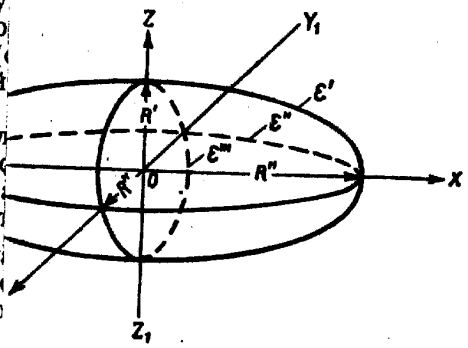

Рис. 71

проекция малой полуоси b на тот же

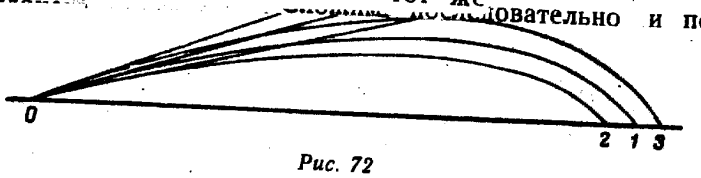

Рис. 72

Если стрельбу ведут при малых углах бросания, то можно

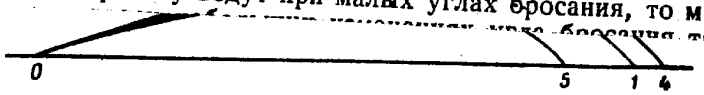

Рис. 73

имели бы различную кривизну (рис. 73). Условимся и здесь, что траектория *1* соответствует случаю полного отсутствия

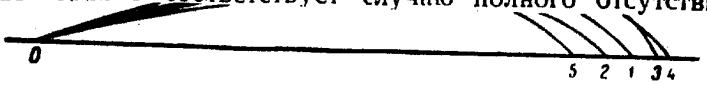

Рис. 74

грузку, то, начиная с некоторого момента, он будет иметь

Рис. 75

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сания (рис. 76): более тяжелый снаряд при большем ...

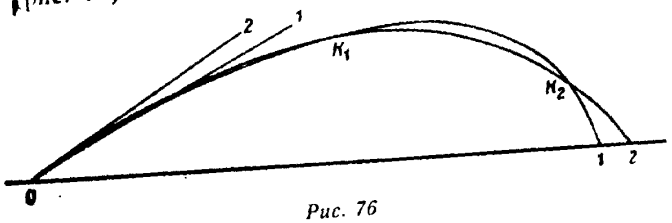
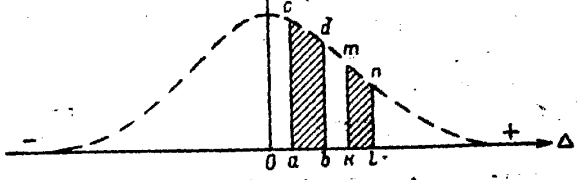


Рис. 76

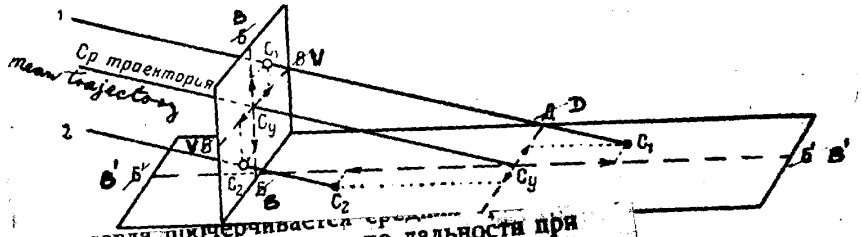
... отношение веса снаряда к его ...

Рис. 77

Средняя траектория — это такая воображаемая траектория, которая была бы получена при отсутствии причин, вызывающих рассеивание траекторий (траектории 1 на ...



чает центр рассеивания C_y (рис. 79).



рассеивания всегда прочерчивается ср...

с. 80 показана шкала рассеивания по дальности при

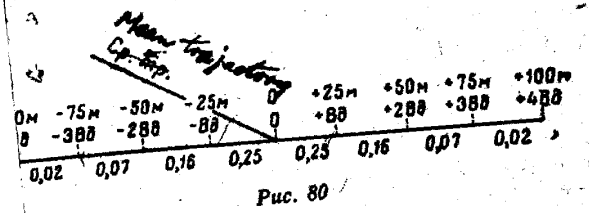


Рис. 80

...главе.

...но конец траек-
...ебольшом протяжении
...ь принят за прямую
...кно легко установить
... между срединным
...м по дальности и сре-
...клонением по высоте

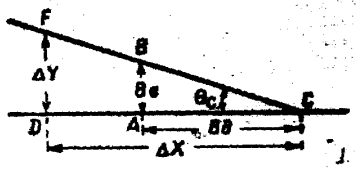


Рис. 81

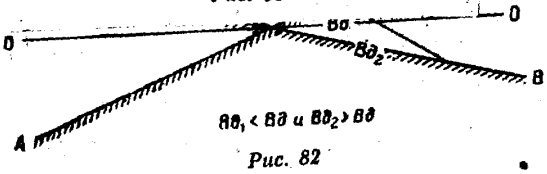


Рис. 82

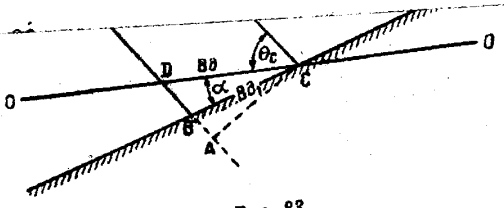


Рис. 83

Approved For Release 1999/09/08 : CIA-RDP78-04861A000100010001-1

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сания (рис. 76): более тяжелый снаряд при большем угле

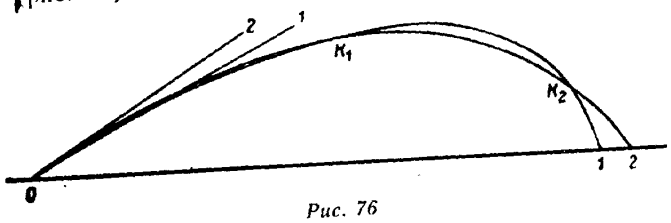

Рис. 76

[1] Поперечной нагрузкой называется отношение веса снаряда к его

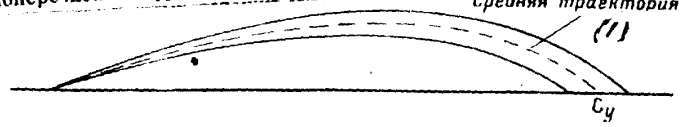

Рис. 77

Средняя траектория — это такая воображаемая траектория, которая была бы получена при отсутствии причин, вызывающих рассеивание траекторий (траектории 1 на

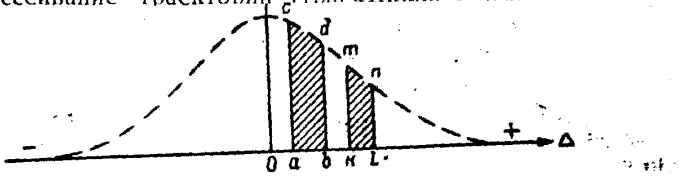

Рис. 78

Применительно к рассеиванию закон Гаусса может быть

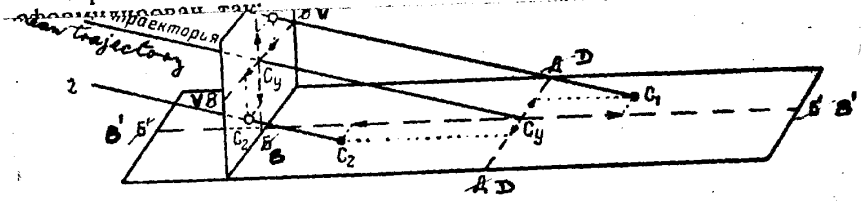

Рис. 79

15— Зак. 911

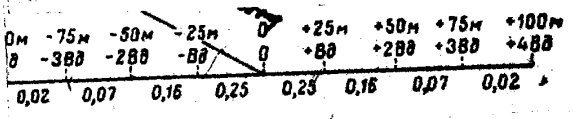

Рис. 80

...конец траек-
...большом протяжении
...принят за прямую
...легко установить
...между срединным
...по дальности и сре-
...клонением по высоте

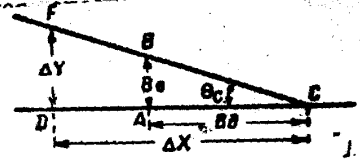

Рис. 81

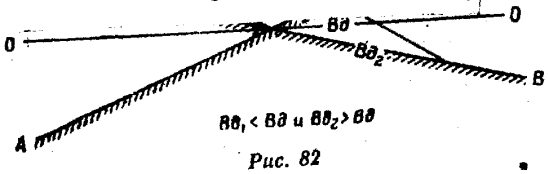

Рис. 82

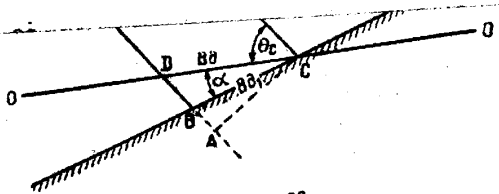

Рис. 83

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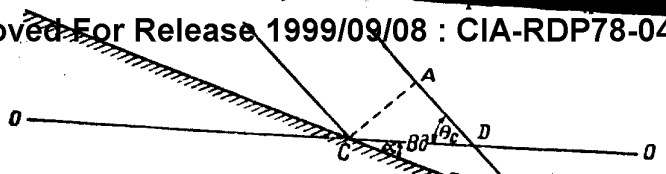
сторону ... от орудия ...



... происходит на некоторой площади.

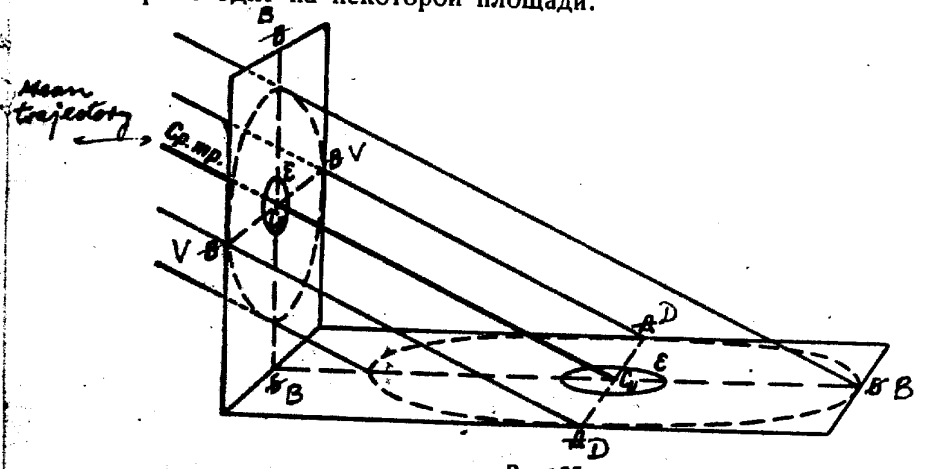


Рис. 85

1	2	7	16	25	25	16	7	2	
2			2	7	16	25	25	16	7	2
3	2/1	7/3,5	16/8	32/16	41/20,5	41/20,5	32/16	16/8	7/3,5	2/1
4	1,9	7,1	16,5	24,5	24,5	16,5	7,1	1,9		

Рис. 86

Батарейный эллипс оказался равным 10 *Вд*, т. е. 200 *м*.

Если рассчитать распределение точек ... вд, что и для

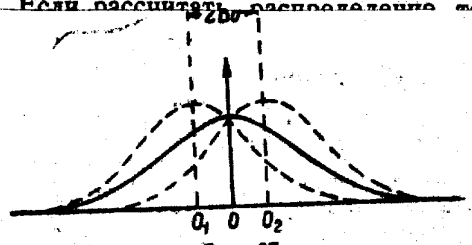


Рис. 87

ния при стрельбе батареей практичес-

2	7	16	25	25	16	7	2				
				2	7	16	25	25	16	7	2
2/1	7/3,5	16/8	25/12,5	27/13,5	23/11,5	23/11,5	27/13,5	25/12,5	16/8	7/3,5	2/1
2,5		10	19,5		18	18		19,5	10		2,5

Рис. 88

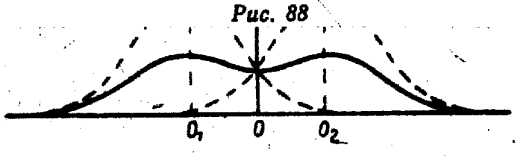


Рис. 89

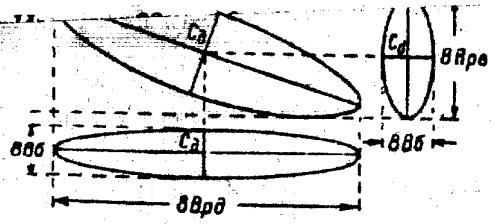


Рис. 90

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сторону ... от орудия ...

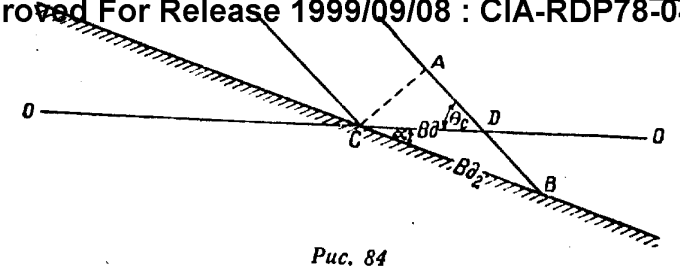

Рис. 84

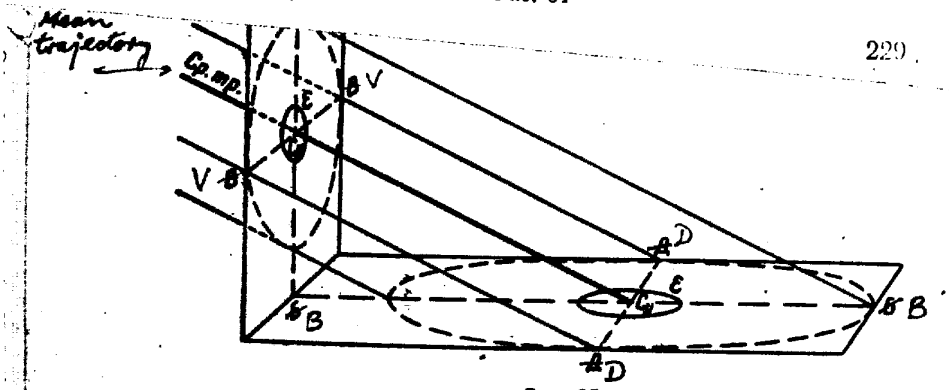

Рис. 85

1	2	7	16	25	25	16	7	2	
2			2	7	16	25	25	16	7	2
3	2/1	7/3,5	18/9	32/16	41/20,5	41/20,5	32/16	18/9	7/3,5	2/1
4	1,9	7,1	16,5	24,5	24,5	16,5	7,1	1,9		

Рис. 86

Батарейный эллипс оказался равным 10 *Вд*, т. е. 200 *м*.

Если рассчитать распределение точек ... батарейном ...

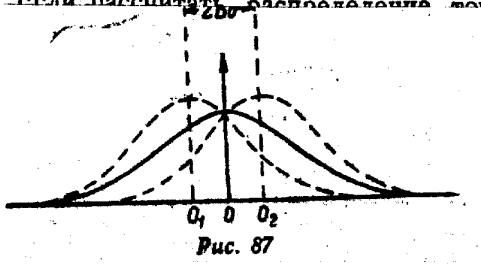

Рис. 87

...д, что и для ...

...ния при стрельбе батареей практичес-

2	7	16	25	25	16	7	2				
				2	7	16	25	25	16	7	2
2/1	7/3,5	16/8	25/12,5	27/13,5	23/11,5	23/11,5	27/13,5	25/12,5	16/8	7/3,5	2/1
2,5	10	19,5	18	18	19,5	10	2,5				

Рис. 88

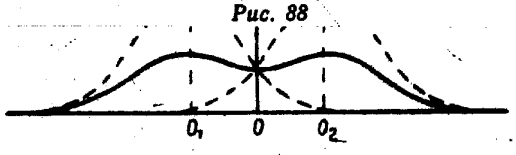

Рис. 89

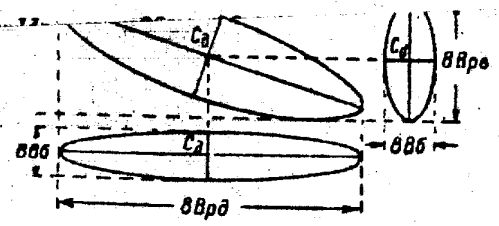

Рис. 90

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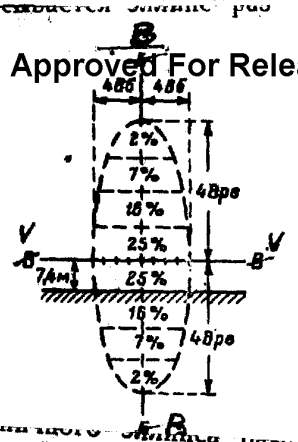

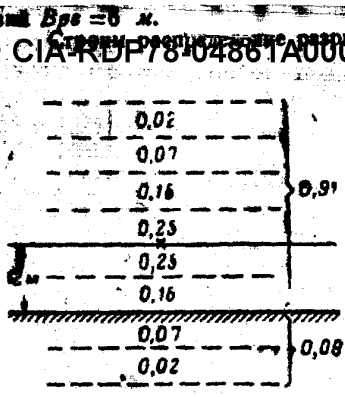

Рис. 92

средней траектории, образуя с ней некоторый угол (рис. 93).

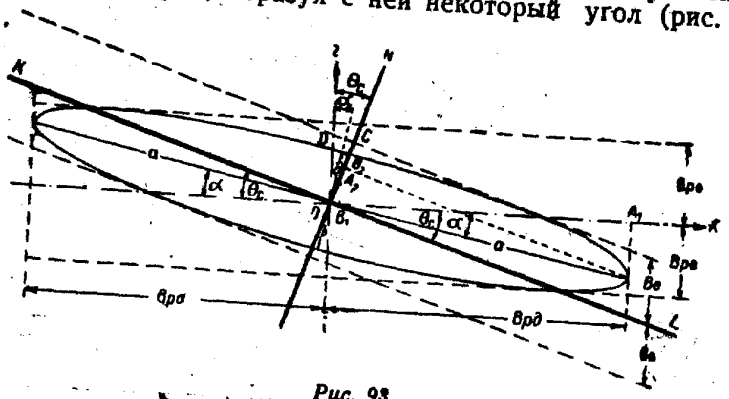

Рис. 93

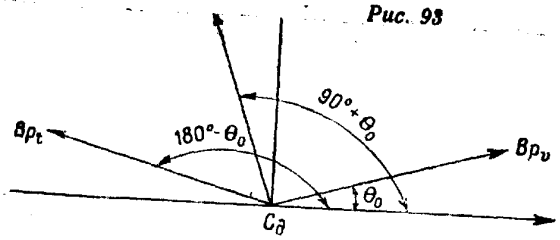

Рис. 94

положителен.

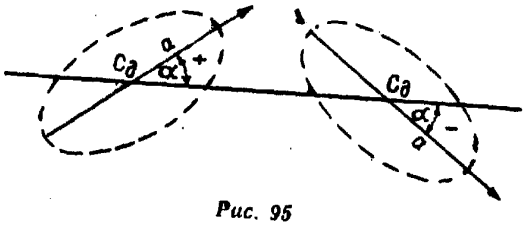

Рис. 95

Пример. Стреляет 122-*мм* гаубица обр. 1938 г. осколочно-фугасной

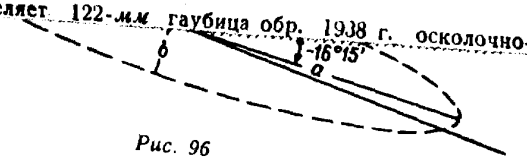

Рис. 96

Рис. 97

На рис. 97 показано, как уменьшается вероятность попада-

Рис. 98

тории относитель
равномерности расс
что вероятность п
наибольшей в том
средняя траектори
центр цели. С уд
траектории от центра цели вероятность попада

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вероятность попадания практичес

OVER

CPYRGHT Approved For Release 19[illegible]09/08 : CIA-RDP78-04861A000100010[illegible]

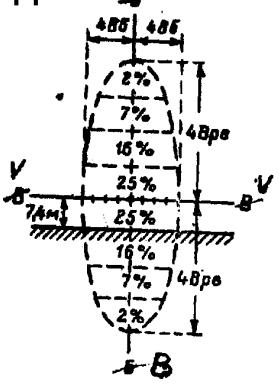

Рис. 91

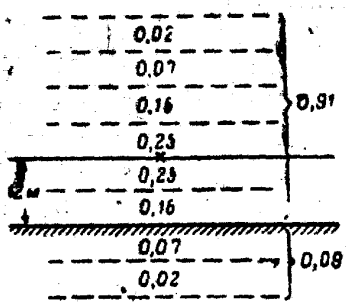

Рис. 92

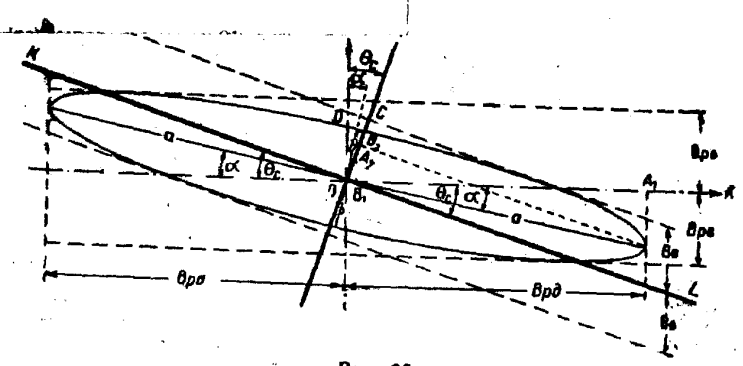

Рис. 93

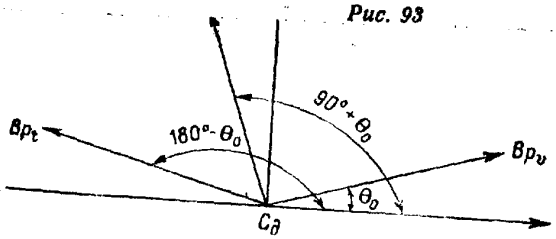

Рис. 94

Рис. 95

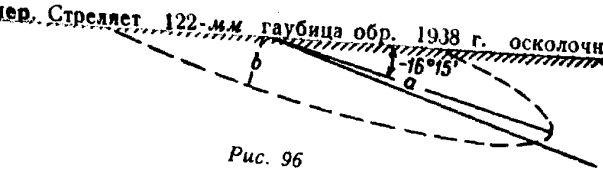

Рис. 96

Рис. 97

Рис. 98

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CPYRGHT
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нулю (рис. 99).

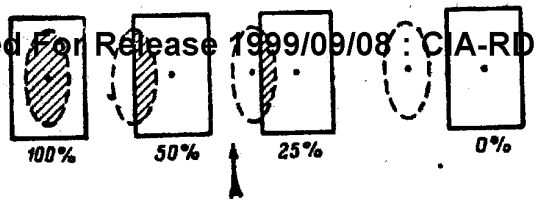


Рис. 99

4. *Направление стрельбы.* Направление стрельбы опреде-

можно сде-
ки важных

рассеивания
сти стрель-
льность, тем
е. Поэтому,
еньшее рас-
и наиболь-
попадания, огневые позиции батарей следует
жно ближе к противнику. Опыт Великой

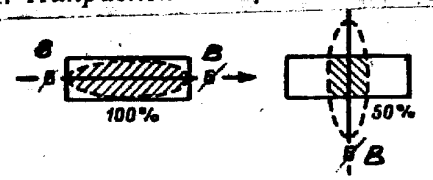


Рис. 100

лице значений Φ (β).
01)

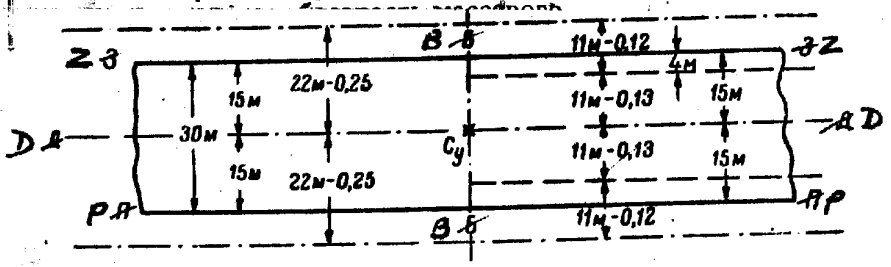


Рис. 101

Проведем две параллельные линии *ПП* и *ЗЗ*, расстояние между

Рис. 102

255

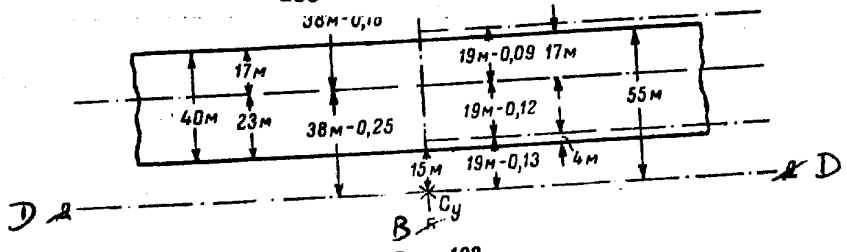


Рис. 103

1. Решение по шкале рассеивания (рис. 103)

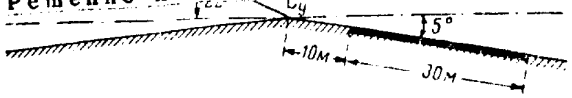
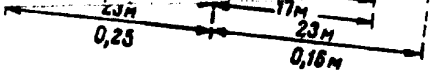


Рис. 104

Решение. Прежде всего определим величину срединного отклоне-



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ОVER

примеров 1, 2, 3 и 4 были показаны

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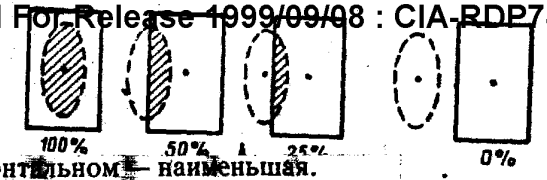
нулю (рис. 99).



ibольшая, при фронтальном — наименьшая.

то, что
кторах, оп-
ну вероят-
можно сде-
ки важных

рассеивания
сти стрель-
льность, тем
е. Поэтому,
еньшее рас-
и наиболь-

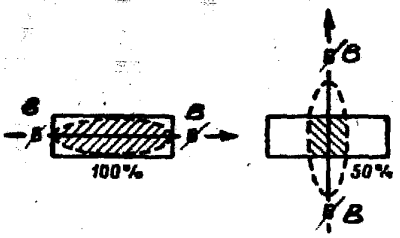


Рис. 100

ение стрельбы опреде-

попадания, огневые позиции батарей следует
жно ближе к противнику. Опыт Великой

ице значений Ф (β).
01)

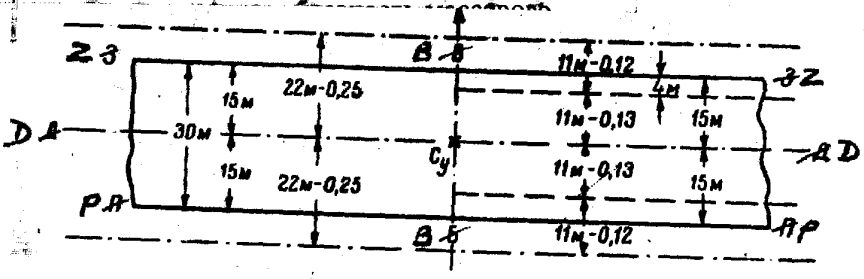


Рис. 101

нно о

Проведем две параллельные линии ПП и ЗЗ, расстояние между

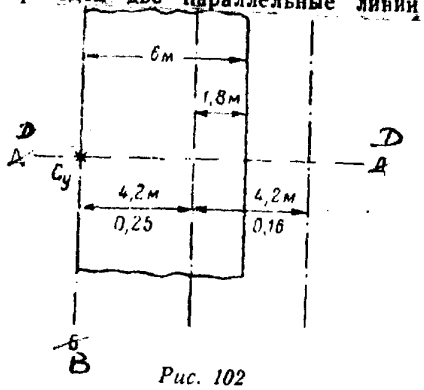


Рис. 102

Ф (β),

97 =

огонь
равна
попа-
раек-
край,

255

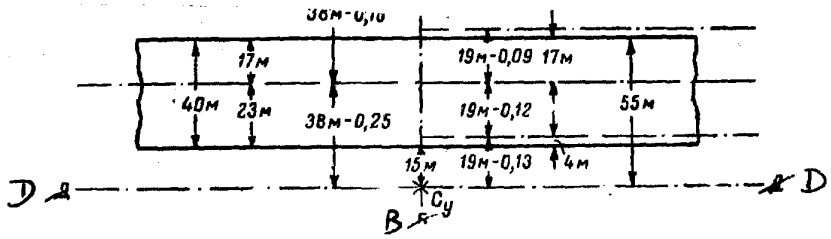


Рис. 103

1. Решение по шкале рассеивания (рис. 103)

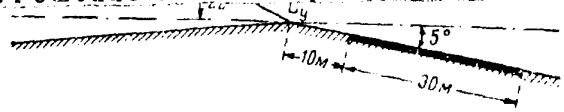


Рис. 104

Решение. Прежде всего определим величину срединного отклоне-

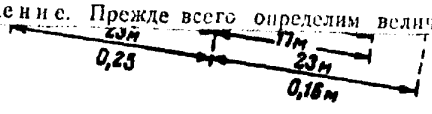


Рис. 105

Approved For Release 1999/09/08 : CIA-RD

номер 1 2 3 и 4 были показаны

OVER

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представляющей собой полосу бесконечной длины, зани
такое положение, как показано на рис. 106.

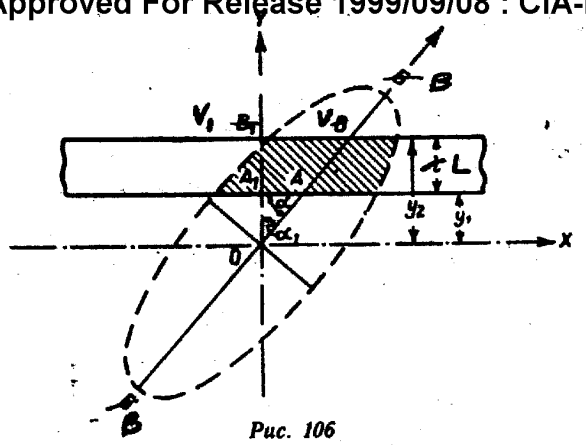


Рис. 106

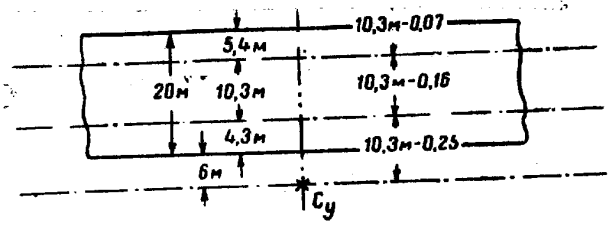


Рис. 107

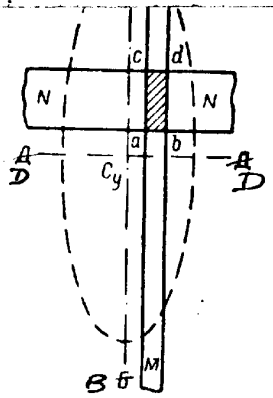


Рис. 108

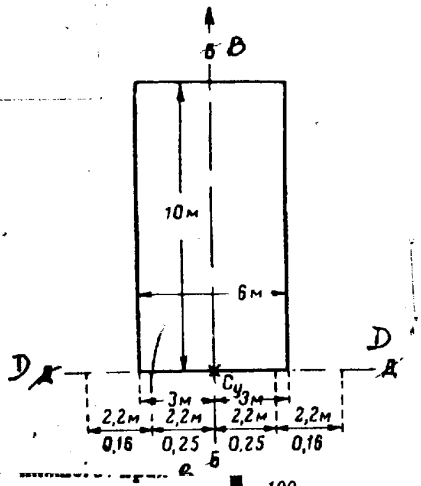


Рис. 109

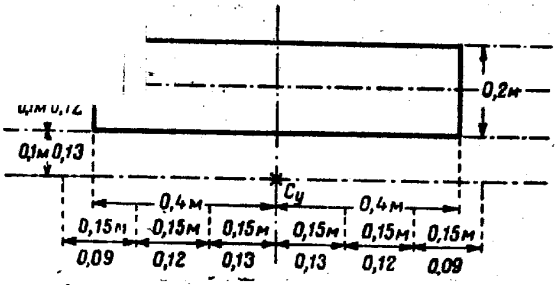
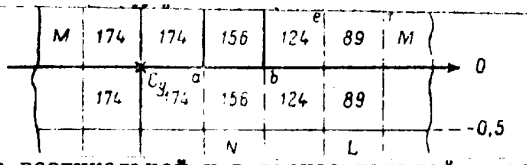
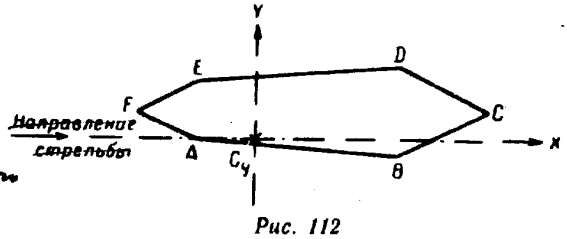


Рис. 110



щую и в вертикальной и в горизонтальной плоскости.



Direction of fire

Рис. 112

Approved For Release 1999/09/08 : CIA-RDP78-04861A0001000

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CPYRGHT

представляющей собой полосу бесконечной длины, зани
такое поло[illegible] как показано на рис. 106.

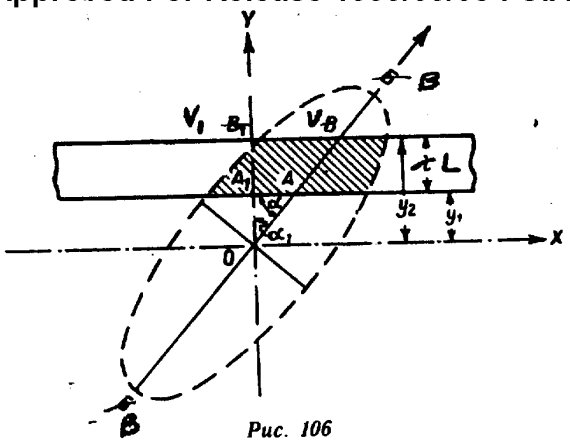


Рис. 106

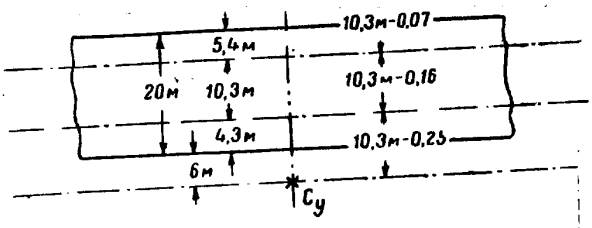


Рис. 107

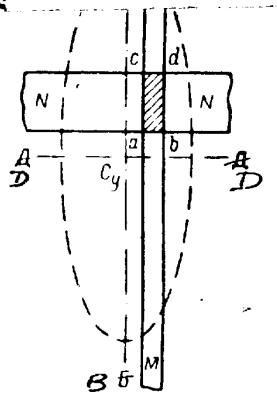


Рис. 108

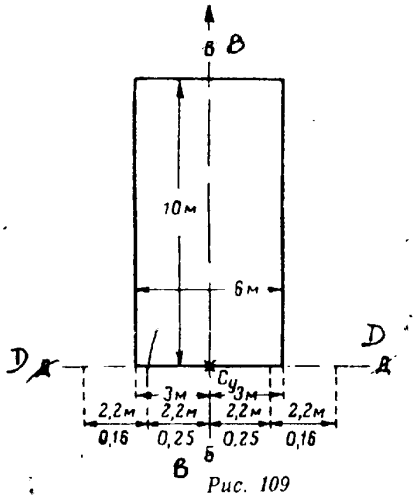


Рис. 109

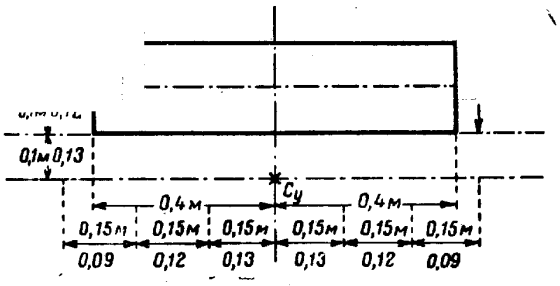


Рис. 110

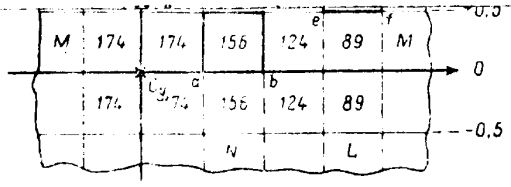
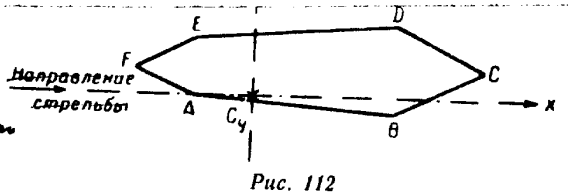


Рис. 111



Direction of fire

Рис. 112

OVER

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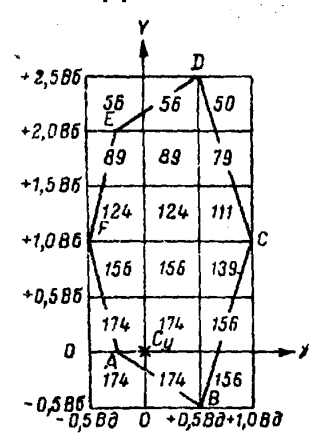

Рис. 113

265

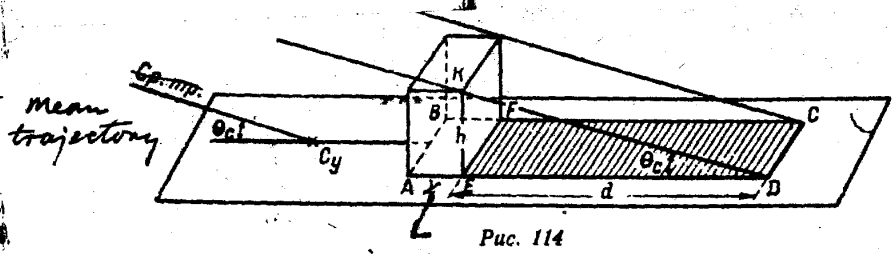

Рис. 114

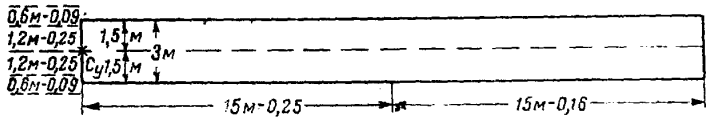

Рис. 115

Решение. Принимая фигуру танка за прямоугольный параллелепипед

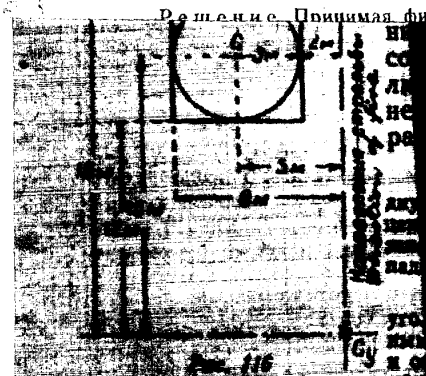

Рис. 116

Ответ. 11%.

Задача 161. Определить вероятность
при фронтальном огне (рис. 117), если сре
точку O,
этом: AM
$BM = 1{,}8$
$DE = 4$

Отв

Зада
веером ве
ной стене
Чему рав
стену, ес
средняя
стеной?

Отв

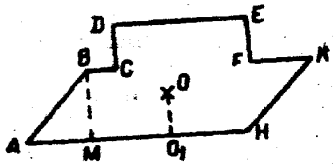

Рис. 117

Задача 163. Определить вероятность

OVER

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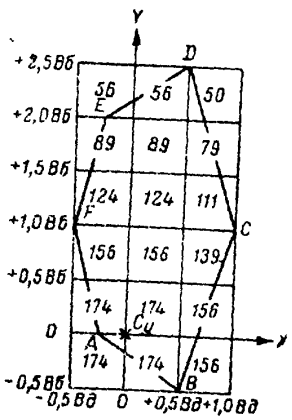
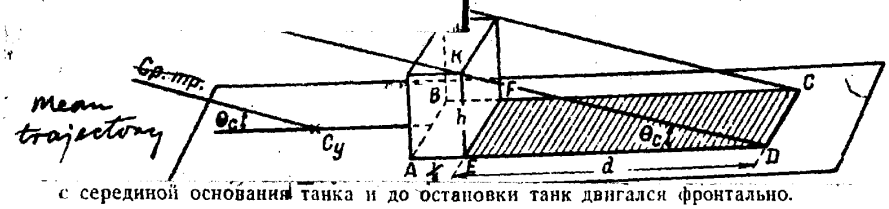


Рис. 113



с серединой основания танка и до остановки танк двигался фронтально.

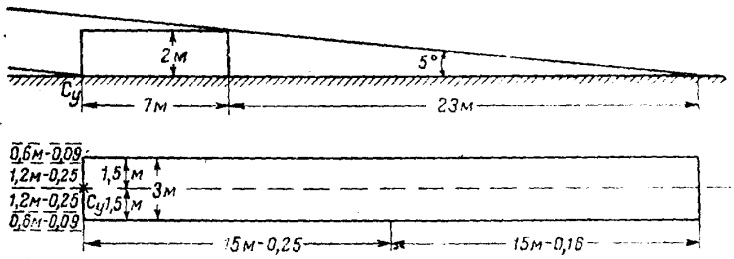


Рис. 115

Решение. Принимая фигуру танка за прямоугольный параллелепипед

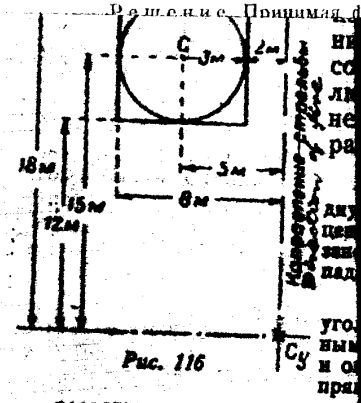


Рис. 116

вероятность попадания по да

Ответ. 11%.

Задача 161. Определить вероятность
при фронтальном огне (рис. 117), если сре
точку *O*.
этом: *AM*
BM = 1.8
DE = 4
Отв

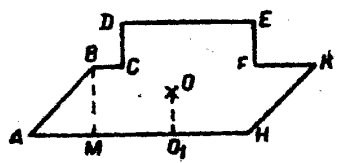


Рис. 117

Задач
веером вед
ной стене
Чему рав
стену, есл
средняя т
стеной?
Отв

Задача 163. Определить вероятность п

OVER